Overall Frequency Distribution by Total Score

Grade 4
Mean=26.51; S.D.=8.67
### Level Frequency Distribution Chart and Frequency Distribution

**2003 - Numbers of students tested:** 9399

#### Grade 4 2000 - 2001

<table>
<thead>
<tr>
<th>Level</th>
<th>% at ('00)</th>
<th>% at least ('00)</th>
<th>% at ('01)</th>
<th>% at least ('01)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30%</td>
<td>100%</td>
<td>18%</td>
<td>100%</td>
</tr>
<tr>
<td>2</td>
<td>37%</td>
<td>77%</td>
<td>23%</td>
<td>82%</td>
</tr>
<tr>
<td>3</td>
<td>26%</td>
<td>33%</td>
<td>39%</td>
<td>58%</td>
</tr>
<tr>
<td>4</td>
<td>7%</td>
<td>7%</td>
<td>19%</td>
<td>19%</td>
</tr>
</tbody>
</table>

#### Grade 4 2002 - 2003

<table>
<thead>
<tr>
<th>Level</th>
<th>% at ('02)</th>
<th>% at least ('02)</th>
<th>% at ('03)</th>
<th>% at least ('03)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8%</td>
<td>100%</td>
<td>8%</td>
<td>100%</td>
</tr>
<tr>
<td>2</td>
<td>27%</td>
<td>92%</td>
<td>14%</td>
<td>92%</td>
</tr>
<tr>
<td>3</td>
<td>37%</td>
<td>66%</td>
<td>36%</td>
<td>78%</td>
</tr>
<tr>
<td>4</td>
<td>28%</td>
<td>28%</td>
<td>42%</td>
<td>42%</td>
</tr>
</tbody>
</table>

### Frequency Distribution Chart

<table>
<thead>
<tr>
<th>Frequency</th>
<th>0-12</th>
<th>13-19</th>
<th>20-29</th>
<th>30-40</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Minimal Success</td>
<td>717</td>
<td>1304</td>
<td>3417</td>
<td>3961</td>
</tr>
<tr>
<td>2 Below Standard</td>
<td>20-29</td>
<td>30-40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### 4th grade  
**Task 1**  
**Shapes with Straws**

<table>
<thead>
<tr>
<th>Student Task</th>
<th>Solve problems about divisors and multiples in a geometric context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core Idea 2</td>
<td><strong>Number Operations</strong></td>
</tr>
<tr>
<td></td>
<td>Understand the meanings of operations and how they relate to each other, make reasonable estimates, and compute fluently.</td>
</tr>
<tr>
<td></td>
<td>• Develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems</td>
</tr>
<tr>
<td>Core Idea 3</td>
<td><strong>Geometry and Measurement</strong></td>
</tr>
<tr>
<td></td>
<td>Use characteristics, properties, and relationships of two-dimensional geometric shapes and apply appropriate techniques to determine measurements.</td>
</tr>
<tr>
<td></td>
<td>• Recognize geometric ideas and relationships and apply them to problems. (3rd grade)</td>
</tr>
<tr>
<td></td>
<td>• Identify and compare attributes of two-dimensional shape and develop vocabulary to describe the attributes.</td>
</tr>
</tbody>
</table>
Shapes with Straws

This problem gives you the chance to:
• solve problems about divisors and multiples in a geometric context

Anna’s class is making decorations using drinking straws. The students thread the straws together with string and then spray them with silver and gold paint. The shapes the students have made are shown below.

1. How many straws are needed to make these shapes?
   
   square = _______________  
   pentagon = _______________  
   triangle = _______________  
   hexagon = _______________

2. Anna has 24 straws. How many decorations of each shape can she make? 
   _____ squares  or  _____ triangles  or  _____ hexagons  or  _____ pentagons

Explain how you figured this out.

3. Anna’s class wants to make 9 triangles and 5 hexagons. How many straws do the students need in all? Show how you figured this out.

---

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# Shapes with Straws

## Test 4 Form A Rubric

The core elements of performance required by this task are:
- solve problems about divisors and multiples in a geometric context

Based on these, credit for specific aspects of performance should be assigned as follows:

<table>
<thead>
<tr>
<th>Points</th>
<th>Section Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

### 1. Gives correct answers as:
- square = 4
- triangle = 3
- pentagon = 5
- hexagon = 6

All four correct answers: 2 points

**Partial credit:**
- Three or two correct answers: 1 point

<table>
<thead>
<tr>
<th>Points</th>
<th>Section Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>(1)</td>
</tr>
</tbody>
</table>

### 2. Gives correct answers as:
- 6 squares or 8 triangles or 4 hexagons or 4 pentagons

All four correct answers: 2 points

**Partial credit:**
- Three or two correct answers: 1 point

Gives explanation such as:
- I divided 24 by the number of sides in each shape.
- or
- I divided 24 by 4 then 3 then 6 then 5.

**Accept repeated subtraction/addition strategies.**

**Partitioning must use exactly 24 straws.**

### 3. Gives correct answer as:
- 57

Shows work such as:
- \(9 \times 3 = 27\)
- \(5 \times 6 = 30\)
- \(27 + 30 = 57\)

**Allow 1 point for each correct multiplication if there is no blatant error in the explanation.**

<table>
<thead>
<tr>
<th>Points</th>
<th>Section Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points**

| 8      |                |
Looking at Student Work - Shapes with Straws

Many students did very well on this task. Student A counts and labels the sides of the geometric shapes to organize information for later steps in the problem. The student can use division to find out how many equal groups will fit in 24. The student draws, counts and uses multiplication to clearly explain his thinking. The layout of the work in part 3 shows a clear, logical progression of thought to solve the problem. Student A does confuse pentagon and hexagon in working through the problem.

Student A

The shapes the students have made are shown below.

1. How many straws are needed to make these shapes?
   - square = 4
   - triangle = 3
   - pentagon = 5
   - hexagon = 6

2. Anna has 24 straws. How many decorations of each shape can she make?
   - squares = \[ \frac{24}{4} = 6 \] or 6 squares
   - triangles = \[ \frac{24}{3} = 8 \] or 8 triangles
   - hexagons = \[ \frac{24}{6} = 4 \] or 4 hexagons
   - pentagons = \[ \frac{24}{5} \] or \[ \frac{24}{5} \]

   Explain how you figured this out.
   
   I did \[ \frac{24}{4} \], \[ \frac{24}{3} \], \[ \frac{24}{6} \] and there were \[ \frac{24}{5} \] left over.
   
   Last I did \[ \frac{24}{5} \].

3. Anna’s class wants to make 9 triangles and 5 hexagons. How many straws do the students need in all? Show how you figured this out.

   \[ \frac{9}{2} \] triangles
   \[ \frac{5}{6} \] hexagons

   \[ 52 \text{ straws} \]
Student B has not made the transition to using division for solving problems. The student still relies on repeated addition and counting to solve the problem. In part 3 the student correctly finds the number of straws to make the triangles and the number of straws to make the hexagons. Student B does not complete the problem by finding the total number of straws.

