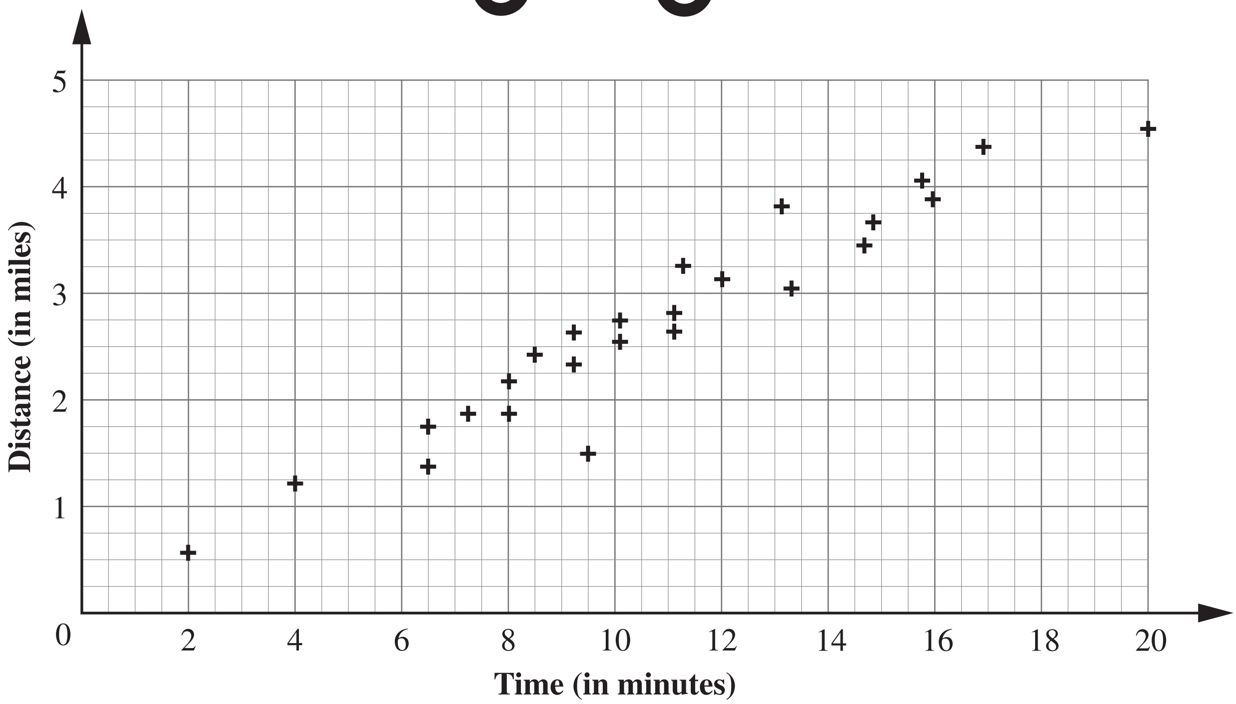
Taxi Times- MAC Core Ideas

Student Task	Use a scatterplot to find lowest average speed and average speed for a taxi drive. Draw and use line of best fit to plan fare rates for the taxi driver.
Data Analysis	**Select and use appropriate statistical methods to analyze data.** Find, use, and interpret measures of center. Understand the relationship between two sets of data (bivariate), display such data in a scatterplot, and describe trends and shape of the plot including correlations (positive, negative, and no) and lines of best fit.
Algebraic Properties and Representations	Recognize and use equivalent graphical and algebraic representations of lines with their geometric characteristics, such as slope.

Taxi Times

This problem gives you the chance to:
- interpret and use information on a scatter plot



A taxi driver used a scatter plot to record the times and distances of all his fares on a typical day.

1. On the scatter plot, draw a circle around the point that shows the lowest average speed.

What was the average speed, in miles per hour, for this trip?
Show how you figured it out. ________________ mph

2. On the scatter plot, draw a line that best fits the data recorded by the taxi driver.

Estimate the average speed of all the taxi driver's trips. ________________ mph

3. On future days, the driver wants to make $45 per hour.
How much should he charge each passenger per mile? ________________

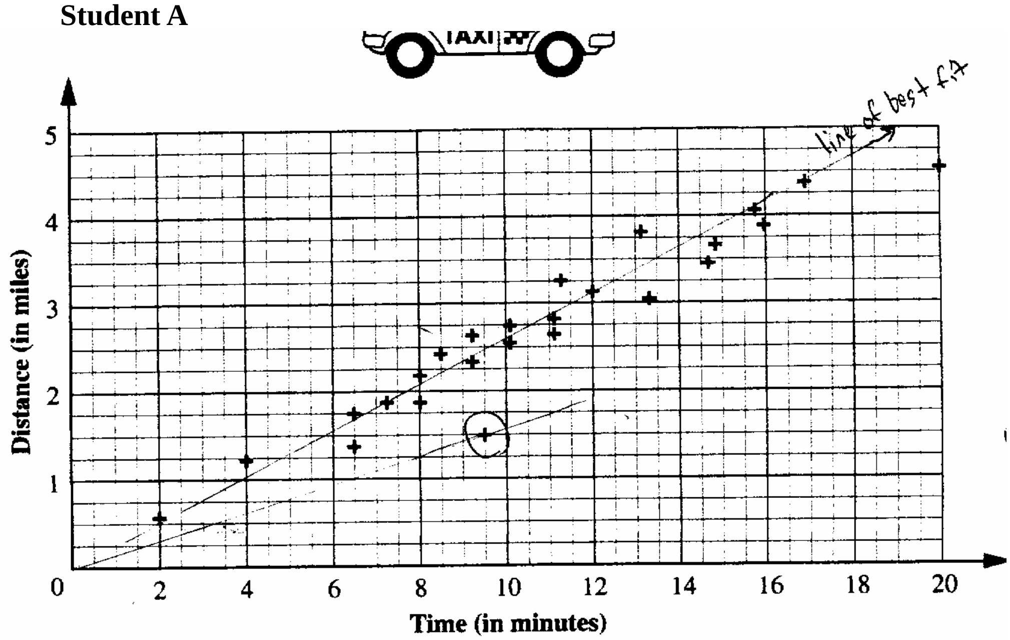
Show how you obtained your answer.

8

Looking at Student Work – Taxi Times

Most students really struggled on this problem. They did not have an understanding of how the shape of the data related to speed. Student A is one of the few exceptions. She uses slope to find the lowest average speed and the average speed. She also understands the relationship between the outlier and the line of best fit.

Student A



A taxi driver used a scatter plot to record the times and distances of all his fares on a typical day.

1. On the scatter plot, draw a circle around the point that shows the lowest average speed

What was the average speed, in miles per hour, for this trip? 9.375 m

Show how you figured it out.

Draw line from (0,0) to pt : slope is $\approx \frac{1.25}{8} \rightarrow \frac{1.25}{2/15} \rightarrow 9.375$

time ÷ 60 gives hours

distance remains same unit

Student A

2. On the scatter plot, draw a line that best fits the data recorded by the taxi driver. ✓ 1

Estimate the average speed of all the taxi driver's trips. ____15____ mph

Draw line of best fit: slope is $\approx \frac{1}{4} \rightarrow \frac{1}{1/15} \rightarrow 15$

time ÷ 60 gives hrs, distance remains same unit

3. On future days, the driver wants to make $45 per hour.
How much should he charge each passenger per mile? ____$3 /mile____ ✓

Show how you obtained your answer.

Taxi Driver can drive 15 miles in 1 hour

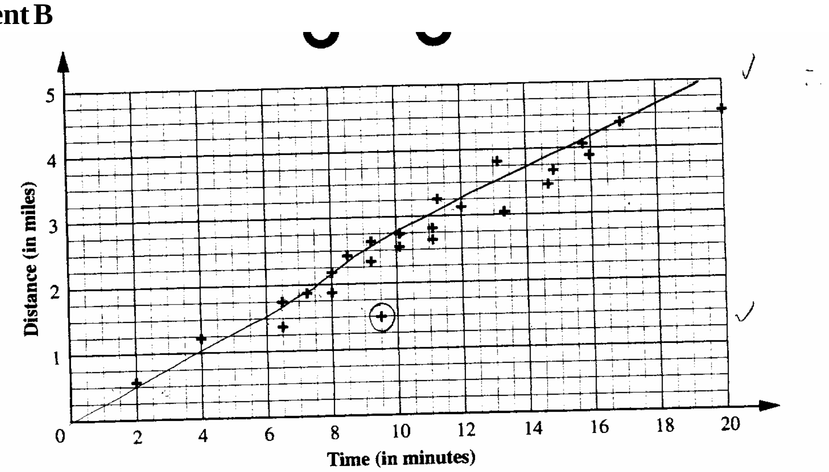
$x = \$/\text{mile}$

$45 = 15x$

$\$3 = x$

Student B manages to meet all the demands of the task by using the distance formula. In part 1 of the task, this is done efficiently by recognizing that the outlier is the lowest speed. Because Student B does not understand the purpose of the line of best fit, the student finds the average of all the points to find average speed in part 2.

Student B



A taxi driver used a scatter plot to record the times and distances of all his fares on a typical day.

1. On the scatter plot, draw a circle around the point that shows the lowest average speed.

What was the average speed, in miles per hour, for this trip? 9.5 mph

Show how you figured it out.

Page 2

Taxi Times Test 10: Form A

Some students did not recognize the relationship between average speedand the money the driver could make in an hour. Student C is just using money and hours to find the fare rate.

Student C

2. On the scatter plot, draw a line that best fits the data recorded by the taxi driver.

Estimate the average speed of all the taxi driver's trips. 15 mph

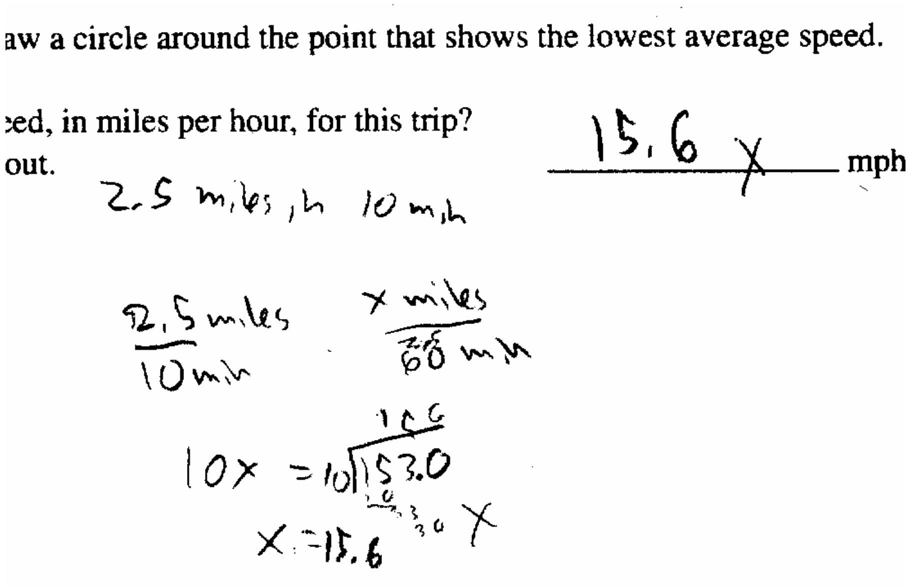
3. On future days, the driver wants to make $45 per hour.
How much should he charge each passenger per mile?

Show how you obtained your answer.

$\frac{45}{60}$

While many students could recognize and circle the point representing the lowest average speed, they did not use that point for calculations. Generally they would pick a point in the middle of the line of best fit (10, 2.5) and use that in the distance formula. See the work of Student D below.

Student D


aw a circle around the point that shows the lowest average speed.

ed, in miles per hour, for this trip?
out.

15.6 X mph

2.5 miles, h 10 m,h

2.5 miles / 10 m,h   x miles / 60 m,h

10x = 10)153.0

x = 15.6

Some students showed an understanding of the problem, but did not convert from minutes to hours. Student E picks a point on the line of best fit for problem 2, but forgets to multiply by 60. This leads to confusion in part 2. While the student puts an incorrect fare in part three, reading the further work the student seems to have a partial understanding of the rates in the problem and how those should be used in solving for fare. The student is not sufficiently comfortable with the ideas to pick out the correct thinking from experimentation.

Student E

2. On the scatter plot, draw a line that best fits the data recorded by the taxi driver. 1

Estimate the average speed of all the taxi driver's trips. ___13___ mph X

3. On future days, the driver wants to make $45 per hour.
How much should he charge each passenger per mile? ___$3.50___ X

Show how you obtained your answer.