**Student B**

1. How many straws are needed to make these shapes?

   \[
   \begin{align*}
   \text{square} &= \frac{4}{2} \quad \text{pentagon} &= \frac{5}{1} \\
   \text{triangle} &= \frac{3}{1} \quad \text{hexagon} &= \frac{6}{1}
   \end{align*}
   \]

2. Anna has 24 straws. How many decorations of each shape can she make?

   6 squares or 8 triangles or 4 hexagons or 4 pentagons

   Explain how you figured this out.

   \[
   \text{I added how many times } \checkmark
   \]

   \[
   \text{you can put } 4,3,5,6 \text{ into } 24.
   \]

3. Anna’s class wants to make 9 triangles and 5 hexagons. How many straws do the students need in all? Show how you figured this out.

   \[
   \begin{align*}
   \text{triangle} &= 2 \times 9 \\
   \text{hexagon} &= 3 \times 5
   \end{align*}
   \]

Student C can use multiplication to find the number of shapes that can be made from a group of 24 straws. The student has a completely correct strategy in part 3, but confuses pentagons and hexagons even though they were correctly identified in part 1.
Student C

1. How many straws are needed to make these shapes?

- square = \( \frac{4}{\sqrt{}} \)
- pentagon = \( \frac{5}{\sqrt{2}} \)
- triangle = \( \frac{3}{\sqrt{}} \)
- hexagon = \( \frac{6}{\sqrt{}} \)

2. Anna has 24 straws. How many decorations of each shape can she make?

- 6 squares or 8 triangles or 4 hexagons or 4 pentagons

Explain how you figured this out.

\[
\begin{align*}
6 \times 4 &= 24 \\
3 \times 8 &= 24 \\
4 \times 6 &= 24
\end{align*}
\]

3. Anna's class wants to make 9 triangles and 5 hexagons. How many straws do the students need in all? Show how you figured this out.

\[
\begin{align*}
9 \times 3 &= 27 \\
5 \times 5 &= 25 \\
27 + 25 &= 52
\end{align*}
\]

Student D misinterprets what is asked in part two. The student finds the number of straws to make 24 shapes instead of the number of shapes that can be made with 24 straws. In part 3 the student does not complete the final step of combining the straws needed to make triangles with the straws needed to make hexagons.

Student D

1. How many straws are needed to make these shapes?

- square = \( \frac{4}{\text{straws}} \)
- pentagon = \( \frac{15}{\text{straws}} \)
- triangle = \( \frac{5}{\text{straws}} \)
- hexagon = \( \frac{16}{\text{straws}} \)

2. Anna has 24 straws. How many decorations of each shape can she make?

- 14 squares or 72 triangles or 14 hexagons or 120 pentagons

Explain how you figured this out.

\[
\begin{align*}
14 \times 24 &= 336 \\
72 \times 2 &= 144 \\
14 \times 14 &= 196 \\
120 \times 2 &= 240
\end{align*}
\]

3. Anna's class wants to make 9 triangles and 5 hexagons. How many straws do the students need in all? Show how you figured this out.

\[
\begin{align*}
9 \times 3 &= 27 \text{ straws} \\
5 \times 5 &= 30 \text{ straws}
\end{align*}
\]
Student E knows how to use division to find the number of equal groups that will fit into 24. The student confuses the idea of “in all” with addition, not recognizing that the problem is about 9 sets of triangles and 5 sets of hexagons. The student may have been taught to look for key words or may not completely clear about the relationship between equal groups and multiplication.

**Student E**

1. How many straws are needed to make these shapes?
   - square = \[ \frac{4}{2} \sqrt{ } \]
   - pentagon = \[ \frac{5}{2} \sqrt{ } \]
   - triangle = \[ \frac{6}{2} \sqrt{ } \]
   - hexagon = \[ \frac{6}{2} \sqrt{ } \]

2. Anna has 24 straws. How many decorations of each shape can she make?
   - \( \left[ \frac{10}{4} \right] \) squares  or  \( \left[ \frac{9}{3} \right] \) triangles  or  \( \left[ \frac{4}{4} \right] \) hexagons  or  \( \left[ \frac{5}{5} \right] \) pentagons

   Explain how you figured this out.

   \[
   \begin{align*}
   4 & \div 2 \; 6 \div 2 \; 10 \div 2 \; 12 \div 2 \; 15 \div 2 \; 18 \div 2 \\
   \hline
   2 & \; 3 \; 5 \; 6 \; 7 \; 9
   \end{align*}
   \]

3. Anna’s class wants to make 9 triangles and 5 hexagons. How many straws do the students need in all? Show how you figured this out.

   \[
   \begin{align*}
   + \frac{9}{3} & + \frac{12}{2} & + \frac{6}{2} & \times \sqrt{} \\
   \frac{11}{2} & \div 2 & \times \sqrt{} & \frac{8}{2} \sqrt{}
   \end{align*}
   \]

**Teacher Notes:**

________________________________________________________________________________

________________________________________________________________________________

________________________________________________________________________________

________________________________________________________________________________

________________________________________________________________________________

Fourth Grade                                                                                                         pg. 9
The maximum score available on this task is 8 points.
The cut score for a level 3 response is 4 points.

Most students (more than 98%) could identify the number of straws to make each of the geometric shapes. About 77% of the students met standards. They could generally name the shapes, find the number of straws needed to make squares and triangles, and do some of thinking for finding straws for 9 triangles. Many students (about 67%) could identify the number of straws to make the shapes, find the number of straws to make 9 triangles and 5 hexagons (but not necessarily the total), and answer some part of question 2. A little less than half the students (about 40%) could find the straws needed for each shape, find the number of shapes that could be made with 24 straws and find the number of straws to make 9 triangles and 5 hexagons (but not necessarily the total). More than 25% could complete all the demands of the task. Less than 1% of the students scored no points on this task.
# Shapes with Straws

<table>
<thead>
<tr>
<th>Points</th>
<th>Understandings</th>
<th>Misunderstandings</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Students with this score tried the problem.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Students with this score could find the number of straws to make each shape in part 1.</td>
<td>Some students confused part 2 with part one and put the same answers. Other students (9%) tried a combination of several shapes to equal 24 straws. About 1/3 of those students could do that successfully.</td>
</tr>
<tr>
<td>4</td>
<td>Students could name the shapes, find the number of triangles and squares that can be made with 24 straws, and find the number of straws needed for 9 triangles.</td>
<td>Many students did not know the number of sides in hexagon. They may not have added the two answers together.</td>
</tr>
<tr>
<td>5</td>
<td>Students could do all of part 1 and find the number of straws needed for 9 triangles and for 5 hexagons. They could use counting, drawing, repeated addition, multiplication or division to find the number of triangles and squares that could be made with 24 straws.</td>
<td>Some students cannot successfully use division to solve problems. They have difficulty choosing the best operation to fit the situation. About 11% of the students still rely on drawing, 5% showed evidence of counting, and 3% relied on repeated addition, another 3% showed no work at all.</td>
</tr>
<tr>
<td>6</td>
<td>Students with this score could complete part 1 and 2 of the task, but missed all of part 3.</td>
<td>11% of all students confused pentagons and hexagons in part 3 to get a total of 52 straws instead of 57. About 6% of all the students in the sample added 9 and 5 to get 14 in part three. They confused “in all” with a total instead of understanding the problem as making 9 equal groups and 5 equal groups. About 4% of all students multiplied 9 by 5 to get 45 straws in all. They didn’t think about the numbers in the shape names like 3 for triangle and 6 for hexagon. They just used the visible numbers.</td>
</tr>
<tr>
<td>8</td>
<td>Students could recognize the number of sides in common geometric shapes and use multiplication and division to solve problems.</td>
<td></td>
</tr>
</tbody>
</table>
Based on teacher observations, this is what fourth grade students seemed to know and be able to do:

- Find the number of straws needed to make squares and triangles.
- Use counting, drawing, repeated addition, multiplication or division to find the number of shapes that could be made with 24 straws.
- Use multiplication to find the number of straws to make 9 triangles.

Areas of difficulty for fourth graders, fourth grade students struggled with:

- Finding the number of straws needed to make hexagons and pentagons.
- Finding the number of hexagons or pentagons that could be made with 24 straws.
- Finding the total number of straws to make 9 triangles and 5 hexagons. They could generally find the amount of each, but missed the logic of combining the answers to find the total.