0.5, 1.25, 1.3, 1.75, 1.9, 1.9, 2.2, 2.5, 2.75, 2.5, 3, 3.2, 3.25
3.75, 3.5, 3.6, 3.75, 4.0, 4.5, 4.5 (add them all together
= 55.6 / 25 = 2.224
↑ # of taxi trips

$\frac{2.224}{x} = \frac{10}{60\text{min.}}$

$\frac{10x}{10} = \frac{133.44}{10}$ 1 f.t

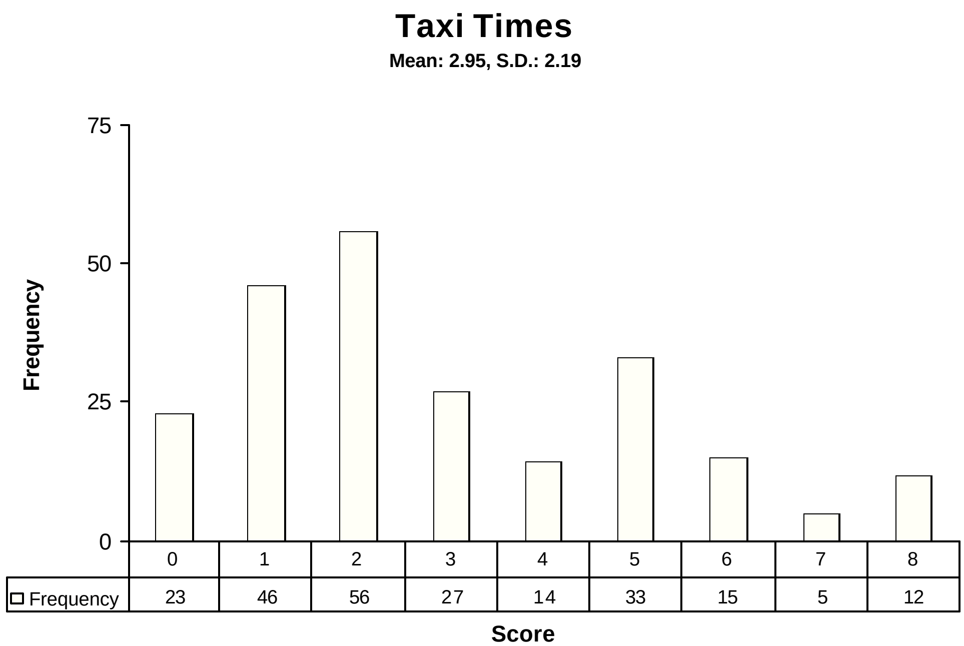
x = 13.3 or 13 mph.

$\frac{1\text{ mile}}{13\text{ mile}} = \frac{x}{\$45}$

$\frac{45}{13} = \frac{13x}{13}$

x = 3.46 or 3.5

Frequency Distribution for each Task – Grade 10
Grade 10 – Taxi Times



Score:	0	1	2	3	4	5	6	7	8
% < =	10.0%	29.9%	54.1%	65.8%	71.9%	86.1%	92.6%	94.8%	100.0%
% > =	100.0%	90.0%	70.1%	45.9%	34.2%	28.1%	13.9%	7.4%	5.2%

The maximum score available on this task is 8 points.
The cut score for a level 3 response is 4 points.

Most students (about 90%) could draw a line of best fit. Almost half the students could also identify the point on the scatterplot with the lowest average speed and do some correct calculation for using the rate/distance formula. Less than 20% of the students find both of the average speeds and relate speed to fare. 10% of the students scored no points on this task.

Taxi Times

Points	Understandings	Misconceptions
0	Most students with this score tried the problem. About half the students attempted all the parts of the problem. The others usually only attempted the line of best fit or circling the lowest average speed.	Almost 10% of the students do not know that line of best fit should be straight. They are still trying to connect all the points.
1	Most students with this score correctly drew the line of best fit.	A little less than half the students chose (2, 0.5) as the lowest average speed; probably thinking about its vertical position on the graph, rather than thinking about distance formula or the relationship between the variables.
2	Students with this score could circle the lowest average speed and draw the line of best fit.	While most of these students identified the lowest ave. speed, they picked a different point on the graph substitute into the distance formula. About 25% of the students showed no work for calculating the lowest average speed. About 15% tried to calculate an average using several points on the graph. Some used all, some just tried a high and low speed
3/4	Students with this score could usually get some credit for calculating fares in part three. With about equal numbers showing 45 divided by the correct speed in two, divided by an incorrect speed in two, or showing no work.	Many students did not differentiate between lowest average speed and average speed. Students often used their answer to part 1 for part 2. More than half the students did not show calculations for their answers. Therefore it is difficult to identify misconceptions from guessing.
5	Students could use the distance formula to find the average speed and find the fare for the driver.	Students with this score were not successful with part one of the task. They couldn't identify the lowest average speed on the graph or do the calculations.
6/7	Students with this score could not calculate the lowest average speed.	
8	Less than 5% of the students could complete all the parts of the task. Most of them used distance formula to find speeds. A few were able to use slope.	

Based on teacher observations, this is what Course Two students seemed to know and be able to do:

- Draw a line of best fit
- Find the outlier or lowest average speed on a scatterplot

Areas of difficult for Course Two students:

- Understanding the relationship between slope and speed on a time/ distance graph
- Applying distance formula to numbers in a graph
- Converting from miles per minute to miles per hour
- Understanding the difference in meaning of points lying on the line of best fit and those not on the line
- Understanding average in a graphing situation

Questions for Reflection on Taxi Times:

- How many of your students could correctly identify the lowest average speed on the scatterplot?
- Did they use that number when calculating lowest average speed? Or did they use one of the following points:

(2, 1/2)	(9.5,1.5)	(20,4.5)	(10,2.5)	(13, 3.??)	Other

- What strategies did students use to find speeds?

Slope	D=RT	Average of several points	No work	Other?

- What experiences have your students had with scatterplots this year?
- What experiences have your students had with understanding slopes?
- What types of activities or questions might you add to activities you already do to help students gain a deeper understanding of the types of relationships being tested in the task?
- What are the core ideas you want students to know and to understand about scatterplots?

Teacher Notes

Instructional Implications:

1. Distance/Time graphs are excellent examples of problems to focus on the concept of slope in a real context. The average speed is distance divide by time or the change

in y's divided by the change in x's, which of course is the slope of the line. If you start with a horizontal rays from the origin it would have zero slope and zero speed. As you rotate the ray counterclockwise the slope increase and thus the average speed increases, so the first point you hit in the graph would have the lowest average speed. As you continue rotating the average speed increase until it is vertical or undefined. This is the powerful idea behind slope that has many applications.

2. Students had trouble identifying the scale of the graph. On the vertical scale the interval distance was 1/4 in length and the horizontal scale interval was 1/2 in length. This contributed to some misreadings of the coordinates and ultimately incorrect calculations. Students should have experience with graph that have different vertical and horizontal scales.

3. Students had trouble converting from miles per minutes to miles per hour. Some work on dimensional analysis might be helpful.
i.e. mile/min x min/hour = mile/hour

4. Students had trouble with the concept of a line that best fits the data. Students need experience with scatterplots and using best fit type lines to determine trends and estimate functional relationships between the axes.

5. The last question requires students to attack the problem from an inverse perspective in relationship to the first sets of questions. Students need experience with determining and developing an inverse approach (undoing or working backwards) when confronted with a problem in context.

6. Baker's Choice, by Key Curriculum Press, is a good replacement unit for slope, inequalities, optimizing outcomes. This could be an addition to the Algebra One Program.

Number Patterns

Student Task	Explore patterns on a hundreds chart. Use algebra to prove why patterns hold true for all cases.
Mathematical Reasoning and Proof	Show mathematical reasoning in solutions in a variety of ways, including words, numbers, symbols, pictures, charts, and models.
Algebraic Properties and Representations	Use symbolic algebra to represent and explain mathematical relationships. Use symbolic expressions to represent relationships arising from various contexts.

Number Patterns

This problem gives you the chance to:
- use words or algebra to express number patterns

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Tasha draws a rectangle around three numbers. **12, 13, 14**
She multiplies the first number by the third number. $\mathbf{12 \times 14 = 168}$
She squares the middle number. $\mathbf{13^2 = 13 \times 13 = 169}$
She subtracts the first answer from the second answer. $\mathbf{169 - 168 = 1}$

1. Draw a rectangle around any three numbers in a row.

Multiply the first number by the third number. ____________________

Square the middle number. ____________________

Subtract the first answer from the second answer. ____________________

What pattern do you notice? ____

Does this pattern always occur when you draw a rectangle around three numbers in a row?

____

Use algebra to explain why.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Martin draws a rectangle around five numbers. **36, 37, 38, 39, 40**
He multiplies the first number by the fifth number. **$36 \times 40 = 1440$**
He squares the middle number. **$38^2 = 38 \times 38 = 1444$**
He subtracts the first answer from the second answer. **$1444 - 1440 = 4$**

2. Draw a rectangle around any five numbers in a row.

Multiply the first number by the fifth number. ____________________

Square the middle number. ____________________

Subtract the first answer from the second answer. ____________________

What pattern do you notice? ____

Does this pattern always occur when you draw a rectangle around five numbers in a row?

____

Use algebra to explain why.

8

Looking at Student Work – Number Patterns

Student A takes the less usual choice of making the middle number in the sequence x. This makes completing the proof simpler. In part two, student A not only proves the case in the pattern, but only is able to generalize the pattern to any odd sequence and prove what the answer is and why it would work going way beyond the demands of the task.

Student A

Multiply the first number by the third number. $7 \cdot 9 = 63$ ✓

Square the middle number. $8^2 = 64$ ✓

Subtract the first answer from the second answer. $64 - 63 = 1$ ✓

What pattern do you notice? The middle #'s square will be one more that multiplying its adjacent #s. ✓

Does this pattern always occur when you draw a rectangle around three numbers in a row?

$(x-1)(x+1) = x^2 - 1$ x = middle #

multiply $x^2 - 1 = x^2 - 1$

yes ✓

Use algebra to explain why. When both parts of the puzzle are =, the rule will work for any # b/c no # can be substituted for x that won't equal the other side (2)

2. Draw a rectangle around any five numbers in a row.

Multiply the first number by the fifth number. $5 \cdot 9 = 45$ ✓

Square the middle number. $7^2 = 49$ ✓

Subtract the first answer from the second answer. $49 - 45 = 4$ ✓

What pattern do you notice? The square of the middle # is 4 more than multiplying th # −2 by that # +2.

Does this pattern always occur when you draw a rectangle around five numbers in a row?

$(x-2)(x+2) = x^2 - 4$

multiply $x^2 - 4 = x^2 - 4$

yes ✓

Use algebra to explain why. equal ∴ all #'s work

(2)

$(x-3)(x+3) = x^2 - 9$

$x^2 - 9 = x^2 - 9$

$(x-y)(x+y) = x^2 - (y^2)$ → This is the equation f making the # pattern problems.

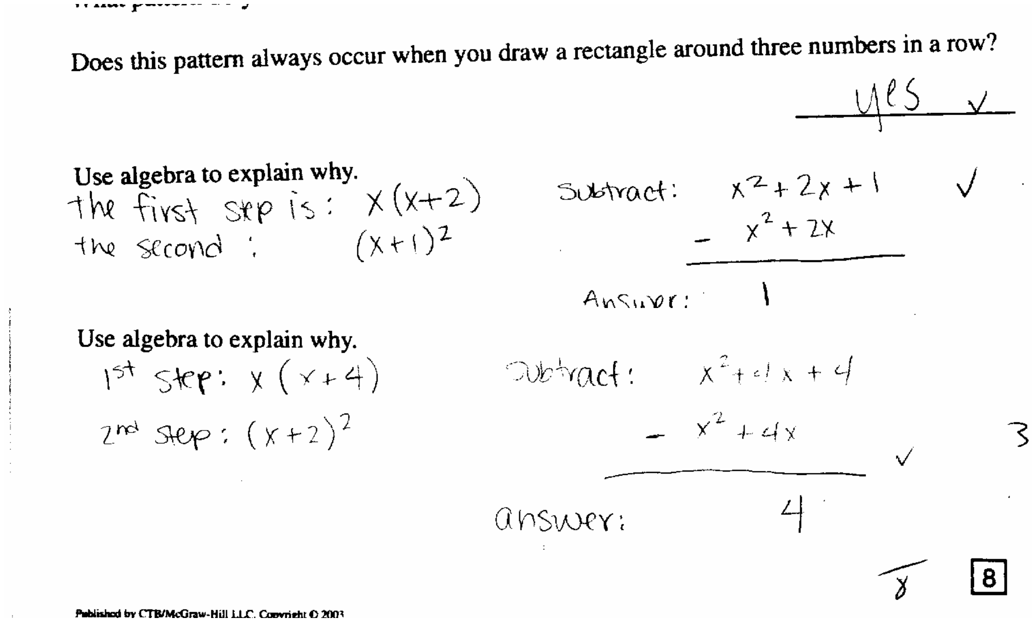
x = middle #

y = # of integers (or distance on line) one intends to dra a rectangle around.

6 **8**

Student B chooses the first number in the sequence for the variable and completes the proof.

Student B



Does this pattern always occur when you draw a rectangle around three numbers in a row?

yes ✓

Use algebra to explain why.

the first step is: $x(x+2)$

the second : $(x+1)^2$

Subtract: x^2+2x+1 ✓

$-\ x^2+2x$

Answer: 1

Use algebra to explain why.

1st step: $x(x+4)$

2nd step: $(x+2)^2$

Subtract: x^2+4x+4

$-\ x^2+4x$ ✓

answer: 4

3

8

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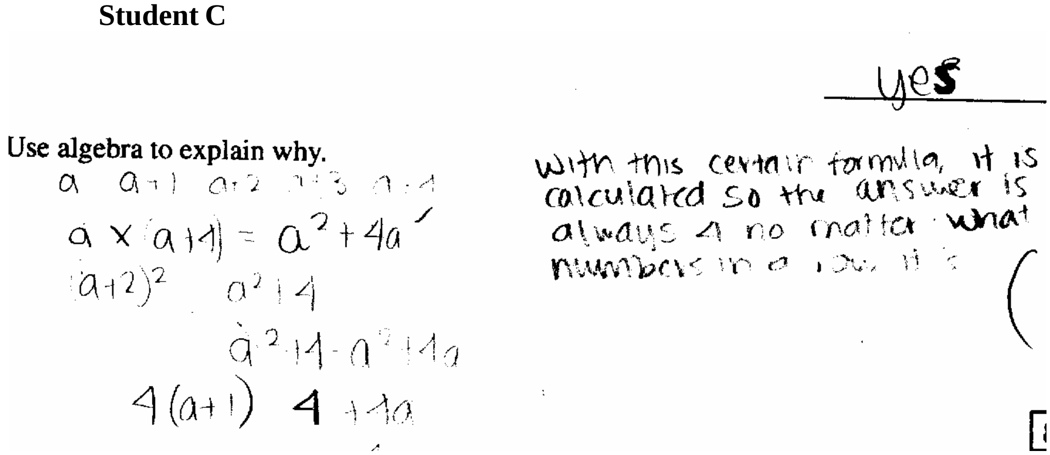
Students, who chose the first number in the sequence as the variable, had a difficult time either expressing the square of the middle number or multiplying it out correctly. Student C had the right equation for the middle number but could not complete the multiplication correctly.

Student C



yes

Use algebra to explain why.

a a+1 a+2 a+3 a+4

$a \times (a+4) = a^2+4a$

$(a+2)^2 \quad a^2+4$

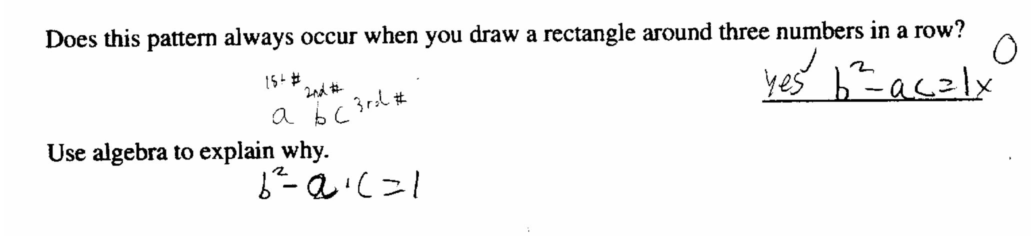
a^2+4-a^2+4a

$4(a+1) \quad 4+4a$

With this certain formula, it is calculated so the answer is always 4 no matter what numbers in a row it is

More than 10% of the students tried to use 3 or more variables. They did not see the relationship between the numbers in the sequence. Therefore their proofs were really just a statement of their pattern, but not based on the structure of the numbers in the sequence. See the work of Student D.

Student D

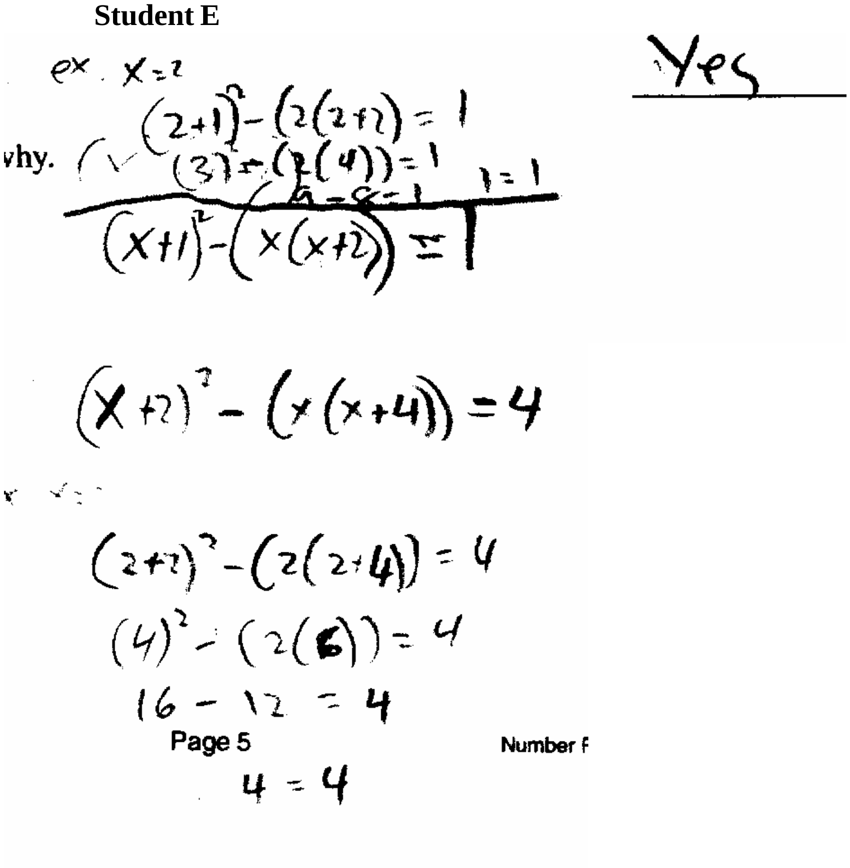

Does this pattern always occur when you draw a rectangle around three numbers in a row? yes $b^2 - ac = 1x$

1st # 2nd # 3rd #
a b c

Use algebra to explain why.
$b^2 - a \cdot c = 1$

Some students, like Student E, use algebra to set up the proper equations. They then use substitution of specific values of x to solve for a specific case, rather than simplifying the expressions to show it is true for the general case. They are not comfortable or familiar with the idea of using algebra for a proof or logic tool. Note that in part two, student E forgets where the sequence started and uses an incorrect value for x.

Student E


ex. $x = 2$

$(2+1)^2 - (2(2+2)) = 1$

$(3)^2 - (2(4)) = 1$

$1 = 1$

$(x+1)^2 - (x(x+2)) = 1$

Yes

$(x+2)^2 - (x(x+4)) = 4$

$(2+2)^2 - (2(2+4)) = 4$

$(4)^2 - (2(6)) = 4$

$16 - 12 = 4$

Page 5 Number F

$4 = 4$

Some students could only do part of the computations for the proof. They either did the final step of subtracting X squared in their heads or did not fully understand the logic needed to complete a proof. See the work of Student F.

Student F

Use algebra to explain why.

$x-1,\ x,\ x+1$

$(x-1)(x+1)=x^2-1$

About half the students made no attempt at all to use algebra. Some students, like Student G, used verbal logic or arguments. Some students just used numerical examples. Others did not attempt this part.

Student G

What pattern do you notice? WHEN YOU SUBTRACT THE FIRST ANSWER FROM THE SECOND, THE ANSWER WILL ALWAYS TURN OUT TO BE 1. ✓

Does this pattern always occur when you draw a rectangle around three numbers in a row? YES ✓

$83 \times 85 = 7055$

$7056 - 7055 = 1$

BECAUSE IT'S ALWAYS IN A ROW TOGETHER. ALSO, THE NUMBER THAT YOU SQUARE WILL ALWAYS BE HIGHER THAN THE 1ST AND SECOND NUMBER YOU MULTIPLY.

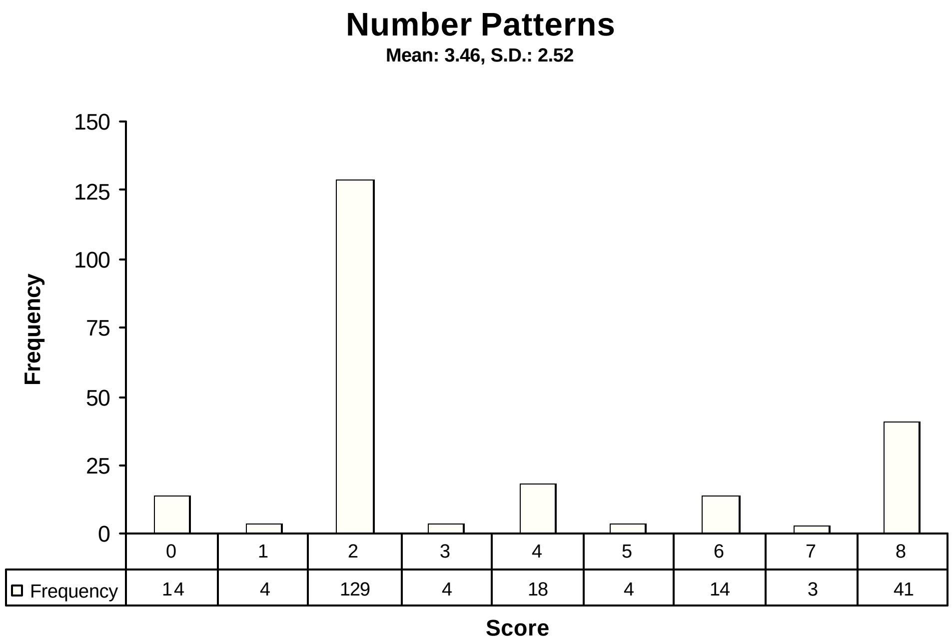
Use algebra to explain why.

X

Page 4 Number Patterns Test 10: Form A

Teacher Notes:____

Grade 10 – Number Patterns



Score:	0	1	2	3	4	5	6	7	8
% < =	6.1%	7.8%	63.6%	65.4%	73.2%	74.9%	81.0%	82.3%	100.0%
% > =	100.0%	93.9%	92.2%	36.4%	34.6%	26.8%	25.1%	19.0%	17.7%

The maximum score available on this task is 8 points.
The cut score for a level 3 responses is 4.

Most students (about 90%) were able to do the arithmetic calculations and describe the number pattern. About 50% of the students made some attempt to use algebra, with out 40 % being able to do at lease one of the equations needed to show why the pattern would work for all cases. About 18% of the students could meet all the demands of the task. About 6% of the students scored 0 on this task.

Number Patterns

Points	Understandings	Misconceptions
0	Most students attempted this problem.	
2	Students with this score could do the arithmetic calculations and describe the number pattern.	About half the students doing this task did not use algebra. Some made verbal arguments about the size of the numbers or used numerical examples. Others did not attempt the algebra at all. More than 10% of the students tried to use 3 or more variables. They did not use the relationships of the numbers in the sequence.
4	Students could find the pattern and express one of the equations in parts one and two.	Students found it more difficult to find an expression equal to the middle number squared, particularly if they defined the variable as the first number squared. They may have had trouble with the multiplication or did not see the need for using the middle number squared to complete the logic of the problem.
6	Students with this score could generally write correct equations for both parts of the number pattern.	Students did not see how to use algebra as a logic tool for proving why something is true. They were not able to combine the two equations to complete the proof.
8	Students could find a number pattern, test it with specific examples, and use algebra to prove why it would always work.	

Based on teacher observations, this is what Course Two students seemed to know and be able to do:

- Follow directions to complete a set of operations.
- Test those operations on different numeric examples.
- Describe the pattern formed by the calculations.

Areas of difficulty for Course Two students:

- Using algebraic to express relationships of numbers in a sequence.
- Using equations to prove a generalization or rule.
- Squaring binomials.

Questions for Reflection on Number Patterns:

Look carefully at student work to see how they tried to prove the patterns. Did they use:

No attempt at algebra (e.g. used words, numerical examples, etc.)	Use more than one variable	Couldn't or didn't express middle number squared	Could set up both equations	No Attempt at this part of the task

- What kinds of difficulties did they have in completing the proof?
- Could students multiply out the middle number squared?
- How often do your students write proofs as part of their classwork? Are their proofs limited to geometry topics?
- Do you think the lack of algebra skills is related to students taking geometry in junior or senior years, students who have had a two year algebra program, or other structural issues? Or is the problem more an issue of learning algebra in a procedural way, that does not give students enough experience at setting up problems and using algebra as a tool to solve problems and prove conjectures.

Implications for Instruction:

Students at this grade level should be comfortable with the idea of a mathematical proof or justification. They should have many opportunities to explain their thinking, make hypotheses, and test them using both algebra and geometric theorems. Students should have enough problems to solve to enable them to see why proof by example is not enough to complete an argument. There are many interesting problems that seem to have one pattern for the first 5 or 6 cases, but if explored further prove to have a different pattern. Mark Driscoll's Fostering Algebraic Thinking or NCTM's Navigations in Algebra might be good reference sources. As part of their algebra curriculum, they should be given problems which require them to determine the number of variables needed to solve the problem and why using fewer variables is often more efficient or necessary. The use of three variables by many students shows a profound lack of understanding around the concept of variable and how it is used to define relationships.