Questions for Reflection on Shapes with Straws:
Looking at student work in part 2 of the problem, how many of your students used:

|----------|-----------|-------------------|-----------------|-----------|---------------------------------------------------------------|

- Do your students seem to be comfortable with the purpose for multiplication and division or are they still relying on more cumbersome strategies?
- What kinds of activities, questions, or experiences can help students make the transition to multiplicative thinking?
- How many of your students saw “in all” as addition? What are the implications for instruction here?
- How many of your students did not add the two answers together in part 3?

Teacher Notes:

---

Implications for Instruction:
Students need to interpret situations that require multiplication and/or division to solve. Students also need experience with demonstrating mathematical reasoning by explaining how they determined their answers. Students at this age should have many experiences solving word problems using multiplication and division. Students need more experiences with word problems in which the numbers are embedded in a
context where the needed information is not laid out for them, but rather where they have to figure out what information they will need and how to use it.

### 4th grade Task 2 Number Trains

<table>
<thead>
<tr>
<th>Student Task</th>
<th>Solve problems about factors and multiples in a toy train context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core Idea 3</td>
<td>Understand patterns and use mathematical models to represent and understand qualitative and quantitative relationships.</td>
</tr>
</tbody>
</table>
| Patterns, Functions, and Algebra | • Use inverse operations to solve multi-step problems  
• Understand and use the concept of equality |
| Core Idea 2  | Understand the meanings of operations and how they relate to each other, make reasonable estimates, and compute fluently. |
| Number Operations | • Develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems |
Number Trains

This problem gives you the chance to:
• solve problems about factors and multiples in a toy context

Sally is making number trains.
Each train has an engine and two boxcars.
Each engine and each boxcar has a number.

Each engine can pull two boxcars **only** when the product of the two boxcar numbers is equal to the engine number.

For example, engine number 12 can pull boxcars with numbers 1 and 12 because \(12 = 1 \times 12\).

Put different pairs of numbers into the empty boxcars so that the engines can pull them. The first number train has been done for you.

1.

\[
\begin{align*}
12 &= 1 \times 12 \\
12 &= 1 \times 12 \\
12 &= 1 \times 12
\end{align*}
\]

2.

\[
\begin{align*}
18 &= 1 \times 18 \\
18 &= 1 \times 18 \\
18 &= 1 \times 18
\end{align*}
\]
3. 

4. Two boxcars have the numbers 2 and 18.

What is the number of the engine that can pull these two boxcars?

List the other 4 pairs of boxcar numbers that can be pulled by this engine.

(a) ___________ \times ___________

(b) ___________ \times ___________

(c) ___________ \times ___________

(d) ___________ \times ___________
## Number Trains

### Test 4 Form A Rubric

The core elements of performance required by this task are:
- solve problems about factors and multiples in a toy context

Based on these, credit for specific aspects of performance should be assigned as follows:

<table>
<thead>
<tr>
<th></th>
<th>Points</th>
<th>Section Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answers as:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2, 6)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(3, 4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Both answers correct: 1 point</td>
<td>1</td>
</tr>
<tr>
<td>2.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answers as:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1, 18)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>(2, 9)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3, 6)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>All three correct answers: 2 points</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Partial credit:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Two or one correct answers: 1 point</td>
<td>(1)</td>
</tr>
<tr>
<td>3.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answers as:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1, 24)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>(2, 12)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3, 8)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4, 6)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>All four correct answers: 2 points</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Partial credit:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Three or two correct answers: 1 point</td>
<td>(1)</td>
</tr>
<tr>
<td>4.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer as:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Engine Number = 36</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answers as:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1, 36)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3, 12)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4, 9)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6, 6)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>All four correct answers: 2 points</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Partial credit:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Three or two correct answers: 1 point</td>
<td>(1)</td>
</tr>
</tbody>
</table>

|   | Total Points | 8 |

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Page 2

Fourth Grade- 2003

pg. 16
Looking at Student Work - Number Trains

Most students did very well on this problem. They demonstrated fluency with multiplication facts and could interpret the rules and constraints of the problem. Student A fills in the boxcar in part 4 and shows the multiplication to find the answer. The student appears to be able to find the alternate solutions mentally.

Student A

3.  

\[
\begin{array}{c}
24 = 6 \times 4 \checkmark \\
36 = 3 \times 12
\end{array}
\]

4. Two boxcars have the numbers 2 and 18.

\[
\begin{array}{c}
36 = 2 \times 18)
\end{array}
\]

What is the number of the engine that can pull these two boxcars?

\[
\begin{array}{c}
36
\end{array}
\]

List the other 4 pairs of boxcar numbers that can be pulled by this engine.

(a) \[5 \times 5 \checkmark \]

(b) \[1 \times 26 \checkmark \]

(c) \[3 \times 12 \checkmark \]

(d) \[4 \times 9 \checkmark \]
Student B repeats 18 x 2 in part 4, but attempts to find the missing fact of 12 x 3 = 36. Student B seems to use a combination of drawing, counting, and repeated addition to solve for the larger products.

4. Two boxcars have the numbers 2 and 18.

What is the number of the engine that can pull these two boxcars? 36

List the other 4 pairs of boxcar numbers that can be pulled by this engine.

(a) 1 \times 36
(b) 18 \times 2
(c) 6 \times 6
(d) 4 \times 9
Student C correctly finds all the missing combinations in parts 1 and 2 of the task. As the product gets larger, the student has difficulty finding all the combinations. When Student C gets to part 4, she just chooses products for which she knows the factors.

Student C

3.

![Three boxcars with products]

4. Two boxcars have the numbers 2 and 18.

![A boxcar with products]

What is the number of the engine that can pull these two boxcars?

![Engine number 3/6]

List the other 4 pairs of boxcar numbers that can be pulled by this engine.

(a) \(3/6 \times 18\)  
(b) \(24 \times 8\)  
(c) \(40 \times 5\)  
(d) \(8 \times 1\)

Fourth Grade
Student D shows the work of a typical student, who knows several combinations to get the same product but is not proficient enough with multiplication facts to make an exhaustive list.

1. 

2. 

3. 

4. Two boxcars have the numbers 2 and 18.

What is the number of the engine that can pull these two boxcars?

List the other 4 pairs of boxcar numbers that can be pulled by this engine.

(a) _______ × _______ 

(b) _______ × _______ 

(c) _______ × _______ 

(d) _______ × _______ 

Fourth Grade- 2003
Student E sees a pattern of using the product as the second factor, because the example is $1 \times 12 = 12$. The student demonstrates no understanding of multiplication.

1. 
   \[ 12 = 1 \times 12 \]

2. 
   \[ 18 = 4 \times 18 \]

   \[ 12 = 3 \times 12 \]

   \[ 18 = 2 \times 16 \]

   \[ 12 = 2 \times 12 \]

   \[ 18 = 1 \times 16 \]

Teacher Notes:
The maximum score available on this task is 8 points. The cut score for a level 3 response is 4 points.

Most students (about 90%) could find the factors for 18 and 24 as well as multiply 2 x 18 to get 36. Many students (about 71%) could find all the factor pairs for 12, 18, and 24. They had difficulty finding all the factor pairs for 36. Almost 40% of the students could meet all the demands of the task. Only 1% of the students scored no points on this task.
## Number Trains

<table>
<thead>
<tr>
<th>Points</th>
<th>Understandings</th>
<th>Misunderstandings</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Students with this score attempted the problem.</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Students with this score understood that they were looking for factor pairs. They usually could get some of the combinations to make a product of 18 and 12.</td>
<td>Most students repeated some of the combinations. In part one 12% of all students forgot 3x4. In part 2, 14% forgot 1x18. 10% forgot 3x6. 7% omitted 9x2.</td>
</tr>
<tr>
<td>4</td>
<td>Students with this score could do some of the factor pairs for 18, 24, and 36.</td>
<td>In part 3 16% of all the students forgot either 2x12 or 24x1. 13% omitted 8x3. Only 6% left out 4x6.</td>
</tr>
<tr>
<td>6</td>
<td>Students with this score could find all the factor pairs for 12, 18, and 24.</td>
<td>13% of the students gave factor pairs not equal to 36 in part 4. 18% omitted 3x12. About 12% omitted either 1x36 or 6x6.</td>
</tr>
<tr>
<td>8</td>
<td>Students could find all the factor pairs to make various products.</td>
<td></td>
</tr>
</tbody>
</table>

### Teacher Notes:

Based on teacher observations, this is what fourth grade students seem to know and be able to do:
- Find factor pairs to equal various products.
- Multiply 2x18.

Areas of difficulty for fourth graders, fourth grade students struggled with:
- Making an exhaustive list of all the factor pairs for a given product.
- Making an organized list.
- Remembering that 1x itself is a factor pair for any number, except 0.
Questions for Reflection on Number Trains:

- Did most of your students understand that they were being asked to find factor pairs?
- Do you think most of your students are fluent with their multiplication facts?
- What activities do you use to help students learn multiplication facts?
- Do you suggest games and activities for parents to use at home to help build fluency in number facts?
- How do you work on building students problem-solving strategies? What resources do you use for good problems? What are some problems students have done to help them learn to make organized lists?

Teacher Notes:

________________________________________________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________

Implications for Instruction:

Students need practice with interpreting problems and their constraints. At this grade level, students need to be fluent with, and recognize, multiple factors of composite numbers. Some students had difficulty recognizing when they had already used a given solution. Students need to be able to develop the logical reasoning to check for differences and compare answers. Students need more experience with the problem solving strategy: make an organized list. This strategy will help them to notice or eliminate duplicates. The Lane County Problem Solving in Mathematics by Dale Seymour Publications is a good resource for teaching problem solving strategies.

Teacher Notes:

________________________________________________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________
4th grade  Task 3  Hexagon Desks

<table>
<thead>
<tr>
<th>Student Task</th>
<th>Find, extend, and analyze a number pattern involving chairs placed around hexagonal desks. Plot and use a graph to display the number pattern information.</th>
</tr>
</thead>
</table>
| Core Idea 3 Patterns, Functions, and Algebra | Understand patterns and use mathematical models to represent and to understand qualitative and quantitative relationships.  
  - Represent and analyze patterns and functions using words, tables, and graphs  
  - Find the results of a rule for a specific value  
  - Use concrete, pictorial, and verbal representations to solve problems involving unknowns. |
Hexagon Desks

This problem gives you the chance to:

• find and extend a number pattern
• plot and use a graph

Sarah finds how many students can sit around a row of desks. The top surface of each desk is a hexagon, and the hexagons are arranged in rows of different shapes.

1 desk 6 students

2 desks 10 students

3 desks 14 students

4 desks

1. Complete Sarah’s table.

<table>
<thead>
<tr>
<th>Number of desks in a row</th>
<th>Number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>
2. On the grid, plot the results from the table you completed in question 1. The first two points have already been plotted for you.

3. Sarah says that 47 students can sit around a row of 11 desks. Without drawing the desks, explain how you know that Sarah is wrong.

_________________________________________________________________________
_________________________________________________________________________
_________________________________________________________________________

How many students can sit around a row of 11 desks? 10
Hexagon Desks

Test 4 Form A Rubric

The core elements of performance required by this task are:
• find and extend a number pattern
• plot and use a graph

Based on these, credit for specific aspects of performance should be assigned as follows:

1. Correctly completes the table:

<table>
<thead>
<tr>
<th>Number of desks in a row</th>
<th>Number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>22</td>
</tr>
<tr>
<td>6</td>
<td>26</td>
</tr>
</tbody>
</table>

All four correct values: 2 points

Partial credit:
Three or two correct values: 1 point

2. Correctly plots the four values from the student’s answer to question 1 on the grid:

Accept points in the correct square that are not on the horizontal grid lines.

3. Correctly continues table or graph.
   or
   States that 47 is not an even number.
   or
   Gives a correct alternative reason.
   Gives correct answer as:
   46 students

Total Points 10
Looking at Student Work - Hexagon Desks

In general students did very well on Hexagon Desks. Many 4th grade students showed a good understanding of algebraic thinking and functions. Student A expresses a rule in part 3 that could translate directly into a formula, $2(x-2) + 10$.

**Student A**

<table>
<thead>
<tr>
<th>Number of desks in a row</th>
<th>Number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>22</td>
</tr>
<tr>
<td>6</td>
<td>26</td>
</tr>
</tbody>
</table>

Sarah says that 47 students can sit around a row of 11 desks. Without drawing the desks, explain how you know that Sarah is wrong.

At each middle table there are 4 \( \checkmark \)

At each end table there are 5 \( \checkmark \)

There are 2 end tables and 9 middle tables so that = 46.

How many students can sit around a row of 11 desks? \( \boxed{46} \)

Fourth Grade
Student B does a great job of explaining why adding a new table is gives four more seats each time. The student is paying attention to how the pattern grows, what elements are growing and how they fit together, which is the logic underlying a generalization or formula.

**Student B**

Without drawing the desks, explain how you know that Sarah is wrong.

I know that Sarah is wrong because the number of students sitting many number of desks all have to be even because when you add a ripple you're adding five and taking one off so you're actually adding four.

How many students can sit around a row of 11 desks? 46 students

Student C has a complete and simplified formula for this situation, but errs in trying to make her own notation. She knows that adding four every time is the same as multiplying by four and that in the first case there is an extra 2. Her rule is multiply by 4 and add 2 that equates to 4x+2. She cannot however put that rule clearly in the form of a number sentence or precise words. The graph has been made by adding four to get each point. The work for 46 is shown next to the writing for part 3.
Student D is part way to this same understanding. The student knows that the first one has an extra 2, but forgets to subtract out the 4 for desk 12. (The procedure should have been amount for 6 desks + the amount for 6 desks – 4 to get amount for 11 desks – 2 for the extra in the first desk.)

3. Sarah says that 47 students can sit around a row of 11 desks. Without drawing the desks, explain how you know that Sarah is wrong.

\[
\text{Since } 6 \text{ desks } + \text{ I multiplied } (2 \times 6) - 2 = 5 
\]
Student E forgets about the 2 extra for the first desk.

Student E

3. Sarah says that 47 students can sit around a row of 11 desks.
   Without drawing the desks, explain how you know that Sarah is wrong.

   Sarah is wrong because on the page before 5 desks is 22, and 6 desks is 26.
   6+5 = 11 desks  22 + 26 = 48, 48 is 1 more than what she thought so she's wrong.

How many students can sit around a row of 11 desks?

48 students
Student F makes some errors on the table, adding 4’s and then adding 6’s. But the student’s procedure in 3 would have been correct if the answer for desk 6 in the table had been correct. If you double the amount for the sixth term and subtract out the first term (6) which includes the constant, you will have the amount for 11 (6+5).
3. Sarah says that 47 students can sit around a row of 11 desks. Without drawing the desks, explain how you know that Sarah is wrong.

Sarah is wrong because on the chart 6 is odd and 11 would be more and it is odd, not even.

How many students can sit around a row of 11 desks? 54
Many students seem to have a firm belief that the points in a graph will lie in a straight line. Student G has the wrong numbers in the table, but shows a graph that matches the correct values for this function. The student does not use the graph to help answer the question, but relies on a drawing and counting strategy. This strategy is not successful because the numbers are large and cumbersome.

Complete Sarah's table.