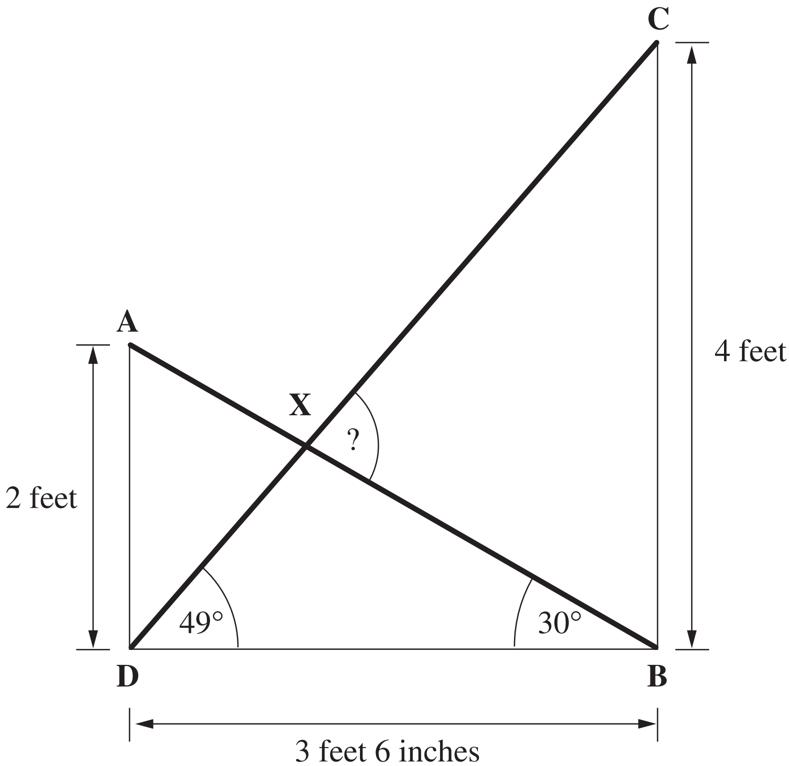
Teacher Notes:

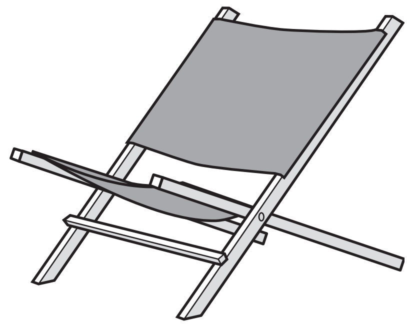
Garden Chair

Student Task	Students analyze a diagram to determine angle sizes, use Pythagorean Theorem to find the hypotenuse of a right triangle, make a mathematical argument for why two triangles are similar, and use proportional reasoning to find the length of one side of a triangle.
Geometry and Measurement	Analyze characteristics and properties of two-dimensional geometric shapes; develop mathematical arguments about geometric relationships, and apply appropriate techniques, tools, and formulas to determine measurements.
Mathematical Reasoning and Proofs	Use synthetic, coordinate, and /or transformational geometry in direct or indirect proof of geometric relationships. Establish the validity of geometric conjectures using deduction; prove theorems, and critique arguments made by others.

Garden Chair

This problem gives you the chance to:
- use a drawing to calculate lengths and angles





Dan is designing a garden chair. The diagram above shows a side view of the chair when it is set up for use. $\overline{AB}$ and $\overline{CD}$ represent two lengths of wood hinged together at X. $\overline{BD}$ is horizontal. A is vertically above D, and C is vertically above B.

1. Calculate the angle between the two lengths of wood, $\angle BXC$. ________________

2. Use the Pythagorean theorem to calculate the length of $\overline{CD}$. ____________
Show your work.

3. Show that triangles AXD and BXC are similar.

____

____

____

____

4. Use the fact that triangles AXD and BXC are similar to calculate $\overline{CX}$, the distance from the top of the seat to the hinge.
Show how you figured it out. ____________

8

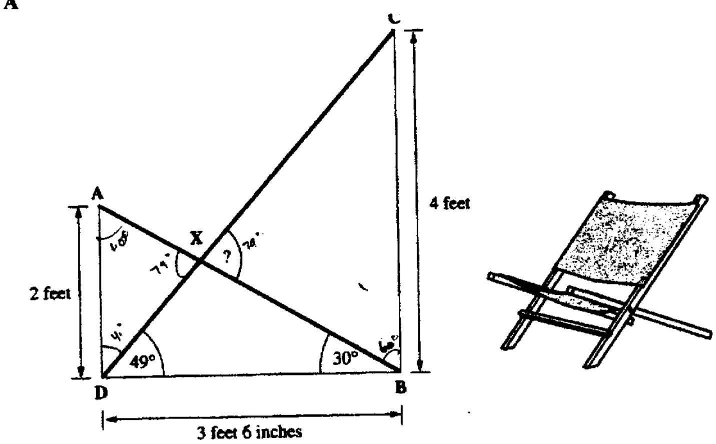
Looking at Student Work – Garden Chair

Student A gives very detailed explanation of calculations and their reasons in part 1 and shows a complete and correct solutions to all parts of the task. Notice the attention to detail in using parallel lines for proving equal angles. Student A also uses good proportional reasoning to find length of CX.

Student A

Student A gives very detailed explanation of calculations and their reasons in part 1 and shows a complete and correct solutions to all parts of the task. Notice the attention to detail in using parallel lines for proving equal angles. Student A also uses good proportional reasoning to find length of CX.

Student A



Dan is designing a garden chair. The diagram above shows a side view of the chair when it is set up for use. $\overline{AB}$ and $\overline{CD}$ represent two lengths of wood hinged together at X. $\overline{BD}$ is horizontal. A is vertically above D, and C is vertically above B.

1. Calculate the angle between the two lengths of wood, ∠BXC. 79° ✓ 2

m∠BXC = m∠DXA (vertical angle theorem)

m∠BXC = 180° − m∠DAX − m∠ADX (sum of ∠'s in △ is 180°)

= 180° − (90° − m∠ABD) − m∠ADX (DAX complimentary to ∠ABD, acute ∠'s of a rt△)

= 180° − (90° − m∠ABD) − (90° − m∠XDB) (∠ADX complimentary to ∠XDB)

m∠BXC = 79°

2. Use the Pythagorean theorem to calculate the length of $\overline{CD}$. 5.3151 feet ✓
Show your work.

$CD^2 = DB^2 + BC^2$ $\quad CD = \sqrt{28.25} = \sim 5.3151$ feet

$= (3.5)^2 + 4^2$

$= 12.25 + 16$

$= 28.25$

3. Show that triangles AXD and BXC are similar.

$\overline{AD} \perp \overline{DB}$ and $\overline{CB} \perp \overline{DB}$ as given. If 2 lines are $\perp$ to the same line they are $\parallel$, $\therefore$ AD $\parallel$ BC. $\angle DAB \cong \angle CBA$ A/A of $\parallel$ lines are $\cong$. $\angle AXD \cong \angle BXC$, VA theorem. $\triangle AXD \sim \triangle BXC$, AA

4. Use the fact that triangles AXD and BXC are similar to calculate $\overline{CX}$, the distance from the top of the seat to the hinge.
Show how you figured it out. 3.5434 feet ✓

$\frac{CX}{(CD-CX)} = \frac{BC}{AD} = \frac{4}{2} = 2$ $\quad CX = 2(CD) - 2(CX)$

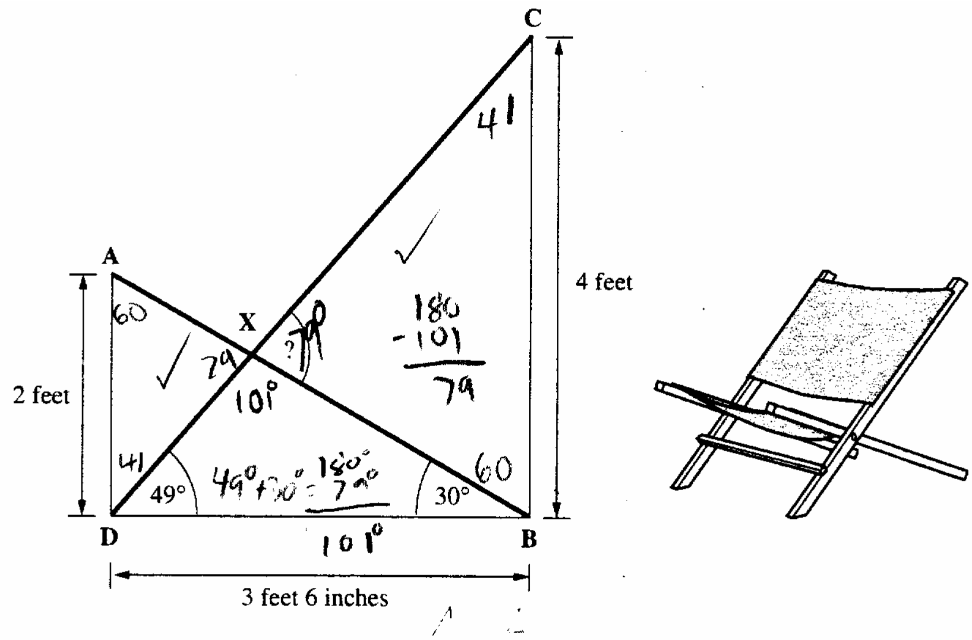
$3(BX) = 2(CD)$

$3(CX) = 2(CD$

$CX = \frac{2}{3}(CD) = \frac{2}{3}(5.3151) = \sim 3.5434$

Student B also shows all calculations needed to solve the problems posed by the task.

Student B



Dan is designing a garden chair. The diagram above shows a side view of the chair when it is set up for use. $\overline{AB}$ and $\overline{CD}$ represent two lengths of wood hinged together at X. $\overline{BD}$ is horizontal. A is vertically above D, and C is vertically above B.

1. Calculate the angle between the two lengths of wood, ∠BXC. = 79° ✓

3. Show that triangles AXD and BXC are similar.

They both have same vertical angle, And all of the interior angles are also the same, therefore the triangles are similar.

4. Use the fact that triangles AXD and BXC are similar to calculate $\overline{CX}$, the distance from the top of the seat to the hinge.
Show how you figured it out.

≈ 3.544 feet
≈ 3½ f

2AD = CB so 2 DX = CX

2(2) = 4
4 = 4

CD = √28.25 ≈ 5.315

x + 2x = 5.315
3x/3 = 5.315/3

x ≈ 1.772 = DX

2(1.772) = CB
3.544 = CX

Many students had difficulty with part three of the task. They could not use the concept of similarity to see this as a proportional relationship or they could see the ratio of one 1:2, but weren’t sure what to do with that information. Some students, like Student C, tried to use sine to solve for the missing length. Others, like student D, lost track of which side they needed to find. Student D has solved the problem of using feet and inches by converting everything to inches. The student sees that segment CD is composed of three segme nts, but forgets that he is solving for the longer segment.

Student C

3. Show that triangles AXD and BXC are similar.

1) ∡DXA ≅ ∡CXB	1) vertical angles are congruent
2) ∡ADB and ∡CBD are perpendicular	2) Given
3) ∡ADB & ∡CBD are 90°	3) def. of perpendicular lines
4) ∡ADX = 41 & ∡CBX = 60	4) Angle addition postulate
5) △AXD ∼ △BXC	5) Angle - angle similarity

2

4. Use the fact that triangles AXD and BXC are similar to calculate $\overline{CX}$, the distance from the top of the seat to the hinge. Show how you figured it out.

$\overline{CX}$ = 4.07 feet. X

$\sin 79 = \frac{4}{z}$

$\frac{4}{\sin 79} = z$ X

$4.07 = z$

0

0

Student D

2. Use the Pythagorean theorem to calculate the length of $\overline{CD}$. Show your work.

63.7 ✓

$42^2 + 48^2 = c^2$ ✓

$\sqrt{4068} = c$

$63.7 = c$

42 × 42: 84, 168, 1764

48 × 48: 384, 192, 2304

4068

1

1

3. Show that triangles AXD and BXC are similar.

all there angles are the same and the angles in similar triangles are congruent X ✓

see figure

1

4. Use the fact that triangles AXD and BXC are similar to calculate $\overline{CX}$, the distance from the top of the seat to the hinge. Show how you figured it out.

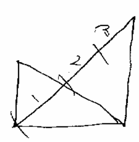
21.2 X

X

2.5 half of 4 feet

the proportion is 1/2

$\frac{63.7}{3} = 21.2$



0

0

A common error students made was to convert 3 feet 6 inches to 3.6 feet instead of 3.5 feet. Student E correctly uses the Pythagorean Theorem and makes sense of the proportional relationships in part 4. The decimal conversion is the only error made by the student.