<table>
<thead>
<tr>
<th>Number of desks in a row</th>
<th>Number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>14 ✓</td>
</tr>
<tr>
<td>4</td>
<td>18 ✓</td>
</tr>
<tr>
<td>5</td>
<td>20 ×</td>
</tr>
<tr>
<td>6</td>
<td>28 ×</td>
</tr>
</tbody>
</table>

3. Sarah says that 47 students can sit around a row of 11 desks. Without drawing the desks, explain how you know that Sarah is wrong.

Sarah is wrong because there could not be 47 students in 11 rows. She has added 3 more and that’s not the right answer. The answer is 44 students.

How many students can sit around a row of 11 desks? 44 students
Other students think that the points on the graph must be in a straight line and the straight line must start at 0. See the graphs of Student I and Student J.

Student I

![Graph from Student I]

Student J

![Graph from Student J]

3. Sarah says that 47 students can sit around a row of 11 desks. Without drawing the desks, explain how you know that Sarah is wrong.

She did not start from 0.
Students did not use their graphs as sense-making tools. Student K saw an incorrect pattern of adding 5’s in the table. Because the student thought points should be in a straight line, his graph actually fits the function of desks in a row. Had he used the graph in part 3, Student K could have gotten the correct answer.

**Student K**

<table>
<thead>
<tr>
<th>Number of desks in a row</th>
<th>Number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
</tr>
</tbody>
</table>

Fourth Grade
The maximum score available on this task is 10 points. The cut score for a level 3 response is 5 points.

Most students (about 85%) could find and continue the pattern correctly for at least 2 points and earn partial credit for graphing. Many students (about 70%) could find all the numbers in the table and graph them correctly. 30% of the students could meet all the demands of the task including making a mathematical justification for why 47 students could not fit at 11 desks. About 3% of the students scored no points on this task.
Hexagon Desks

<table>
<thead>
<tr>
<th>Points</th>
<th>Understandings</th>
<th>Misunderstandings</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Most students with this score tried the problem.</td>
<td>Students made errors in the pattern, usually adding 5 or 6 each time instead of 4. They could not graph their points correctly. They may have extended the line correctly from the given 2 pts on the grid. They did not make a connection between the values in the table and the points in the graph.</td>
</tr>
<tr>
<td>2</td>
<td>Students could correctly continue the table.</td>
<td>Some students thought that the points should line up with zero, so they graphed a pattern that did not match the numbers in their table. Many students at this score point did not attempt the graph.</td>
</tr>
<tr>
<td>5</td>
<td>Students could fill in 2 or 3 points in the table and make a correct graph for their points.</td>
<td>Most students tried to justify why 47 was not possible because it was not a multiple of 4 (10%).</td>
</tr>
<tr>
<td>6</td>
<td>Students could continue the table and graph their points correctly.</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Students could find the correct number of students to sit at 11 desks or explain why 47 was incorrect. Of the students who could explain why 47 was incorrect- most used an argument about even numbers, referred to their graph, or added 4’s.</td>
<td>Students had difficulty with justification, they made arguments like “because 46 students fit” instead of process or logic of how they found the 46.</td>
</tr>
<tr>
<td>10</td>
<td>Students could continue a pattern in a table, graph the points in a table, use the pattern to justify why 47 students could not sit at an arrangement of 11 desks, and find the correct number of students who could sit at 11 desks.</td>
<td></td>
</tr>
</tbody>
</table>

Teacher Notes:
Based on teacher observations, this is what fourth grade students seemed to know and be able to do:

- Continue a pattern in a table
- Graph points on a grid
- Understand the relationship between repeated addition and multiplication

Areas of difficulty for fourth graders, fourth grade students struggled with:

- Relating the numbers in the tables to the points on the grid (many students seemed to be relying on a rule about putting points in a straight line and sometimes thinking the straight line needed to go through zero)
- Connecting the table and the graph as having the same information and being related rather than two separate tasks
- Making sense of the graph, only 14% used the graph to help them make a mathematical argument in part 3
- Making a mathematical argument to explain why 47 students can’t fit at the desks

Questions for Reflection on Hexagon Desks:

- Could most of your students continue the pattern in the table? How many tried to add on some number other than 4 to make their pattern? What do you think they were seeing or paying attention to?
- How many students had graphs that matched the points in their table? When students made an error in the table, did their graphs still go in a straight line or did they graph the data points in the table?
- What types of experiences have students had this year with graphing? Have they inferred things about graphing that aren’t true for all problems, like the lines start at zero? What are some interesting problems you can give students to overcome these misconceptions?
- Students need to see the purpose for making the graph, to use it to help solve problems and to go beyond what they know. What kinds of questions will help promote this kind of thinking by students?
- How many of your students could make rules in part 3 that really matched the situation of the desks, describing what is changing and what is staying the same?
- Did your students have difficulty placing the plotted points? Are they used to working with graphs with different vertical scales?

Teacher Notes:
Instructional Implications:
Students at this grade level need to experience analyzing growing patterns. They also need to be able to correctly plot the information from a table onto a graph with pre-established scales. While many students seemed to know that the plots for growing patterns should be on a straight line, the ordered numbers on their graphs did not match the numbers in their tables. Students need more experiences with linear relationships that are not proportional: i.e., the numbers are not multiples. For example, in this problem the number of students increases by four with each additional desk, but the total number of students is not a multiple of four, so this is not a proportional relationship. Students also need to be able to work backward from a solution. Students might do this by extending the table, extending their graph, or by using repeated addition. An important part of patterning is to recognize the relationship between the two variables. Learning to think and describe what is changing and what is staying the same helps students to find rules that will work for all cases.

Teacher Notes:
### 4th grade  Task 4   Flower Arranging

<table>
<thead>
<tr>
<th>Student Task</th>
<th>Divide a number into parts in order to satisfy given conditions with regard to arranging flowers.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core Idea 1</td>
<td><strong>Number Properties</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Understand numbers, ways of representing numbers, relationships among numbers, and number systems.</strong></td>
</tr>
<tr>
<td></td>
<td>• Develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers (3rd grade)</td>
</tr>
<tr>
<td></td>
<td>• Develop understanding of the relative magnitude of whole numbers and the concepts of sequence, quantity, and the relative positions of numbers.</td>
</tr>
</tbody>
</table>
Flower Arranging

This problem gives you the chance to:
- figure out how many flowers are in an arrangement
- divide a number into parts in order to satisfy given conditions

Tim’s grandmother loves flower arranging.

She always uses an odd number of each flower in each arrangement.

1. The arrangement she is making today has tulips, roses, and lilies.
   Tim’s grandmother uses 9 flowers in all.

   There are more tulips in the arrangement than there are roses,
   and there are more roses than lilies.

   How many tulips are there?  
                             ____________

   How many roses are there?  
                             ____________

   How many lilies are there?  
                             ____________

   Explain how you figured this out.

                             ________________________________
                             ________________________________
                             ________________________________
2. The next arrangement Tim’s grandmother makes also has tulips, roses, and lilies. She uses 11 flowers in all.

**As before, she always uses an odd number of each flower in each arrangement.**

There are more tulips in the arrangement than there are roses, and there are more roses than lilies.

How many tulips are there? ________________

How many roses are there? ________________

How many lilies are there? ________________

Explain how you figured this out.

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________
Flower Arranging

The core elements of performance required by this task are:
• figure out how many flowers are in an arrangement
• divide a number into parts in order to satisfy given conditions

Based on these, credit for specific aspects of performance should be assigned as follows:

<table>
<thead>
<tr>
<th>Points</th>
<th>Section Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Gives correct answers as:</td>
<td></td>
</tr>
<tr>
<td>5 tulips</td>
<td></td>
</tr>
<tr>
<td>3 roses</td>
<td></td>
</tr>
<tr>
<td>1 lily</td>
<td></td>
</tr>
<tr>
<td>All three correct answers: 2 points</td>
<td>2</td>
</tr>
<tr>
<td>Partial credit:</td>
<td></td>
</tr>
<tr>
<td>Gives correct numbers of flowers but in incorrect order, or</td>
<td>(1)</td>
</tr>
<tr>
<td>Gives correct order with two even numbers and the total is 9, or</td>
<td>(1)</td>
</tr>
<tr>
<td>Gives explanation such as:</td>
<td></td>
</tr>
<tr>
<td>1, 3, and 5 are the only 3 different odd numbers that add up to 9.</td>
<td>1</td>
</tr>
<tr>
<td>2. Gives correct answers as:</td>
<td></td>
</tr>
<tr>
<td>7 tulips</td>
<td></td>
</tr>
<tr>
<td>3 roses</td>
<td></td>
</tr>
<tr>
<td>1 lily</td>
<td></td>
</tr>
<tr>
<td>All three correct answers: 2 points</td>
<td>2</td>
</tr>
<tr>
<td>Partial credit:</td>
<td></td>
</tr>
<tr>
<td>Gives correct numbers of flowers but in incorrect order, or</td>
<td>(1)</td>
</tr>
<tr>
<td>Gives correct order with two even numbers and the total is 11, or</td>
<td>(1)</td>
</tr>
<tr>
<td>Gives explanation such as:</td>
<td></td>
</tr>
<tr>
<td>1, 3, and 7 are the only 3 different odd numbers that add up to 11.</td>
<td>1</td>
</tr>
<tr>
<td>Total Points</td>
<td>6</td>
</tr>
</tbody>
</table>
Looking at Student Work – Flower Arranging

This task requires students to pay attention to three different constraints: total number of flowers, odd numbers, and order of flowers. Student A shows how he checked his answer to make sure they fit the constraints. In part 2 the student also explains the logic used to transform the answer in part 1 to the answer in part 2. Look carefully at the depth of logic used in this process.

Student A

How many tulips are there? 5
How many roses are there? 3
How many lilies are there? 1

Explain how you figured this out.

5, 3, 1 are all odd numbers and equal 9.

How many tulips are there? 7
How many roses are there? 3
How many lilies are there? 1

Explain how you figured this out.

I just added 2 more tulips to my other answer (page 6) because there is two more flower being added to my sum. Because there has to be more tulips than roses and roses than lilies, I had to add the two to the tulips so the roses or lilies wouldn't be fixed.
Student B
and there are more roses than lilies.

How many tulips are there?  five
How many roses are there?  three
How many lilies are there?  one

Explain how you figured this out.
I realized there were more tulips than roses, and there were more roses than lilies. Plus I knew there were no even amounts. That gave me the answer.

How many tulips are there?  seven
How many roses are there?  three
How many lilies are there?  one

Explain how you figured this out.
I just added two to the tulips because 9 + 2 = 11.
Student C uses a problem solving strategy of making an organized list and eliminating all other possibilities to arrive at the correct answer.

Student C

There are more tulips in the arrangement than there are roses, and there are more roses than lilies.

How many tulips are there? $\frac{5}{\checkmark}$

How many roses are there? $\frac{3}{\checkmark}$

How many lilies are there? $\frac{1}{\checkmark}$

Explain how you figured this out.

I know there can't be 0, 2, 4, or 6 of any flower so there has to be 5 tulips, 3 roses, and 1 lily.

It was difficult for students to remember all 3 constraints at the same time. Student D has a list of questions to check off to prove her answer is correct. However, she seems to think that the total of roses and lilies must be less than the tulips rather than three different decreasing numbers.

Student D

How many tulips are there? $\frac{9}{\checkmark}$

How many roses are there? $\frac{1}{\checkmark}$

How many lilies are there? $\frac{1}{\checkmark}$

Explain how you figured this out.

First I tried 5 tulips, 3 roses, and 3 lilies. I asked myself, does it equal 11? Yes, are they all odd numbers? Yes. Are the roses + the lilies less than the tulips' amount? No, so I know it was wrong. Then I tried 9 tulips, 1 rose, and 1 lily. I asked myself the same questions and all the answers were yes so I knew it was right.
Some students focused their explanations on the process or calculations they did to get their answers instead of how they made sure the answers matched the constraints. Student E does a nice job of describing a problem solving procedure giving each step, but the explanation doesn’t address the issue of order or odd numbers.

**Student E**

There are more tulips in the arrangement than there are roses, and there are more roses than lilies.

- How many tulips are there? 5
- How many roses are there? 3
- How many lilies are there? 1

Explain how you figured this out.

I thought that 5 + 3, but since there are three answers I used the information and got this.
Student F relates the three groups equals nine to a multiplication strategy, then ignores the differences in quantity for the flower groups. When faced with a prime number in part 2, the multiplication strategy won’t work. Student F then uses a drawing strategy, which allowed her to think more clearly about the constraint of relative size, but does not help with the constraint for odd numbers.

**Student F**

and there are more roses than lilies.

How many tulips are there?  
3 tulips \( \times \) \( \times \)

How many roses are there?  
3 roses \( \times \)

How many lilies are there?  
3 lilies \( \times \) \( \times \)

Explain how you figured this out.

I figured this out by using multiplication. I know there are 9 flowers and 3 different kinds of flowers so I know \( 3 \times 3 = 9 \). So I tried 3 flowers per \( \times \) kind.

<table>
<thead>
<tr>
<th>Tulips</th>
<th>5 tulips ( \checkmark ) ( \checkmark )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roses</td>
<td>4 roses ( \times ) ( \times )</td>
</tr>
<tr>
<td>Lilies</td>
<td>2 lilies ( \times ) ( \times ) ( 1 )</td>
</tr>
</tbody>
</table>

Explain how you figured this out.

I drew a picture and tried the least amount first and got 2. Next I did the greatest amount and did 5. Then I used the rest. I know that 5 was the greatest and 2 was the least.
Student G has rewritten the clues to help think and organize the information. Then the student has made an organized list of all the combinations that will equal the given sum and used this list to check against the constraints and find the correct solution.

Tim's grandmother loves flower arranging.

She always uses an odd number of each flower in each arrangement.

1. The arrangement she is making today has tulips, roses, and lilies.
   Tim's grandmother uses 9 flowers in all.

   There are more tulips in the arrangement than there are roses, and there are more roses than lilies.

   How many tulips are there? 5
   How many roses are there? 3
   How many lilies are there? 1

   Explain how you figured this out.
   I wrote the possible answers that add up to 9 and then I chose the one that had only odd numbers.

   I wrote the possible answers that equals 9 then I chose the one that had only odd numbers and different.

   6
The most typical error pattern is for students to use a combination of even and odd numbers. See the work of Student H.

Student H
more roses than lilies.

How many tulips are there?

How many roses are there?

How many lilies are there?

Explain how you figured this out.

I drew 11 lines and circled 5, 4, and 2 because that makes 11.
The maximum score available on this task is 6 points. The cut score for a level 3 response is 3 points.