Student E

2. Use the Pythagorean theorem to calculate the length of $\overline{CD}$. Show your work.

CD = 5.3814 ✗

$a^2 + b^2 = c^2$

$3.6^2 + 4^2 = c^2$ ✓

$12.96 + 16 = c^2$

$28.96 = c^2$

$c = \sqrt{28.96}$

3.6 × 3.6 = 216 + 1080 = 12.96

$2\sqrt{7.24}$

3. Show that triangles AXD and BXC are similar.

∠DXB = 101° because 180 − the other 2 angles equals it. By linear pair, ∠CXB = 79. By vertical ∠'s, ∠CXB ≅ ∠AXD (both are 79°). ∠ADX = 41° & ∠CBX = 60°. by complimentary ∠'s. Because △s ∠'s add up to 180°, the remaining ∠ in △AXD must be 60° & the remaining ∠ in BXC must be 41°. By AAA, the AXD ~ BXC.

4. Use the fact that triangles AXD and BXC are similar to calculate $\overline{CX}$, the distance from the top of the seat to the hinge. Show how you figured it out.

$\overline{CX}$ = 3.586 ✱

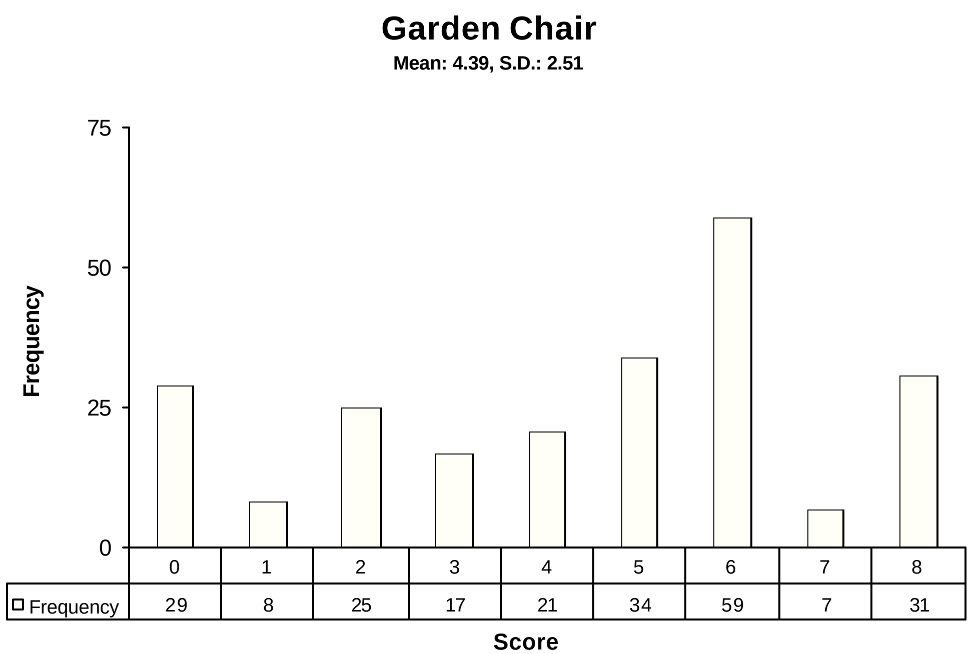
$\frac{2}{4} =$ 3.586

$5.38 = x + \frac{1}{2}x$ ✗

$5.38 = \frac{3}{2}x$

Teacher Notes:

Grade 10 – Garden Chair



Score:	0	1	2	3	4	5	6	7	8
% < =	12.6%	16.0%	26.8%	34.2%	43.3%	58.0%	83.5%	86.6%	100.0%
% > =	100.0%	87.4%	84.0%	73.2%	65.8%	56.7%	42.0%	16.5%	13.4%

The maximum score available on this task is 8 points.
The cut score needed for a level 3 response is 5.

Most students (about 87%) could use Pythagorean theorem to solve for an unknown side. About have the students could find angle X and use Pythagorean theorem. About 17% of the students could also prove similarity and use that property for proportional reasoning. About 13% of the students scored zero points on the task.

Garden Chairs

Points	Understandings	Misconceptions
0	More than 90% of the students with this score attempted the problem.	One common error was to know that there are 180 degrees in a triangle, subtract the other angles to get 101 degrees, and assume it was the missing angle. Students did not see that they were trying to find the exterior angle.
2	Students with this score point correctly used Pythagorean theorem to solve for side CD. About half of them used 3.6 instead of 3.5 for 3 feet 6 inches. These students could generally recognize vertical angles in discussing similarity in part 3.	Students struggled with converting inches to decimals. They could not make a complete argument about similarity, usually forgetting the one of the opposite interior angles or misapplying a congruence theorem.
4	Students with this score could find the measure of angle CXB. They could also do part of the Pythagorean Theorem and complete part of the argument for similarity.	Students struggled with converting inches to a decimal and using proportional reasoning.
6	Students could find angle CXB, use Pythagorean theorem and convert inches to decimals, and make a complete argument for similarity.	Students could not use proportional reasoning to find the length of CX. Some tried to use sine ratios. Others couldn't figure out what to do with the 1/2.
8	Student could use proportional reasoning to find the length of CX. They could apply the information about similarity to the diagram to find the missing length.	

Based on teacher observations, this is what Course Two students seemed to know and be able to do:

- Use Pythagorean Theorem to solve for a hypotenuse.
- Use properties of a triangle and measure of a straight line to find a missing angle.
- Show that vertical angles are equal.

Areas of difficulty for Course Two students:

- Converting 3 feet 6 inches to 3.5 feet.
- Comparing all three angles to make a case for similarity.
- Use similarity and proportional reasoning to solve for a missing side of a triangle.

Questions for Reflection on Garden Chair:

- How many of your students could not solve for angle CXB?
- What types of errors were students making?
- What types of experiences do students need to help them analyze diagrams for unknown angles? What types of experie nces have they had in your class this year?
- Why do you think students struggled with the proportional reasoning in part 4 of the task? What types of errors were they making and what type of logic led to those errors?
- What changes or lessons might you want to add next year to help students understand how to apply similarity to problem solving situations? What resources might be helpful?

Teacher Notes:____

____

____

____

Implications for Instruction:

Students need to be asked a variety of questions to check for understanding around a concept. So of those questions need to push students beyond recall and procedure, into analysis. For example, students learning about relationships of angles need a variety of complicated diagrams to push them to use multiple definitions or relationships to find the missing value. Students learning a concept, such as similarity, need to see the application of the rules and tests for similarity, as a useful tool for solving problems. Students need to make connections between procedures and definitions to applications. Students at this grade level should frequently be given challenging problems to integrate a variety of skills and procedures, where the steps are not broken down into smaller bits and that require longer chains of reasoning. A good resource for problems might be NCTM's Navigating Through Geometry or some of Key Curriculum Press's IMP materials.

Lampshade

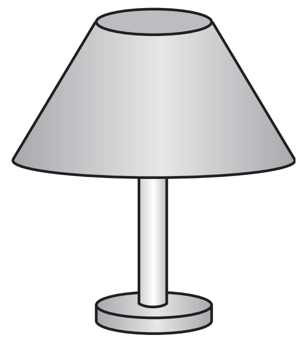
Student Task	Students compare and analyze two diagrams of a lampshade to determine circumference of the shade, circumference and diameter of the pattern, fabric size for making the shade and percentage of fabric used.
Geometry and Measurement	Analyze characteristics and properties of two- and three-dimensional geometric shapes; develop mathematical arguments about geometric relationships; and apply appropriate techniques, tools, and formulas to determine measurements. • Understand and use formulas for the area, surface area, and volume of geometric figures, including spheres and cylinders. • Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools.

Lampshade

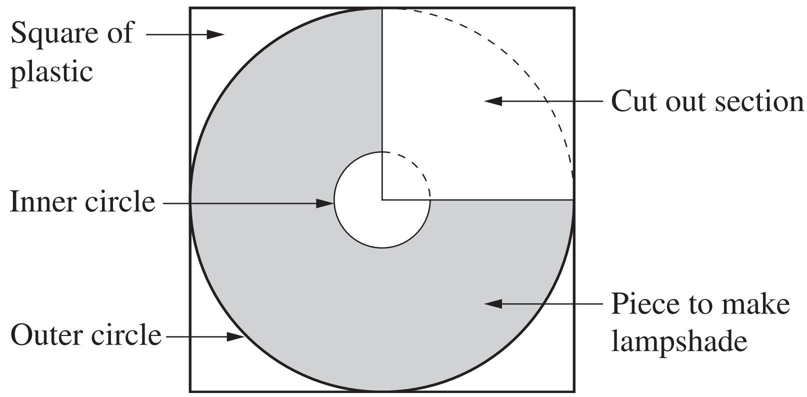
This problem gives you the chance to:

- use the formulas for the circumference and area of a circle
- calculate the area of a composite shape

A lampshade is made by cutting the largest possible circle from a square of plastic.



A smaller circle is cut from the center of the large circle. Then one quarter of the shape is removed, as shown in the diagram below.

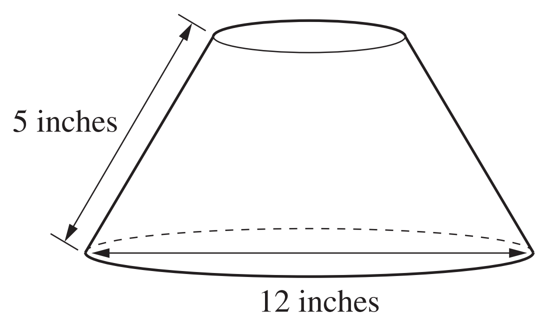


Circumference of a circle = πd

Area of a circle = πr^2

The three-quarter shape is curved around to make a lampshade.

The finished lampshade must have a base diameter of 12 inches and a sloping side of 5 inches.



Calculate the following measurements needed to draw the shape before it can be cut out.

1a. Find the circumference of the bottom of the lampshade. __________________ inches

1b. Use this measurement to find the diameter of the outer cutting circle. __________________ inches
Show your work.

1c. Find the diameter of the small circle. __________________ inches

2a. Calculate the area of the square of plastic. __________________ square inches

2b. Calculate the area of plastic used for the lampshade. __________________ square inches
Show your calculations.

2c. What percentage of the area of the plastic square is used to make the lampshade? __________________

8

Looking at Student Work – Lampshade

Lampshade was a very difficult problem for students. Many had trouble interpreting the relationships between the diagram of the pattern and the diagram of the shade. The Student A had a good conceptual understanding of the situation. The only mark on the diagram was to put a 5 on the pattern to indicate the width of the donut shape. In 1b, Student A knows that the circumference of the shade is 3/4 of the circumference of the larger pattern and can use inverse relationships to find the diameter of the pattern. This was one of the most difficult aspects of the task. In part 2b the student makes an interesting use of proportions to find the area of the large and small circles.

Calculate the following measurements needed to draw the shape before it can be cut out.

1a. Find the circumference of the bottom of the lampshade. π12 = 37.70 inches ✓ 1

1b. Use this measurement to find the diameter of the outer cutting circle.
Show your work.

16 inches ✓ 1

$\frac{3}{4} = \frac{37.70}{x}$ ✓

$x = \frac{50.27}{\pi}$ d = 16

reverse circumferen formula ✓ 1

1c. Find the diameter of the small circle. 6 inches 1

6/2 = 8 8-5 = 3 × 2 = 6

2a. Calculate the area of the square of plastic. 16×16 = 256 square inches ✓ 1

2b. Calculate the area of plastic used for the lampshade.
Show your calculations.

129.63 square inches ✓ 1

$\pi 8^2 = 201.10 u^2$ 150.83 − 21.2

$\pi 3^2 = 28.27$

$\frac{3}{4} = \frac{x}{28.27}$ $\frac{3}{4} = \frac{x}{201.10}$ ✓ 1

x = 21.20 x = 150.83

2c. What percentage of the area of the plastic square is used to make the lampshade? 50.64% ✓ 1

$\frac{129.63}{256} = \frac{x}{100}$

x = 50.64

Student B understands that the pattern is 3/4 of the circle and actually calculates correctly the diameter, which is used to solve part 1c and all of 2. Student B uses diagrams regularly as a tool to make sense of each new situation. However the student puts the area instead of the diameter in section 1b.

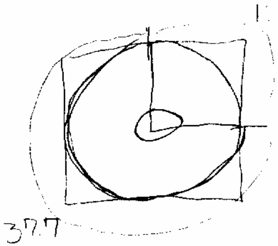
Student B

Calculate the following measurements needed to draw the shape before it can be cut out.

1a. Find the circumference of the bottom of the lampshade. ___37.7___ inches ✓

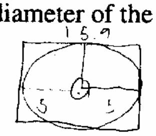
1b. Use this measurement to find the diameter of the outer cutting circle. ___50.2___ inches ✗
Show your work.