Most students (about 82%) could find three numbers to equal the correct total. A little less than half the students (about 47%) could find three numbers for each part that matched all the constraints in the problem. About 28% of the students could also explain how the solution fit the constraints. About 18% of the students scored no points on this task. Most of these students attempted the problem.
Flower Arranging

<table>
<thead>
<tr>
<th>Points</th>
<th>Understandings</th>
<th>Misunderstandings</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>18% of all the students scored no points on this task. 82% of these students attempted the task. For the most part they could find the correct totals.</td>
<td>In order to get partial credit, students needed to have an answer that met two of the three constraints. Generally students ignored relative size and odd. The most common incorrect response was 3 tulips, 3 roses, and 3 lilies and a combination of 4,3 and 2 but given in an incorrect order.</td>
</tr>
<tr>
<td>1</td>
<td>Students could find a solution to part one using even numbers.</td>
<td>The most common incorrect solutions were 4,3,and 2 or 6,2, and 1.</td>
</tr>
<tr>
<td>2</td>
<td>Students could find a solution to part one and part two using even numbers.</td>
<td>The most common solutions for part 2 were 5,4, and 2 and 6,3, and 2. Another common error was for students to use one number for two types of flowers (e.g. 7+1+1 or 5+3+3).</td>
</tr>
<tr>
<td>4</td>
<td>Students could generally find a solution to part one and 2 that met all the criteria.</td>
<td>Students struggled with how to write an explanation. They often discussed the calculation procedures rather than the constraints of the task.</td>
</tr>
<tr>
<td>6</td>
<td>Students could find solutions and give explanations that address the constraints of the problem. 18% showed their addition. 18% addressed two of the three constraints. Only one out of 150 students talked about all three constraints.</td>
<td></td>
</tr>
</tbody>
</table>

Teacher Notes:
Based on teacher observations, this is what fourth grade students seemed to know and be able to do:

- Find 3 numbers to add to a given total
- Arrange numbers from large to small by correctly interpreting the meaning of the word “more”

Areas of difficulty for fourth graders, fourth grade students struggled with:

- Using only odd numbers to make a total
- Using 3 constraints in one problem
- Explaining how they used the constraints to check their solution

Questions for Reflection on Flower Arranging:

Sort your work into piles: those who get full credit, correct solutions to parts one and two with errors in explanations, those with partially correct solutions, and zeros. --

*What types of strategies did the students use who were successful?
  - Make an organized list
  - Show addition
  - Draw pictures or use fingers
  - Eliminate possibilities
  - Other

*Of the students with correct solutions, what kinds of explanations did you like the best? How can you use those examples with students to clarify what you value in an explanation?

*What types of errors did students make in their explanations? Which constraints did they forget to address?

*How many of your students used even numbers?

*How many used a number twice or put the numbers in an incorrect order?

*What types of logic problems have kids worked with in class this year? What are some good resources for building this type of reasoning?

Teacher Notes:

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Instructional Implications:

Students need experience solving problems and satisfying multiple constraints. Students needed to find three different odd numbers to add to a particular sum. The students were also given information regarding the order of the numbers, which type of flower was the greatest, etc. Solving number puzzles with different clues might be one way to help students develop their logical reasoning, as well as computational fluency. Students also need to be able to explain mathematical reasoning and thinking in a way that is clear to others. Learning techniques like making an organized list and eliminating possibilities seemed to be helpful for some students. Students need rich
tasks with lots of information and opportunities to struggle and organize that information in ways that make sense to them. Giving students opportunities to compare strategies with others helps them clarify why a strategy is useful and what makes a strategy useful.

### 4th grade Task 5 Traveling To School

<table>
<thead>
<tr>
<th>Student Task</th>
<th>Interpret a table of travel times. Use the information from this table to solve problems.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core Idea 5</td>
<td>Collect, organize, represent and interpret numerical and categorical data, and clearly communicate their findings.</td>
</tr>
<tr>
<td>Data Analysis</td>
<td>• Interpret data to answer questions about a situation</td>
</tr>
<tr>
<td>Core Idea 2</td>
<td>Reason about and solve problem situations that involve more than one operation in multi-step problems.</td>
</tr>
<tr>
<td>Number Operations</td>
<td></td>
</tr>
<tr>
<td>Core Idea 1</td>
<td>Understand numbers, ways of representing numbers, relationships among numbers, and number systems.</td>
</tr>
<tr>
<td>Number Properties</td>
<td>• Use models, benchmarks, and equivalent forms to judge the size of friendly fractions</td>
</tr>
</tbody>
</table>
Traveling to School

This problem gives you the chance to:
• interpret a table of times and solve problems

Dave, Angela, Mark, and Carrie are all brothers and sisters.

They all go to the same school.

They leave home at the same time, but they travel in different ways.

Look at the chart to see how they travel and how long they take.

<table>
<thead>
<tr>
<th>Student</th>
<th>Way of traveling</th>
<th>Time taken in minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dave</td>
<td>walks</td>
<td>30</td>
</tr>
<tr>
<td>Angela</td>
<td>bicycles</td>
<td>15</td>
</tr>
<tr>
<td>Mark</td>
<td>by car</td>
<td>6</td>
</tr>
<tr>
<td>Carrie</td>
<td>by bus</td>
<td>9</td>
</tr>
</tbody>
</table>

1. Who arrives at school first? ________________

Who arrives at school last? ________________

2. How much longer does it take Angela than Carrie to get to school?
_______________ minutes

3. Carrie decides to travel by car instead of by bus.
How much time does she save? ________________ minutes
4. Dave walks halfway to school. He remembers that he forgot his lunch. He walks back home to get it. He then leaves home to walk to school once more. How long does it take Dave to walk to school today? 

__________________ minutes

Show your work.

5. Tomorrow Dave will run to school. Estimate how long it will take him to do this. __________________ minutes

Explain your answer.

________________________________________________________________________________________

________________________________________________________________________________________
## Traveling to School

The core elements of performance required by this task are:

- interpret a table of times and solve problems

Based on these, credit for specific aspects of performance should be assigned as follows:

<table>
<thead>
<tr>
<th></th>
<th>Points</th>
<th>Session Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Gives correct answers as:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mark</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dave</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Both answers correct: 1 point</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>2. Gives correct answer as:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 minutes</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>3. Gives correct answer as:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 minutes</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>4. Gives correct answer as:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60 minutes (or 1 hour)</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Shows work such as:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 minutes takes him halfway to school</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>15 minutes takes him back home</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>30 minutes is the time he takes to walk to school</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>or</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 + 15 + 30 = 60 minutes (or 1 hour)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Accept estimates between 15 and 25 minutes.</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Gives reasons based on the data such as:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>He would be quicker than if he walked, but slower than if he cycled.</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>or</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gives an explanation that includes reference to speed, rate, or time.</td>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

**Total Points** 8
Looking at Student Work – Traveling to School

Students in the collaborative are improving in their ability to do comparison subtraction. Student A does a nice job of showing the comparison subtraction for part 2 and 3 of this task. Student A also gives a very detailed explanation for finding the total minutes in part 4.

Student A.

<table>
<thead>
<tr>
<th>Student</th>
<th>Way of travelling</th>
<th>Time taken in min</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Dave</td>
<td>walks</td>
<td>30</td>
</tr>
<tr>
<td>3rd Angela</td>
<td>bicycles</td>
<td>15</td>
</tr>
<tr>
<td>5th Mark</td>
<td>by car</td>
<td>6</td>
</tr>
<tr>
<td>6th Carrie</td>
<td>by bus</td>
<td>9</td>
</tr>
</tbody>
</table>

1. Who arrives at school first?  
   Mark

   Who arrives at school last?  
   Dave

2. How much longer does it take Angela than Carrie to get to school?
   \[
   \frac{15}{2} - \frac{9}{3}
   \]

3. Carrie decides to travel by car instead of by bus. How much time does she save?
   \[
   \frac{15}{2} - \frac{9}{3}
   \]
Student A, part 2

How long does it take Dave to walk to school today?

Show your work.

60 ÷ 2 = 30 because he only went half way.

15 + 15 = 30 because he went back.

30 + 30 = 60 because he went to school from home.

5. Tomorrow Dave will run to school. Estimate how long it will take him to do this.

Explain your answer.

I got 15 minutes because if he

runs he can get there so he

will get at least half as fast.

Student B

How long does it take Dave to walk to school today?

Show your work.

half way 15

to school 30 minutes

5. Tomorrow Dave will run to school. Estimate how long it will take him to do this.

Explain your answer.

I think it is 15 minutes to get to school because when you run you go twice as fast so it would only take Dave 15 minutes.
A common misconception for students is to forget to count going halfway (15 minutes) once, but not count the going back to the house. Student C shows the thinking for getting the fifteen minutes. Student D attempts to make a diagram, but doesn’t interpret it correctly.

**Student C**

How long does it take Dave to walk to school today?

Show your work:

\[
\begin{align*}
15 & \quad \text{minutes} \\
\underline{+ 30} & \quad 45 \\
\underline{+ 15} & \quad 60 \\
-30 & \quad 30 \\
\underline{+ 15} & \quad 45
\end{align*}
\]

**Student D**

How long does it take Dave to walk to school today?

Show your work:

\[
\begin{align*}
30 \quad \text{minutes} \\
\underline{+ 15} & \quad 45 \\
\frac{15}{5} & \quad \frac{1}{3}
\end{align*}
\]
Other students seem to think of halfway, not in a mathematical way, but as a description of going part of the way to school. See the work of Student E and Student F.

Student E

How long does it take Dave to walk to school today?

Show your work.

Show your work.

5. Tomorrow Dave will run to school. Estimate how long it will take him to do this.

Explain your answer.

I think it is twenty minutes \( \checkmark \) because he's going running. If he goes, walking it takes 30, so I think I am right.

Student F

How long does it take Dave to walk to school today?

Show your work.

\[
\text{half} = 5 \quad + \quad \frac{30}{5} \quad 30 \text{ minutes} = 55
\]
Finally, some students understood the 3 legs of the journey, but did not take into account the halfway of two of the legs. See the work of Student G, who has a good diagram of the action, but the wrong distances.

Student G

How long does it take Dave to walk to school today?

40 \text{ minutes}

Show your work.

\[ 30 \rightarrow 30 \times x \]

5. Tomorrow Dave will run to school. Estimate how long it will take him to do this. \( \text{About } \frac{3}{4} \text{ minutes} \)

Explain your answer.

I think it will take him about \( \frac{3}{4} \) min, because he would be faster at running than walking.

Teacher Notes:

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________
Traveling to School
Mean: 5.15, S.D.: 2.29

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>427</td>
</tr>
<tr>
<td>1</td>
<td>318</td>
</tr>
<tr>
<td>2</td>
<td>472</td>
</tr>
<tr>
<td>3</td>
<td>886</td>
</tr>
<tr>
<td>4</td>
<td>1470</td>
</tr>
<tr>
<td>5</td>
<td>1809</td>
</tr>
<tr>
<td>6</td>
<td>720</td>
</tr>
<tr>
<td>7</td>
<td>1168</td>
</tr>
<tr>
<td>8</td>
<td>2129</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Score</th>
<th>% ≤ 4.5</th>
<th>% &gt; = 7.9</th>
<th>% &gt; = 12.9</th>
<th>% &gt; = 22.4</th>
<th>% &gt; = 38.0</th>
<th>% &gt; = 57.3</th>
<th>% &gt; = 64.9</th>
<th>% &gt; = 77.3</th>
<th>% &gt; = 100.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.5%</td>
<td>7.9%</td>
<td>12.9%</td>
<td>22.4%</td>
<td>38.0%</td>
<td>57.3%</td>
<td>64.9%</td>
<td>77.3%</td>
<td>100.0%</td>
</tr>
<tr>
<td>1</td>
<td>100.0%</td>
<td>95.5%</td>
<td>92.1%</td>
<td>87.1%</td>
<td>77.6%</td>
<td>62.0%</td>
<td>42.7%</td>
<td>35.1%</td>
<td>62.0%</td>
</tr>
</tbody>
</table>

The maximum score available on this task is 8 points.
The cut score for a level 3 response is 4 points.

Most students (about 87%) could answer questions about a table and do comparison subtraction. Many students (about 77%) could answer questions about the graph, do comparison subtraction, and estimate the time to run to school. More than half the students (62%) could answer questions about the table, find the time it took to travel partway to school, return home, and then make it to school, and make an estimate and explain the reasoning behind it for the time it would take to run to school. About 22% of the students could meet all the demands of the task. About 4.5% of the students scored no points on this task.
## Traveling to School

<table>
<thead>
<tr>
<th>Points</th>
<th>Understandings</th>
<th>Misunderstandings</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>About half the students with this score tried the problem.</td>
<td>Students who attempted the problem and received this score did not make sense of number operation or comparison subtraction. They added to compare. They also were more likely to be students with reading difficulties. They might put down words, when asked a question requiring a number and vice versa.</td>
</tr>
<tr>
<td>1</td>
<td>Students with this score could identify which student arrived first and last to school.</td>
<td>Students could not do comparison subtraction.</td>
</tr>
<tr>
<td>3</td>
<td>Students could answer simple questions from a table, interpret the inverse relationship between time to travel and time of arrival, and do comparison subtraction.</td>
<td>Students had difficulty calculating the time for the boy who forgot his lunch. About 25% of the students could find the time for half way, but forgot to double that amount for the return home.</td>
</tr>
<tr>
<td>4</td>
<td>Students with this score could answer questions from a table, including comparison subtraction. They could also give an estimate for the time it would take to run to school within the acceptable range.</td>
<td>61% of the students could pick the number 15. But students had a very difficult time giving a good mathematical reason for their answer. They did not compare rates to values on the table. They gave answers like “I guessed” or “I divided it in half”, giving no rationale or relating it to a personal experience.</td>
</tr>
<tr>
<td>5</td>
<td>Students could answer questions from the table, give an estimate for running, and relate running to some other rate in the table.</td>
<td>Students could not find the time for the trip to get the forgotten lunch. More students are attempting to use diagrams to help them solve the problem, but they have difficulty interpreting their diagrams.</td>
</tr>
<tr>
<td>7</td>
<td>Students could not give an acceptable explanation for their estimate.</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Students could answer questions from the table, figure the length of trip for the forgotten lunch and explain how they solved it, give an estimate and provide a justification by relating it to some other rate in the problem.</td>
<td></td>
</tr>
</tbody>
</table>
Based on teacher observations, this is what fourth grade students seemed to know and be able to do:

- Answer questions from a table
- Comparison subtraction
- Estimate a reasonable rate

Areas of difficulty for fourth graders, fourth grade students struggled with:

- Giving a justification for an estimate by relating it to other information provided in the problem
- Making diagrams to track information in a word problem
- Finding the time for a trip with multiple stops

Questions for Reflection on Traveling to School:

- How many of your students could do the comparison subtraction in part 2 and 3 of the task?
- What strategies did your students use in part 4? Did they:

<table>
<thead>
<tr>
<th>Go straight to a number sentence?</th>
<th>Draw a picture or diagram?</th>
<th>Use diagram correctly?</th>
<th>Could they find a correct number for half of the trip?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- What opportunities have students had to learn problem solving strategies like draw a picture or diagram? What are some of your favorite problems?
- What types of explanations did students provide for their estimates in part 5? What opportunities do students have to justify their answers or explain their thinking in class? Do they have the opportunity to hear the explanations of others or see models of good explanations? In what ways do you help make this more explicit for students what is valued in an explanation or justification?

Teacher Notes:
**Instructional Implications:**
Students need practice reading and interpreting information from a variety of tables with different formats. They also need to be able to choose from a variety of problem-solving strategies to correctly solve a word problem. Students need experience with problems that require them to draw a picture or diagram. Students need more experience with analyzing and solving multi-step word problems with units of time, elapsed time, etc. Students also need frequent opportunities to make estimates and justify their estimates. Both the teacher and the students need to articulate what is valued in a good explanation. In this particular case, the estimate should have compared or showed a relationship between running and some other rate in the table. Good mathematicians find connections between different pieces of the information.

**Teacher Notes:**

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________