12.5
3)37.7



37.7
+ 12.5
50.2

1c. Find the diameter of the small circle. ___5.9___ inches

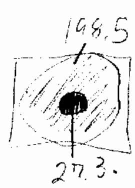


15.9
16
5.9

2a. Calculate the area of the square of plastic. ___252.81___ square inches ✓

2b. Calculate the area of plastic used for the lampshade. ___128.4___ square inches ✓
Show your calculations.

15.9 π 7.95² 198.5



128.4
252.81

171.2
42.8
128.4

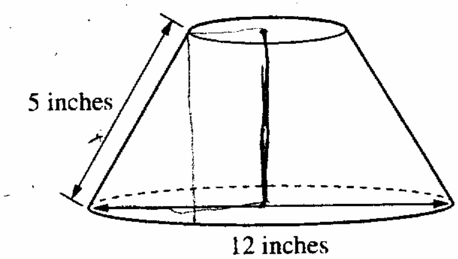
198.5
− 27.3
171.2

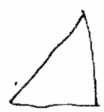
2c. What percentage of the area of the plastic square is used to make the lampshade? ___51%___ ✓

Student C uses Pythagorean triples to find the diameter of the small circle. Look at the combination of sketches on page one and page two. In part two the student fails to notice that the pattern is only 3/4 of the total shape. The student also ignores the fact that the small circle is not part of the fabric used for the lampshade.

Student C

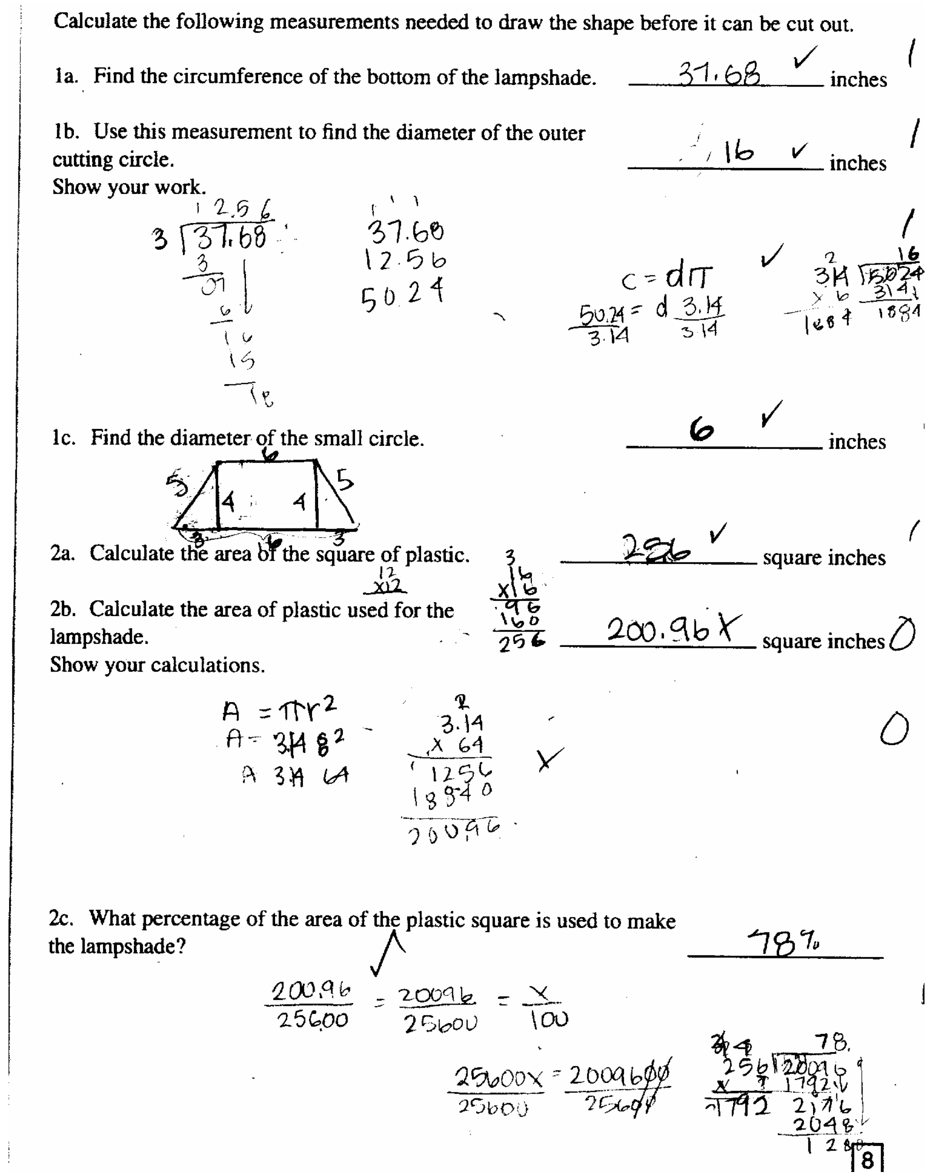
.de must have a base
; and a sloping side





$2\pi r = C$

3.14
× 12
628
3140
37.68



Calculate the following measurements needed to draw the shape before it can be cut out.

1a. Find the circumference of the bottom of the lampshade. 37.68 inches

1b. Use this measurement to find the diameter of the outer cutting circle. Show your work. 16 inches

1c. Find the diameter of the small circle. 6 inches

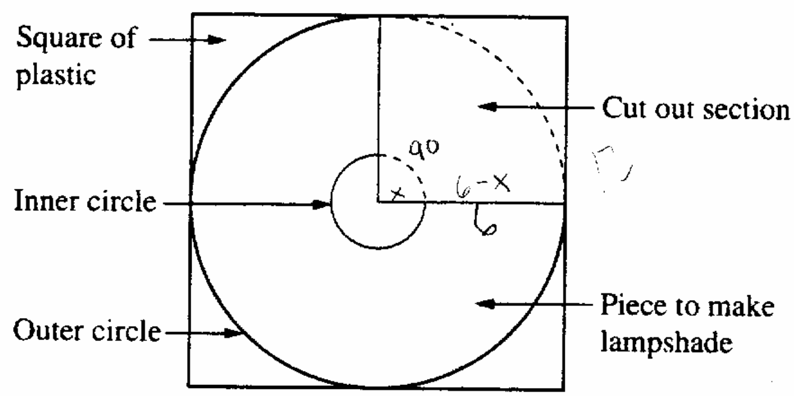
2a. Calculate the area of the square of plastic. 256 square inches

2b. Calculate the area of plastic used for the lampshade. Show your calculations. 200.96 square inches

2c. What percentage of the area of the plastic square is used to make the lampshade? 78%

A common error for students was to think that the diameter of the lampshade was the same as the diameter for the pattern. Looking carefully at the diagram on page 1 for Student D, the student struggles with the idea that the radius of the large circle is 6, but also has an understanding that the radius of the donut shape must be 6-x. The student can't resolve this conflict and leaves the diameter of the small circle blank. The student does not seem to make sense of answers calculated in part 2, ignoring the fact that the area of the lampshade is larger than the area of the circle.

Student D


Square of plastic
Cut out section
90
x
6-x
6
Inner circle
Piece to make lampshade
Outer circle

Student D

Calculate the following measurements needed to draw the shape before it can be cut out.

1a. Find the circumference of the bottom of the lampshade. 37.7 ✓ inches

C = π d   C = π 12

1b. Use this measurement to find the diameter of the outer cutting circle. 12 X inches

Show your work.

C = π 12 X
= 37.7

1c. Find the diameter of the small circle. X inches

2a. Calculate the area of the square of plastic. 144 square inches

12 x 12

2b. Calculate the area of plastic used for the lampshade. 188.5 X square inches

Show your calculations.

A = 5(c)
= 5(37.7) X
= 188.5

2c. What percentage of the area of the plastic square is used to make the lampshade? X 78.47%

113/144 = x/100 = X

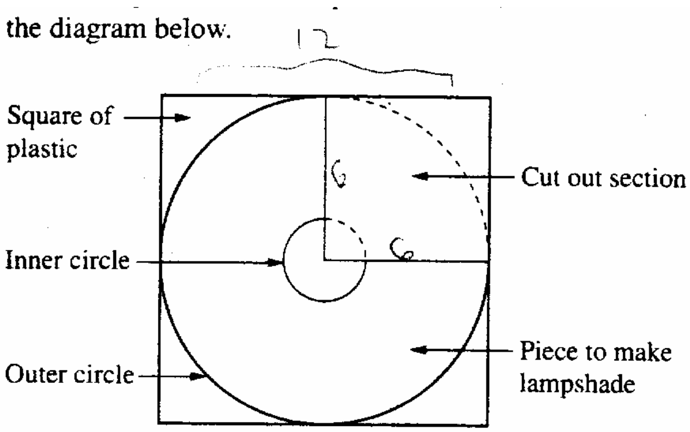
144 x = 11300
x = 78.47 %

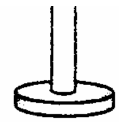
[8]

Student E also has the misconception that the diameter of the pattern is the same as the diameter of the lampshade. In part 1b, Student E cannot use inverse relationships to find the diameter of the larger circle. The student actually confuses the circumference of the lampshade with the circumference of the pattern and calculates the circumference of the large circle minus the unused quarter. In part 2 the student seems to have found the area of the larger circle, but not the area for the square or the small circle.

Student E

the diagram below.



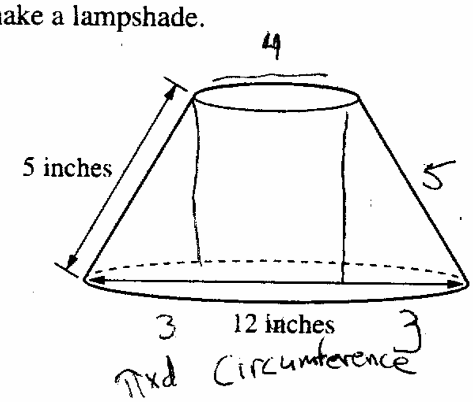


Circumference of a circle :

Area of a circle = πr^2

The three-quarter shape is curved around to make a lampshade.

The finished lampshade must have a base diameter of 12 inches and a sloping side of 5 inches.



Calculate the following measurements needed to draw the shape before it can be cut out.

1a. Find the circumference of the bottom of the lampshade. ≈ 37.699 ✓ inches

1b. Use this measurement to find the diameter of the outer cutting circle.
Show your work. ≈ 28.274 ✗ inches

✗

π 12 ≈ 37.699 · ¾ ≈ 28.274

1c. Find the diameter of the small circle. 4 ✗ inches

πr² = π4 ≈ 12.566·4

2a. Calculate the area of the square of plastic. ≈ 50.265 ✗ square inches

2b. Calculate the area of plastic used for the lampshade.
Show your calculations. ≈ 201.062 ✗ square inches

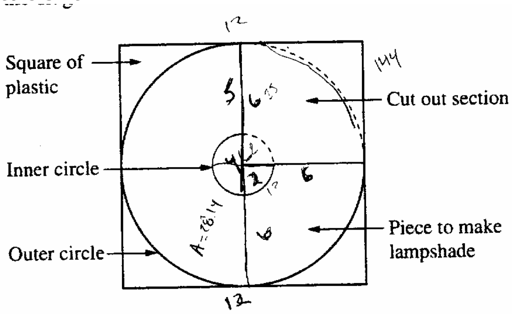
π 4 ≈ 12.566 ·4 ≈ 50.265
π 12 ≈ 37.699 ·4 ≈ 150.796 ✗
≈ 201.062

−¼ of answer

2c. What percentage of the area of the plastic square is used to make the lampshade? 75% ✗

Students at this grade level should be comfortable interpreting basic diagrams. Student F shows no sense making of the pattern diagram. First student F makes the common error of confusing the diameter of the lampshade with the diameter of the pattern. Further the student seems to label similar parts of the diagram with the same numbers. For example 5 and 6 are both used for the radius of the large circle and/or the width of the donut. The diameter of the small circle is both 4 and 2, depending on whether you look at the diagram on page one or the student diagram on page 2 of the task. The student also has trouble determining which square is being described in part 2a.. The student seems to find the area of the small square to answer this section, but realize the area of the large square is needed for finding the percentage.

Student F



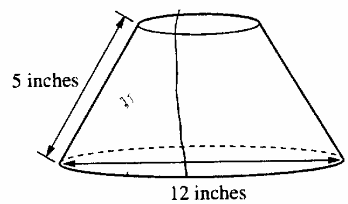
Circumference of a circle =

Area of a circle = πr^2

The three-quarter shape is curved around to make a lampshade.

The finished lampshade must have a base diameter of 12 inches and a sloping side of 5 inches.

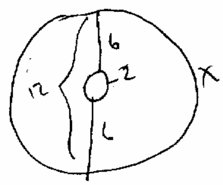
113.04 = 144



Calculate the following measurements needed to draw the shape before it can be cut ou

1a. Find the circumference of the bottom of the lampshade. ✓ 37.68 in

1b. Use this measurement to find the diameter of the outer cutting circle.
Show your work. X in



1c. Find the diameter of the small circle. 4 X in

2a. Calculate the area of the square of plastic. 25 X square in

2b. Calculate the area of plastic used for the lampshade.
Show your calculations. X 30.96 square in

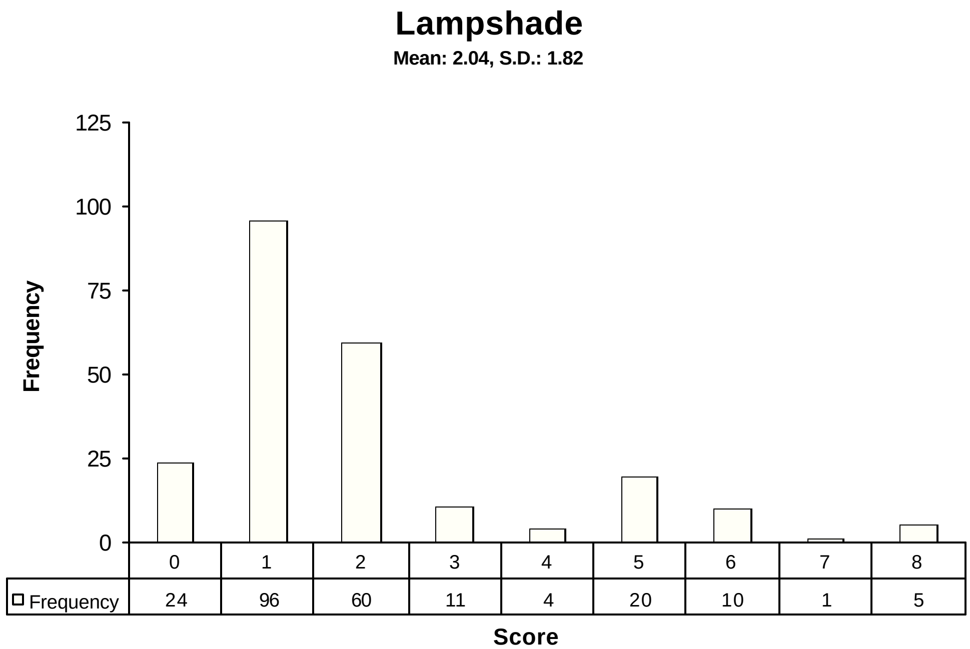
A of Sq. 144
A of Circle. 113.04
30.96 X

2c. What percentage of the area of the plastic square is used to make the lampshade? X ~79%

A = 144 (Square)
A = 113.04 (Circle) X

Teacher Notes:

Grade 10 – Lampshade



Score:	0	1	2	3	4	5	6	7	8
% < =	10.4%	51.9%	77.9%	82.7%	84.4%	93.1%	97.4%	97.8%	100.0%
% > =	100.0%	89.6%	48.1%	22.1%	17.3%	15.6%	6.9%	2.6%	2.6%

The maximum score available on this task is 8 points.
The cut score needed for a level 3 response is 3 points.

Most students (about 90%) could use the formula to find the circumference of the lampshade. About half the students could also find the percentage of the fabric used to make the shade. This may have been based on errors in previous calculations. About 16% of the students could find the circumference of the lampshade, the diameter of the pattern and the small circle, find the area of the total fabric, and make some correct calculation of percentages. More than 10% of the students scored no points on this task.

Lampshade

Points	Understandings	Misconceptions
0	Approximately 80% of the students with this score attempted the problem. About 13% of the remaining students appeared to run out of time as they did not attempt Lampshade or the final task.	Students at this score point showed no consistent error. They did not seem to use any approach or logic strategy that would have led to a correct solution.
1	Students with this score could use the formula to find the circumference of the shade.	Students may have done a fair amount of correct procedures for later parts of the problem, but they did not see the relationship between the shade and the pattern. Usually they thought the diameter of the shade was the same as the diameter of the pattern.
2	Students could find the circumference of the lampshade and could find the percentage of the fabric used to make the shade based on errors in previous calculations. A few correctly found the diameter of the small circle.	Students could not find the diameter of the large circle. Many thought the diameter of the pattern was the same as the shade. They did not see the circumference of the shade as 3/4 the circumference of the pattern. If they did see this relationship, they didn't have the ability to use it to correctly find the diameter.
5	Students with this score could find the circumference of the shade, diameter of the large and small circle, find the area of the square, and calculate percentage of fabric used to make the pattern.	Most of these students did not show their work for finding diameter of the large circle. They struggled with the multiple calculations necessary to find the area of the pattern. Most found the area of the large circle, but either did not subtract out the area for the small circle or did not remember to find 3/4 of the area.
8	Students could analyze and synthesize two diagrams to find the dimensions of the pattern and the percentage of the fabric used to make the shade.	

Based on teacher observations, this is what Course Two students seemed to be know and be able to do:

- Use the formula to find circumference of a circle.
- Calculate percentages.

Areas of difficulty for Course Two students:

- Analyzing and synthesizing diagrams
- Keeping track of what information results from their calculations
- Working multi-step problems
- Using inverse operations on formulas

Questions for Reflection on Lampshade:

- What percentage of your students did not attempt this problem? Do you think time was a factor?
- Looking at your students who scored zero:
    - How many attempted the problem?
    - How many did not attempt the problem?
- How many of your students used 12 as the diameter of the large circle?
- Look carefully at the reasoning of your students in finding the area of the pattern. How many of them could complete each of the following steps:

Find area of large circle using diameter of 16.	Find area of large circle using some other diameter.	Find area of small circle using diameter of 6.	Find area of small circle using some other diameter.	Attempted problem with no correct logic.	Did not attempt this section.

- Do you think students would have had more success if they were labeling their answers at each step of the process, so they could see more clearly what was done and what still wasn't accounted for? What other strategies might have helped them through all the steps needed in this part of the task? What strategies did students use who were successful to keep track of their information?

Implications for Instruction:

Students in middle grades frequently compute area, circumference and diameter of circles and interpret simple diagrams. As students move through the grades the types of thinking and reasoning need to get progressively more complex. What opportunities are provided in the curriculum to go beyond the reasoning of simple calculations and push students to this level? Students should frequently be given complex diagrams to interpret. New Elementary Mathematics Course 2 and 3, by Dr. Wong Khoon Yoong and Sin Kwai Meng, Pan Pacific Publications, might be a good source of problems. Students also need some strategy to help them make sense of problem situations. Students who were successful seemed to make their own diagrams. Perhaps labeling answers as they work through multi-step problems would also be a helpful tool; as students seemed to lose track of where they were in the process. A study in Focus Magazine (Volume 10, Number 1, 2003) shows that high school teachers who worked at not breaking down problems into smaller parts for students quadrupled test scores. It is important for students to struggle with organizing their information and doing longer chains of reasoning.

Teacher Notes:

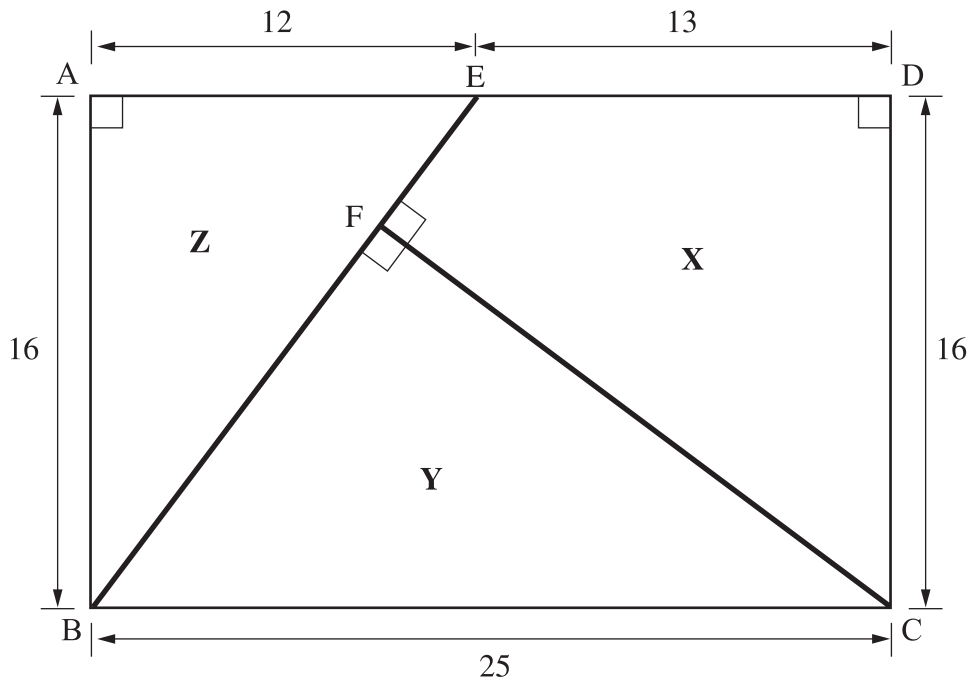
Rectangle and Square

Student Task	Find missing lengths in a diagram using Pythagorean Theorem and similarity. Use understanding of area and properties of a square to solve a spatial puzzle.
Geometry and Measurement.	• Analyze characteristics and properties of two-dimensional geometric shapes; develop mathematical arguments about geometric relationships; apply transformations a; and apply appropriate techniques, tools, and formulas to determine measurements. • Understand relationships among the angles, side lengths, perimeter, and areas, and volumes of similar figures. • Create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.

Rectangle and Square

This problem gives you the chance to:
- solve a problem using similarity and the Pythagorean theorem

A rectangle 25 centimeters long and 16 centimeters wide is divided into two right triangles and a quadrilateral, as shown below. The two triangles Y and Z are similar.



1. Calculate the lengths of the sides of the two triangles Y and Z, and the sides of the quadrilateral X.

BE = ___________________ cm BF = ___________________ cm

FE = ___________________ cm FC = ___________________ cm

2. Draw a diagram to show how the two triangles and the quadrilateral can be rearranged to make a square.

What is the length of each side of the square? __________________ cm

Explain your reasoning.

____

____

____

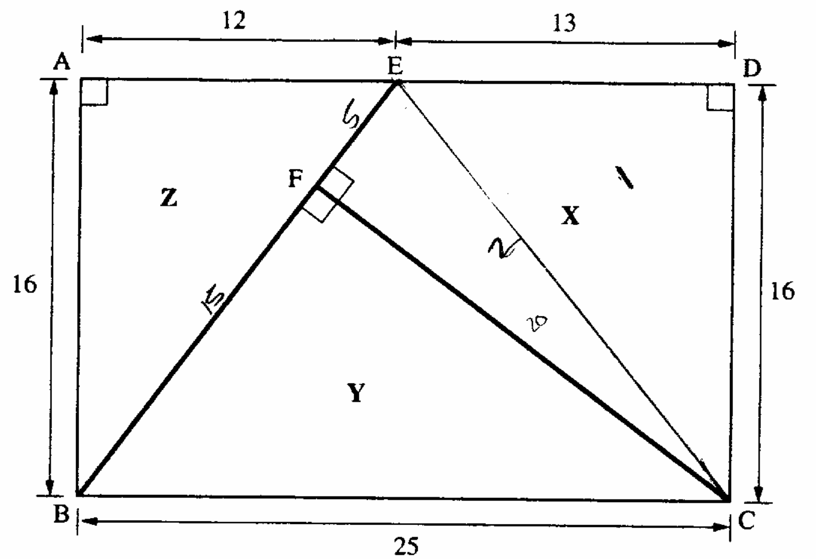
____

8

Looking at Student Work – Rectangle and Square

Student A uses Pythagorean Theorem to find length BE and CF. Then uses similarity to correctly set up a proportion to solve for BF. The student used area to find the side length for the square. Only 10% of the students made this connection.

Student A



1. Calculate the lengths of the sides of the two triangles Y and Z, and the sides of the quadrilateral X.

BE = 20 ✓ cm   BF = 15 ✓ cm

FE = 5 ✓ cm   FC = 20 ✓ cm

BE : $16^2 + 12^2 = BE^2$
$256 + 144 = 400$
$BE = \sqrt{400}$
$BE = 20$

FE : BF = 15
FE = BE − BF =
20 − 15 = 5

FC . $BF^2 + FC^2 = BC^2$
$15^2 + FC^2 = 25^2$
$FC^2 = 400$
$FC = \sqrt{400} = 20$

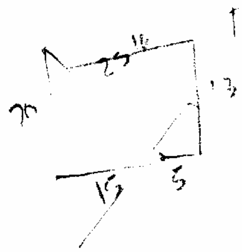
BF : ratio of corresponding sides :

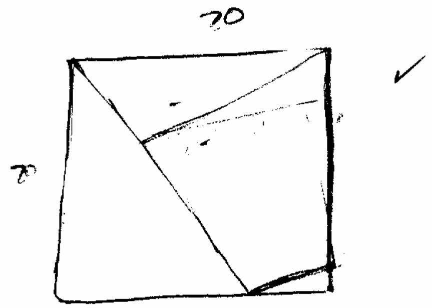
$\frac{BF}{AE} = \frac{BC}{BE} = \frac{BF}{12} = \frac{25}{20}$

$20BF = 300$

$BF = 15$

Student A





What is the length of each side of the square? 20 cm
Explain your reasoning.

The area of all the pieces add up to 400 cm. Therefore the length of each side is 20 cm because 20 • 20 = 400

area 2 = ½ 16(12) 96

4 = ½ (15)(20) 150

2 = ½ 5 (20) 50

1 = 104

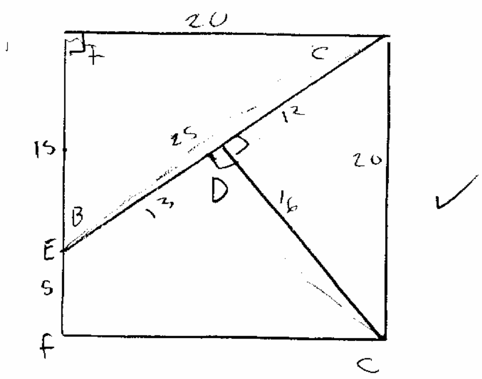
400

or 16 · 25 = 400

$\sqrt{400} = 20$

More than 20% of the students found the side length for the square by looking for matching sizes in the original figure. See the work of Student B and C.

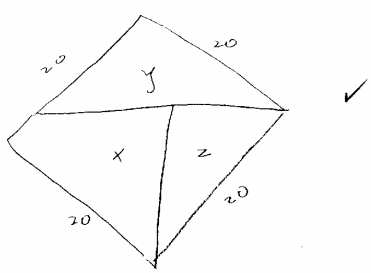
Student B



What is the length of each side of the square? 20 ✓ cm
Explain your reasoning.

I added FE to BF so they equaled 20. Since BE and FC already equaled 20 it seemed most likely. There were 2 FC's one for the quad. and another for a Δ, BE, and by adding BF + FE all four sides would equal 20. ✓

Student C



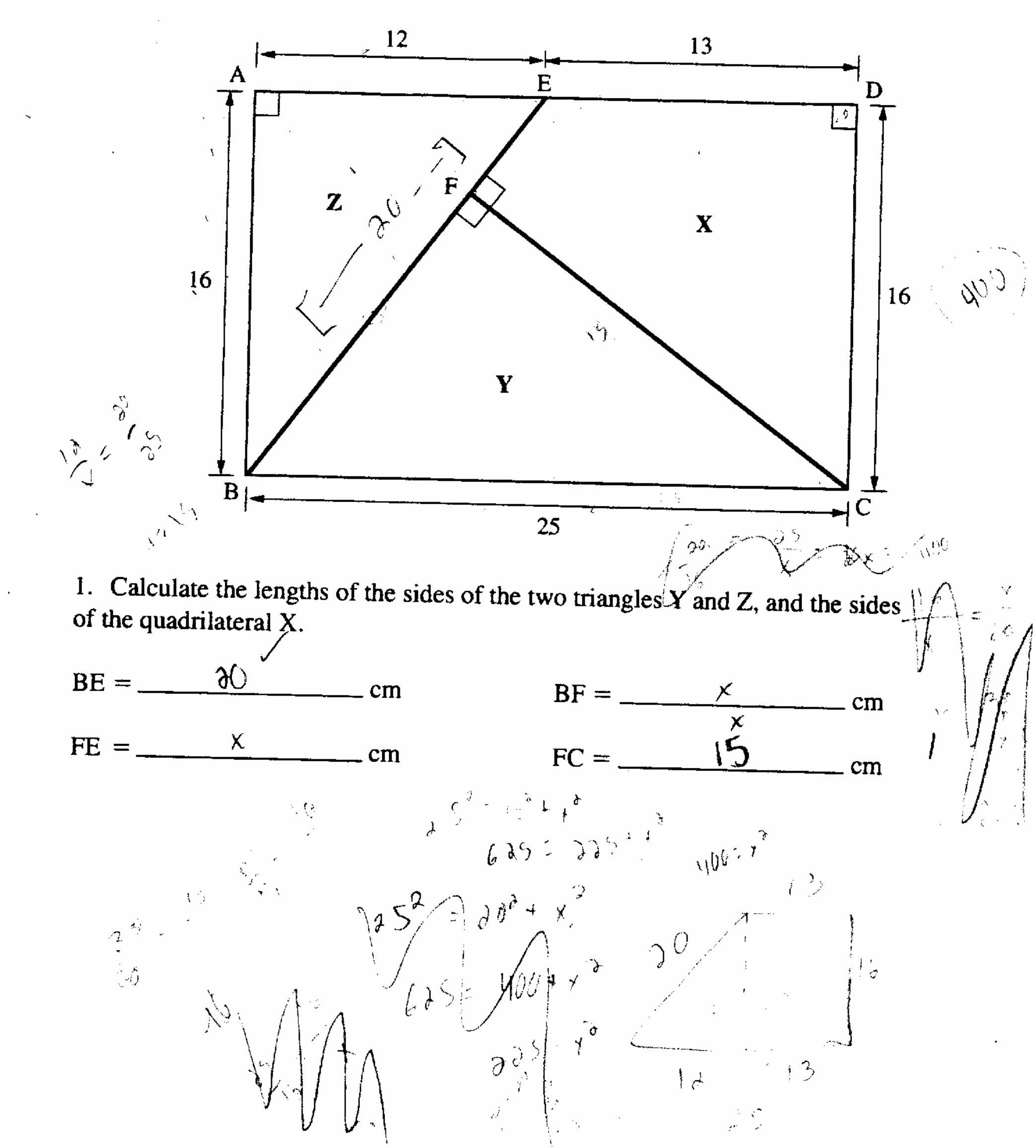
2

What is the length of each side of the square? 20 ✓ cm
Explain your reasoning.

Each side is 20 cm because the definition of a square is that all 4 sides are congruent. Also, 20 is the only length common to all 3 figures.

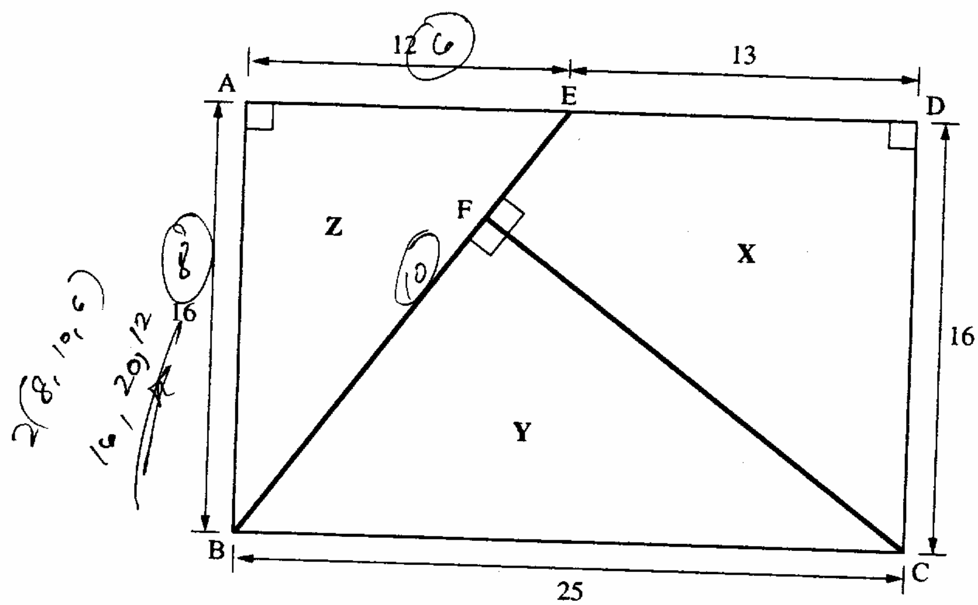
While more than 40% of the students could use Pythagorean Theorem to find the length of BF, they did not seem to understand how to use the properties of similarity to find the other missing dimensions. Student D attempts to use Pythagorean Theorem on a quadrilateral.

Student D



A few students used Pythagorean triples to solve for missing sides. But still could not use similarity to find the other dimensions. See the work of Student E.

Student E



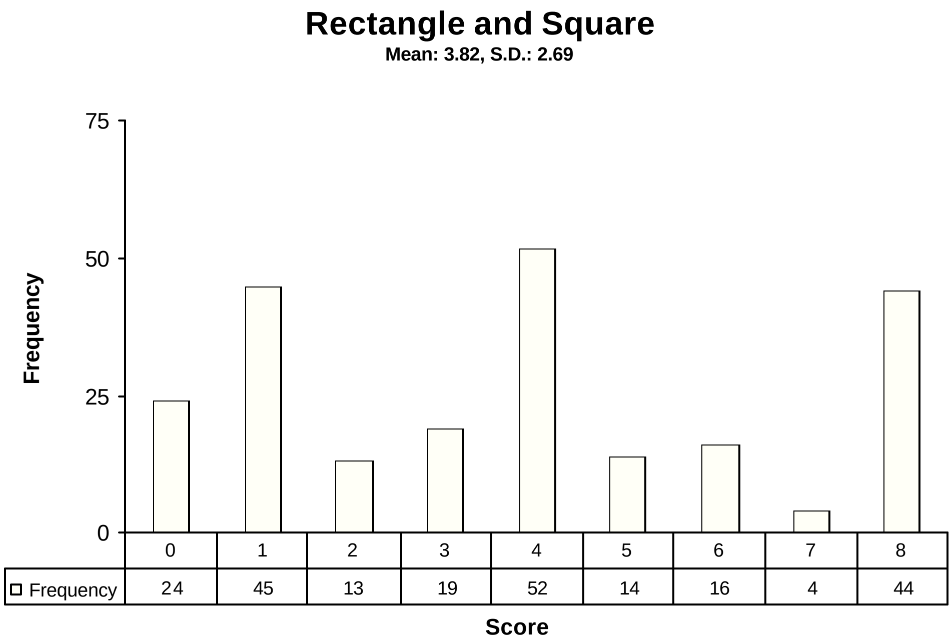
1. Calculate the lengths of the sides of the two triangles Y and Z, and the sides of the quadrilateral X.

BE = 20 cm ✓ BF = ______ cm

FE = ______ cm FC = 15 ✗ cm

Teacher Notes:

Grade 10 – Rectangle and Square



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% > =	100.0%	89.6%	70.1%	64.5%	56.3%	33.8%	33.8%	20.8%	19.0%

The maximum score available for this task is 8 points.
The cut score needed for a level 3 response is 4.

Most students (about 90%) could use Pythagorean theorem and solve for side BF. More than half the students could use similarity and Pythagorean theorem to find all the missing dimensions in the diagram. Almost 21% of the students could use information about the properties of a square to solve the puzzle and explain their reasoning in part 2. About 10% of the students scored zero points on this task.

Rectangle and Square

Points	Understandings	Misconceptions
0	More than half the students with this score attempted the problem.	Students with this score often put down numbers with no work shown. They usually attempted all of part 1 and tried to draw the square.
1	Students with a score of one could use Pythagorean Theorem to solve for BE.	Many tried to use Pythagorean theorem to solve for the missing dimensions of the quadrilateral or matched incorrect parts when setting up proportions using similar triangles.
4	Students could use Pythagorean theorem and properties of similarity to find missing dimensions in a diagram.	About 33% of the students did not attempt part 2. Many of those who tried to use 16 as a side length for the square.
6	Students could find the missing dimensions and rearrange the shapes to make a square.	Students had difficulty articulating how they found the side length of the square. They made statements like how the pieces fit together, need a shorter side, or made no attempt at the explanation.
8	Students could find the missing dimensions, rearrange the shapes to make a square, and give an explanation about how they determined the length of square. Of all the students surveyed about 30% found the side of the square by looking for matching sides or knowing all the sides of the square are the same. About 10% used area of the rectangle to find the side of the square.	

Based on teacher observations, this is what Course Two students seemed to know and be able to do:

- Use Pythagorean theorem
- Use properties of similarity

Areas of difficulty for Course Two students:

- Drawing and rotating geometric shapes, spatial visualization
- Using geometric properties to reason or to make an argument about a square

Questions forReflection on Rectangles and Squares:

- How many of your students could use similarity to set up proportions for finding missing dimensions?
- How many students could not put the corresponding sides together when making their proportions?
- How often do students get to apply geometric properties or definitions to problem solving situations? Give some examples.
- What types of activities do students have to build their spatial visualization skills?
- How often are students required to reason about geometric relationships and explain their thinking? What are some good problems that you use in the classroom?
- Do you regularly or routinely give long problems for students to solve, like a problem of the week? What sources do you use for these problems? What type of grading scheme do you use for evaluating their work?

Implications for Instruction:

Students need to have many experiences working with similar figures, particularly when the shapes have different orientations. Students need to be able to understand and represent translations, reflections, and rotations of objects in the plane by using sketches. While spatial visualization is difficult for students, facility with this can be improved with practice. Some research says that lack of spatial visualization skills prevents students from being successful in more advanced math courses. Students at this grade level need many opportunities to use geometric relationships to solve problems and explain their thinking.

Teacher Notes: