Blue Print for Course One - MAC
Alignment of Tasks to NCTM Content and Process Standards

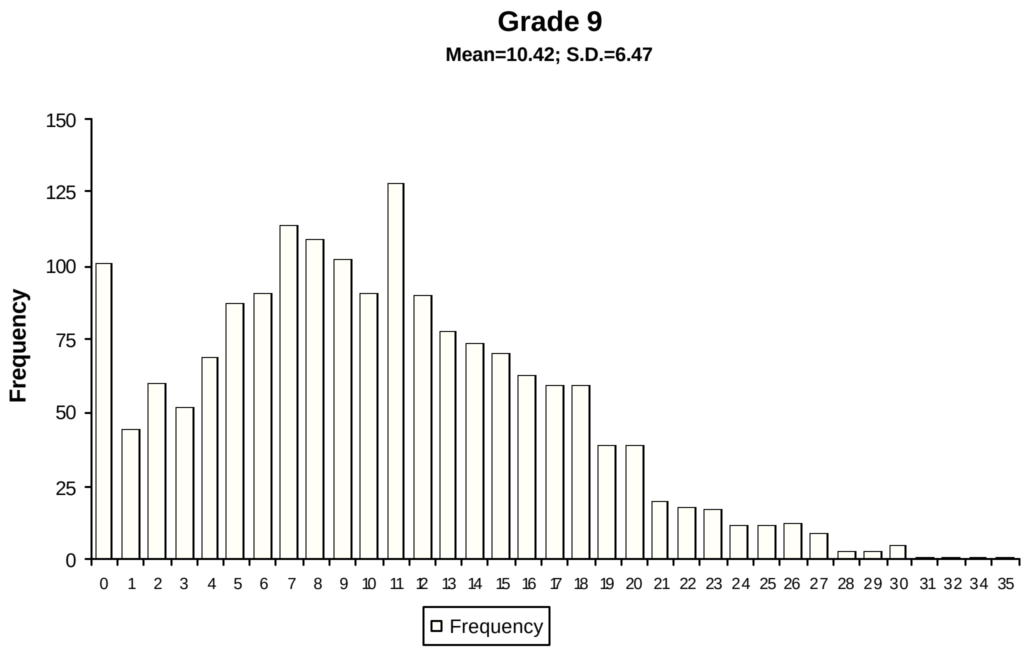
	Vacuum Cleaning	Snakes	Crisscross Numbers	Conference Tables	Number Towers
NUM			**S**		**S**
ALG		**S**	**P**	**P**	**P**
GEO	**P**			**S**	
MEAS	**S**				
DATA		**P**			
PS					
REAS	*	*	*	*	*
COMM			*	*	*
CONN		*		*	
REP	*		*	*	*

NUM = Number and Operations, ALG = Algebra, GEO = Geometry, MEAS = Measurement, DATA = Data Analysis and Probability, PS = Problem Solving, REAS = Reasoning and Proof, REP = Representation, CONN = connections, COMM = Communication

P denotes Primary NCTM Content Standard
S denotes Secondary NCTM Content Standard

- denotes NCTM Process Standard

Overall Frequency Distribution by Total Score



Level Frequency Distribution Chart and Frequency Distribution

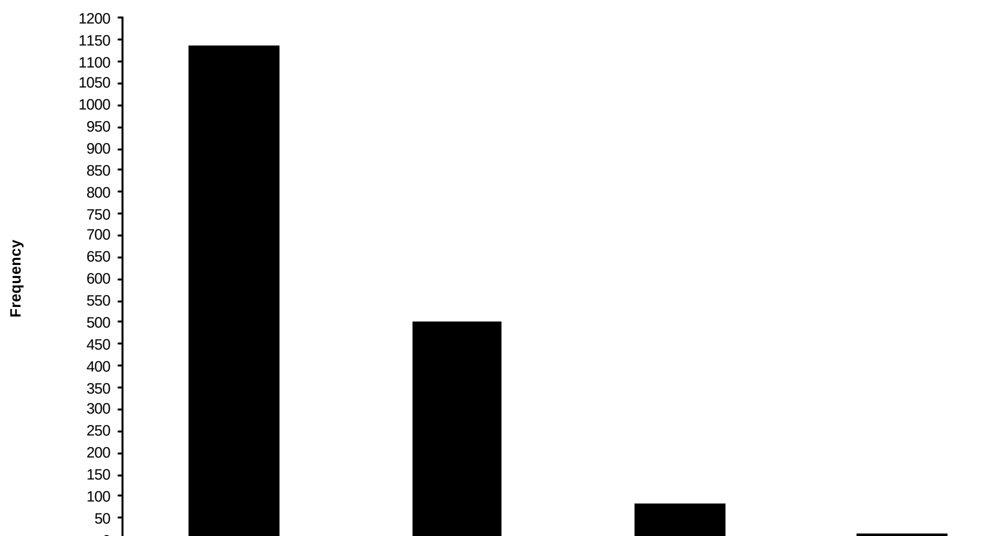
2003 - Numbers of students tested: Course One: 1736

Course One 1999 and 2001

Level	% at ('99)	% at least ('99)	% at ('01)	% at least ('01)
1	37%	100%	22%	100%
2	43%	63%	62%	78%
3	13%	20%	15%	16%
4	7%	7%	1%	1%

Course One 2002 - 2003

Level	% at ('02)	% at least ('02)	% at ('03)	% at least ('03)
1	18%	100%	66%	100%
2	61%	82%	29%	34%
3	20%	22%	5%	6%
4	2%	2%	1%	1%



	0-12 1 Minimal Success	13-21 2 Below Standard	22-28 3 At Standard	29-40 4 High Standard
Frequency	1139	501	84	12

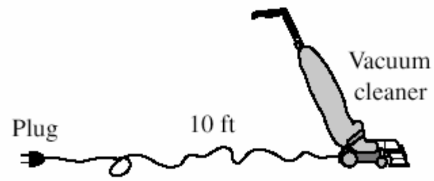
Vacuum Cleaning

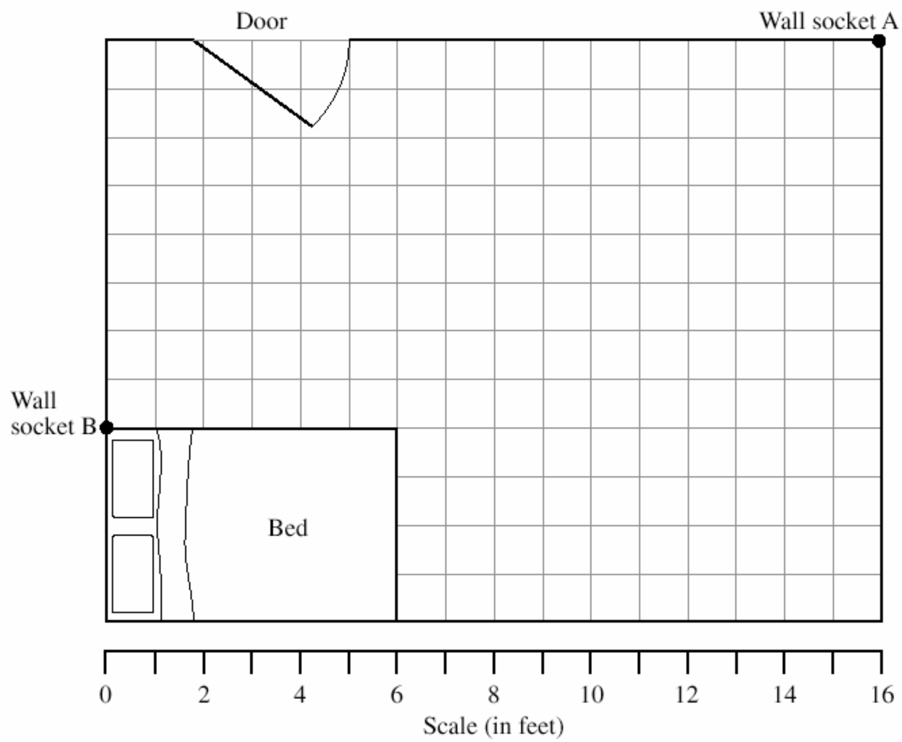
Student Task	Given a radius, make an arc to show area covered by a vacuum cleaner. Use information about furniture to draw areas accessible to vacuum cleaner and uncleaned area on a scaled diagram.
Core Idea 4 Geometry/Meas.	**Understand measurable attributes of objects; develop mathematical arguments about geometric relationships.** Solve problems that involve measurement units and scale factors. Use geometric models to gain insights into, and answer questions in, other areas of mathematics.
Core Idea 2 Mathematical Reasoning	Show mathematical reasoning in a variety of ways including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models. Draw reasonable conclusions about a situation being modeled.

Vacuum Cleaning

This problem gives you the chance to:
- draw and identify accessible regions in a practical context

José wants to vacuum his bedroom floor.





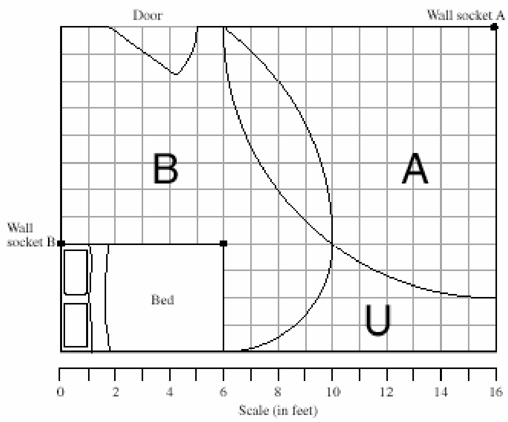
The vacuum cleaner has a cord that is 10 feet long; it can be plugged into the wall in two places, at wall sockets A and B. José does not move the bed or lift the cord when he vacuums his room.

On the diagram above, outline the area of the floor that José can vacuum using wall socket A. Label this area with the letter A. Then outline the area that José can vacuum using wall socket B. Label this area with the letter B.

Label with the letter U the area of the floor that José cannot clean using either wall socket.

5

Vacuum Cleaning	Test 9 Form A Rubric	
The core elements of performance required by this task are: • draw and identify accessible regions in a practical context Based on these, credit for specific aspects of performance should be assigned as follows:	**Points**	**Section Points**
Correctly identifies and labels vacuumed areas on the diagram:		

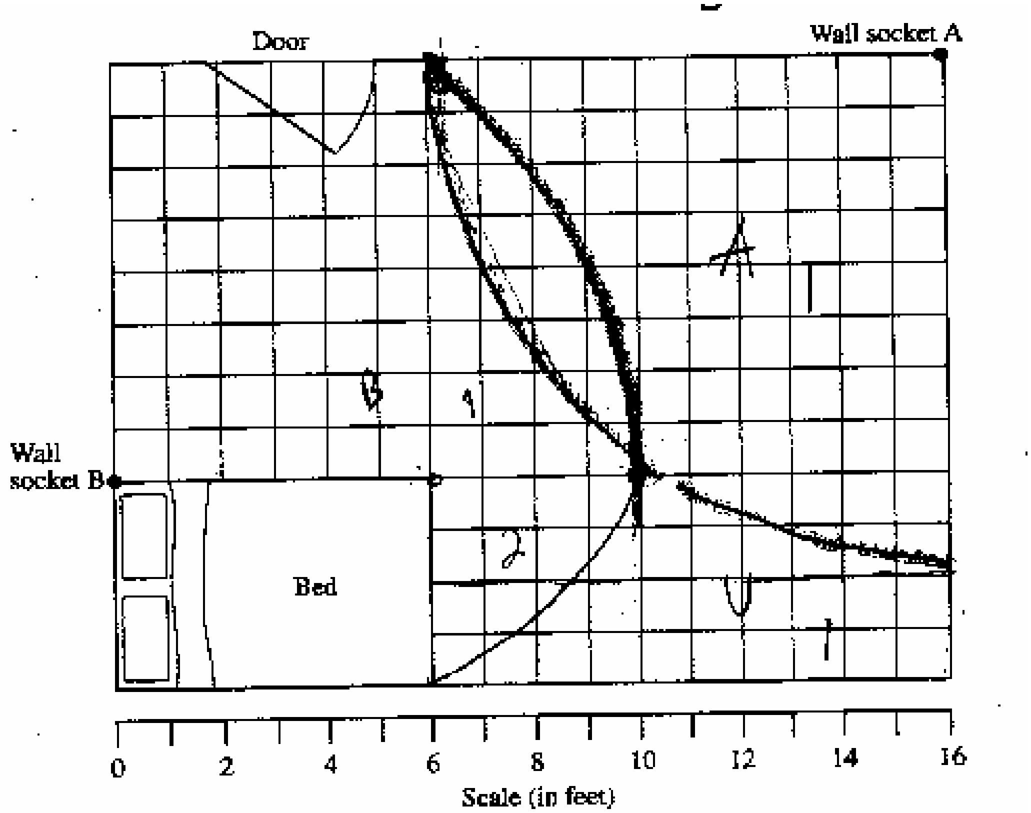


Quadrant shown, center A radius 10 feet labeled A.	1	
Arc shown, center wall socket B radius 10 feet and Quadrant shown, radius 4 feet center corner of bed (as shown); this total area labeled B.	1 2	
Uncleaned area correctly identified. *(allow follow through)*	1 f t	5
Total Points		**5**

Looking at Student Work – Vacuum Cleaning

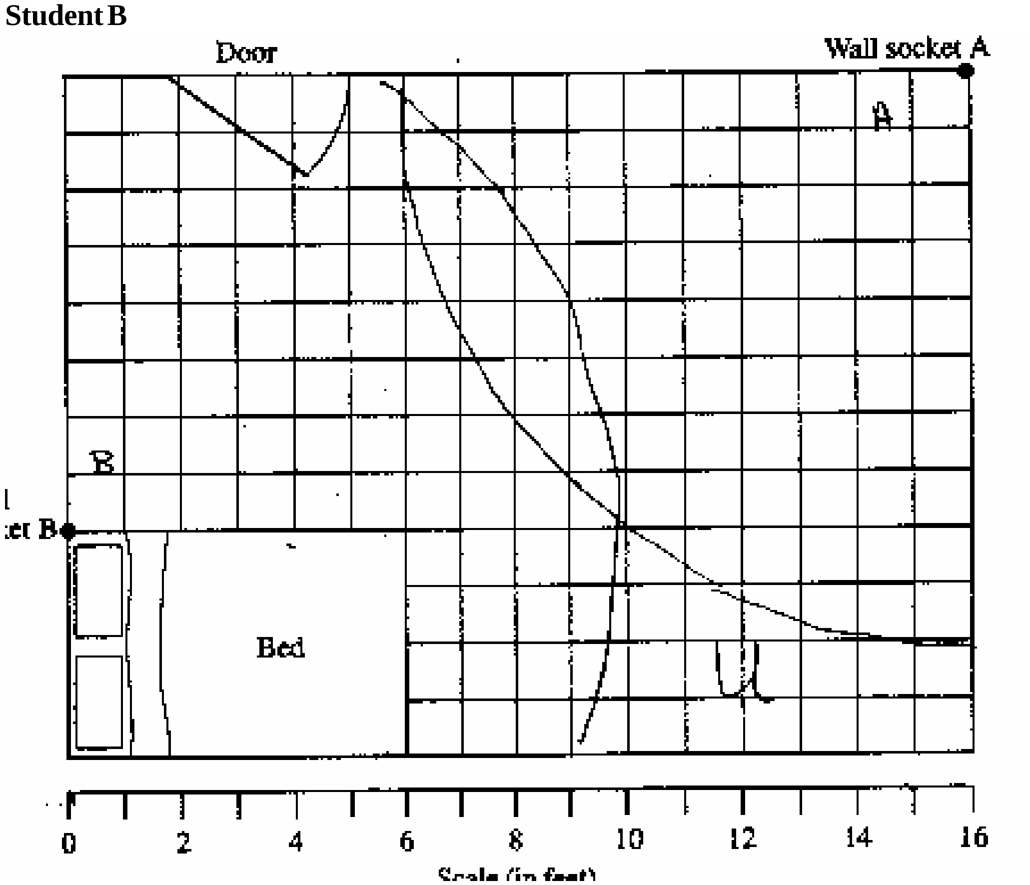
Vacuum Cleaning provides the opportunity to uncover several important mathematical understandings. The work of Student A appears to show the use of a compass with an understanding that mathematics should be done accurately and the ability to choose an appropriate tool. Student A delineates the overlap between regions A and B. In thinking about B, Student A understands that the radius of the circle will change when the vacuum cord reaches the end of the bed.

Student A



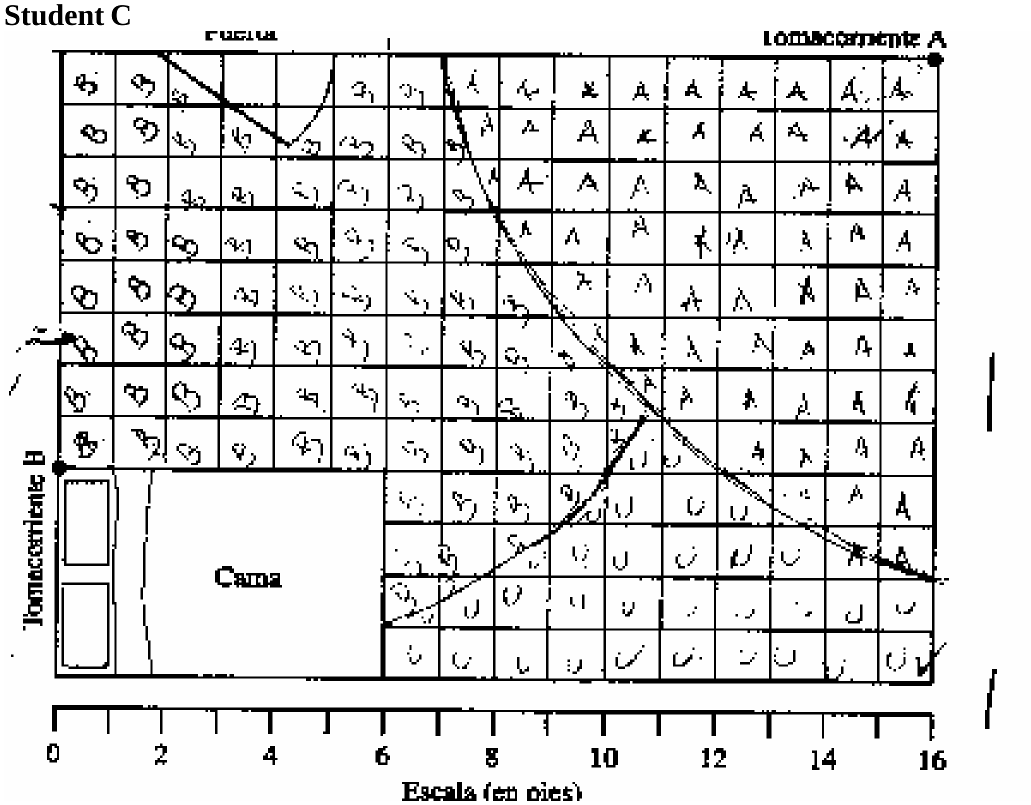
Student B does not use a compass, but demonstrates measurement in choosing the end points for the arc on A and choosing the horizontal extension for B. The student also acknowledges the overlap of the two regions. Student B understands that the cord acts as a radius for a circle and that within the context the area cleaned will be the area within the arc of the circle. The student does not account for the effect on the cord when turning the corner at the end of the bed.

Student B



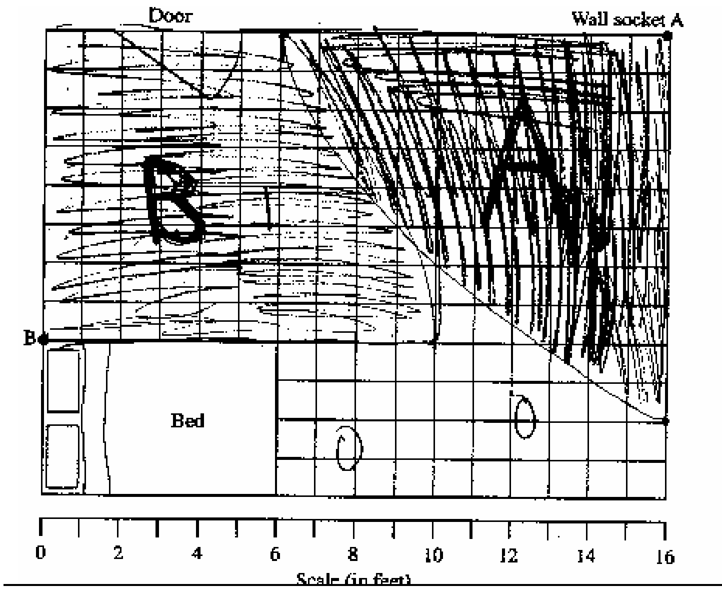
Student C understands that the cord will act as the radius of a circle. The student is not accurate with measurements. While the intersection on the vertical axis for A is located correctly, the intersection on the horizontal axis is at 9 instead of 10. Further questioning might reveal if the student is measuring by counting spaces between points or counting points along the grid. In section B the student has correctly identified a point on the horizontal line from B which should mark the limit of the cord, but appears to account for the effect of the bed on the radius. However on closer inspection the arc is drawn as if B were in the top left corner instead of being drawn from point B.

Student C



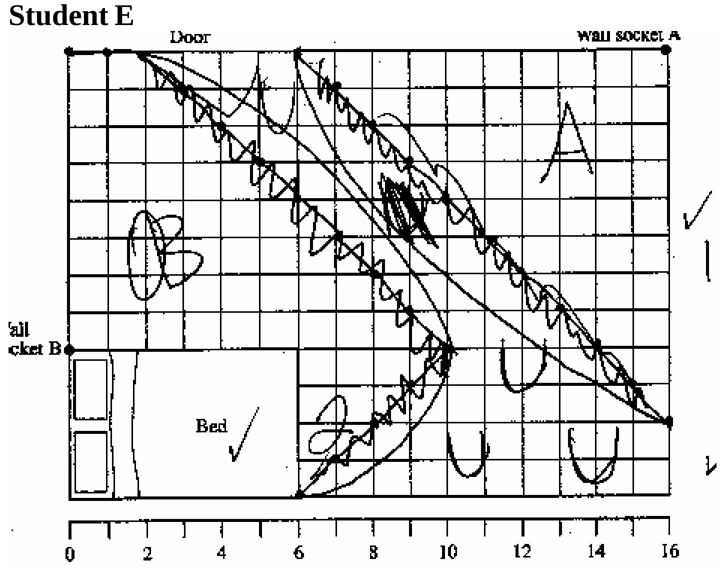
Many students do not have a clear understanding of the cord acting as a radius or how the bed effects the areas cleaned. Student D seems to be struggling with the shape of the area. For A the student attempts to draw an arc, but for region B there is evidence of an arc but as stronger indication of making a rectangular region. Student D does not see the cord as being flexible and able to turn downward at the end of the bed. Within the solution presented, measurements are accurate.

Student D



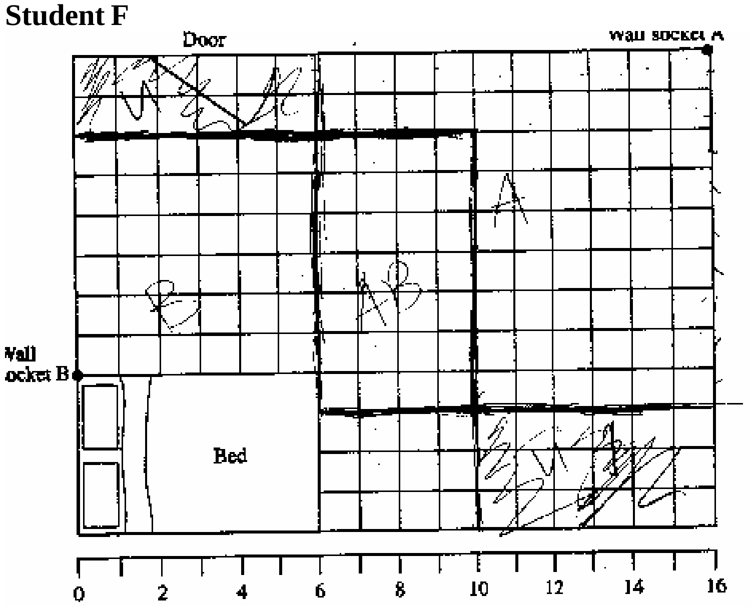
Some students seem to be handicapped by lack of tools or not choosing to use a tool to solve the problem. Student E has made accurate chooses for end points for the arc on A, identified accurately the length of the horizontal radius from point B and shown correctly the effect of the bed on the radius. The student's solution is flawed by the inaccurate estimate of the upper arc for region B.

Student E



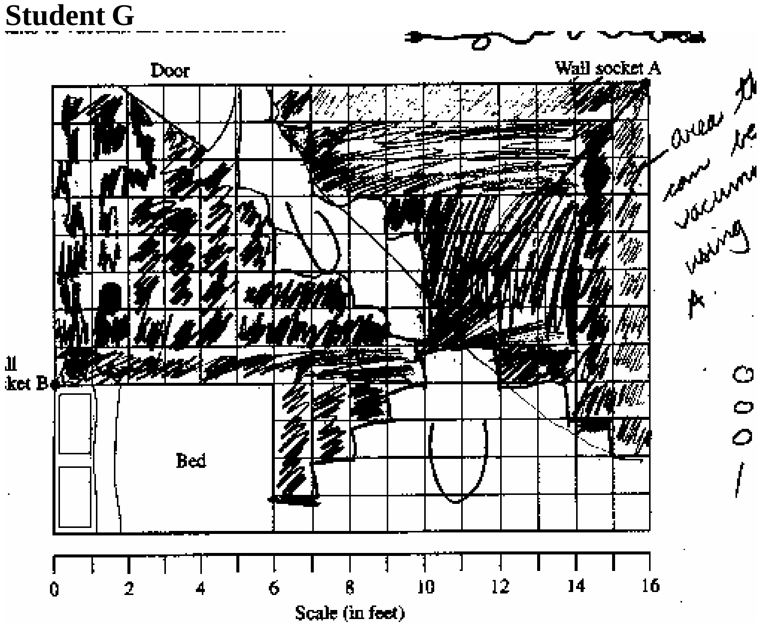
Many student solutions did not use the cord as a radius. Their solutions involve filling in whole squares. Student F measures the correct end points for A of the outer grid and the horizontal length from B, it shows the overlap area, and turns the corner at the bed. The region chosen for B appear to come from (0,0) on the grid instead of the actual location of the point B therefore creating an uncleaned area in the upper portion of the diagram.

Student F



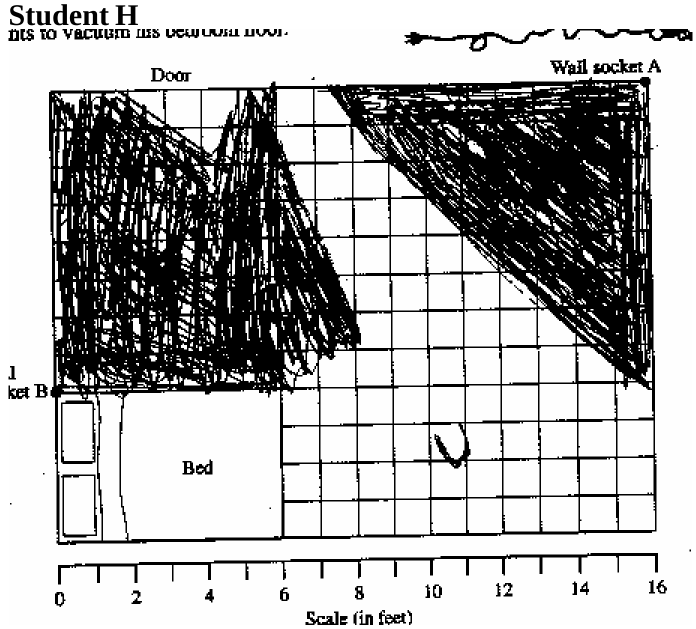
Student G is starting to think about the reach of the cord as it sweeps around from the starting points. The thinking is not yet solidified into the mathematical notion of an arc. Some points in the diagram show use of measurement.

Student G



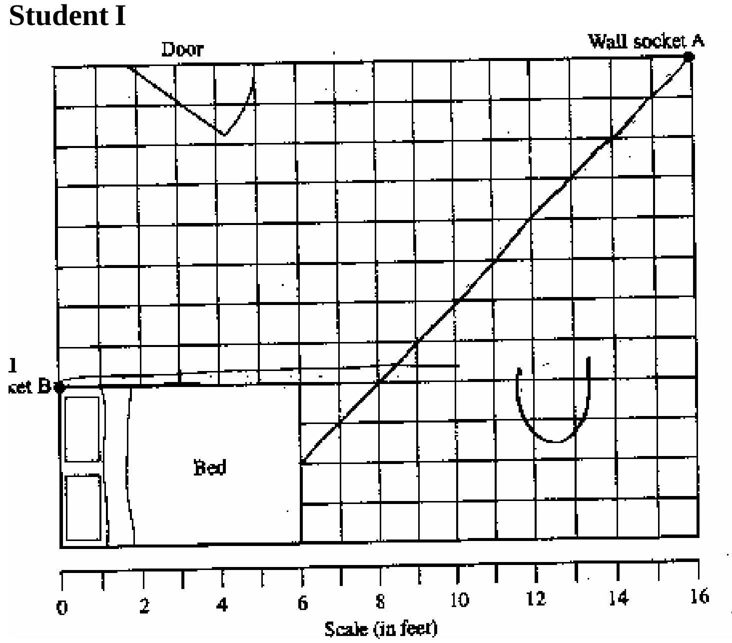
Research shows that U.S. students are weak in measurement. Student H shows a diagram with no use of measurement in thinking about the solution. Region A might show some indication of thinking that could lead to an understanding of arc, while in region B the student still appears to be thinking in square units.

Student H

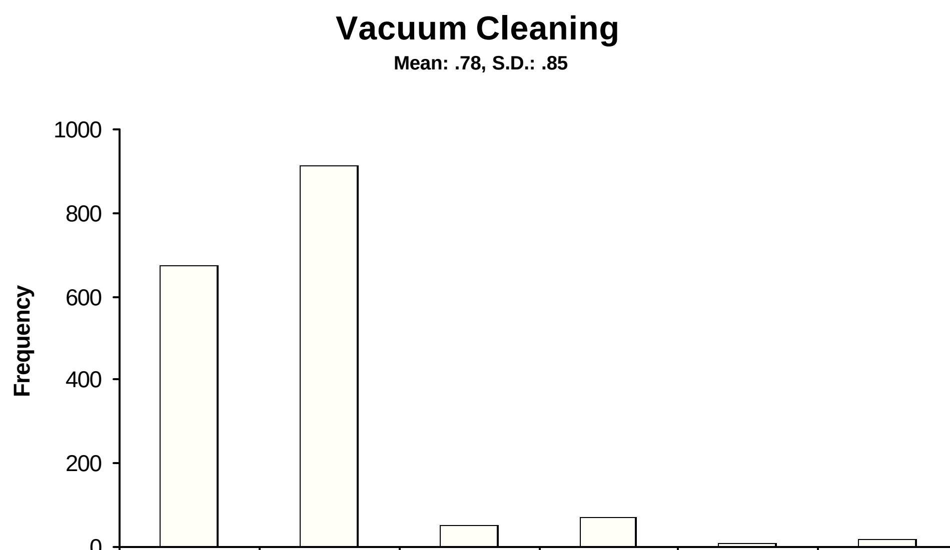


A few students are not thinking of areas cleaned. They seem to be thinking of the reach of the cord as a straight line. Student I shows a line segment drawn horizontally from point B. The student does not account for the difference in length for a diagonal and so inaccurately draws the length of line segment from point A.

Student I



Frequency Distribution for each Task – Grade 9
Grade 9 – Vacuum Cleaning



Score	0	1	2	3	4	5
Frequency	672	912	56	72	7	17

Score:	0	1	2	3	4	5
% < =	38.7%	91.2%	94.5%	98.6%	99.0%	100.0%
% > =	100.0%	61.3%	8.8%	5.5%	1.4%	1.0%

The maximum score available for this task is 5 points.
The cut score for a level 3 response is 3 points.

Many students (approximately 61%) could correctly identify the uncleaned area of the bedroom. Less than 10% of the students could meet the other demands of the task. Over 38 % of the students scored no points on this task. Approximately 80% of those students with a score of 0 attempted the problem.

Vacuum Cleaning

Points	Understandings	Misunderstandings
0	While almost 40% of the students scored no points on this task, most of them were willing to attempt the problem.	About 70% of all responses used regions composed of whole squares instead of regions made by arcs. Almost 32% of the papers with some response showed no use of measurement for any of the dimensions or end points. Over 8% of the students drew straight lines to represent the cord instead of filling in a region.
1	Students could identify the uncleaned area of the bedroom.	Almost 27% of the students did not think that region B turned the corner in any matters when it reached the end of the bed.
3	Students could correctly identify the end points for the arc and draw in region A. Students could draw in the upper arc for B. Students could identify the uncleaned area.	Students could not correctly interpret the effect of the bed on the cord. They generally kept the arc the same for the upper and lower regions of B. Some did not turn the corner at the end of the bed and a few drew the arc as if B were in the upper left corner of the grid.
5	Students understood that the vacuum cord acted as the radius of a circle. They could measure accurately the endpoint for the arcs using the grid. Students understood that the end of the bed would change the length of the radius for B from 10 to 4 and correctly drew in the appropriate arc.	

Teacher Notes:

Based on teacher observations, this is what Algebra One students seemed to know and be able to do:

- Understand the basic relationships between the approximate spaces cleaned by A and B, in order to identify the approximate location of the uncleaned area.
- Demonstrate some use of measurement in making their diagrams.
- Recognize that a cord from B will be able to clean some part of the space around the end of the bed.

Areas of difficulty for Algebra One students, Algebra One students struggled with:

- Identifying the cord as a radius and drawing arcs.
- Use of accurate placement of all end points.
- Identifying the change in radius caused by the bed.

Questions for Reflection on Vacuum Cleaning:

- Do students in your classroom have access to compasses? Are they expected to provide their own or are they available for use as needed? How many of your students' papers showed evidence of using a compass?
- What are your classroom norms around measurement? Do your students know when measurement needs to be accurate and when approximations are appropriate?

Look carefully at student work. Did their arcs meet at intersections or were they "just in the general vicinity"? How many of the students located the following points:

End point for A on the right vertical edge of the grid	End point for A on the upper horizontal edge of the grid	End point for cord B extended horizontally from point B	Good approximation for upper extension of arc B	Correct lower end point for B

- While only a few students drew the cord as a line, many of those who did showed a lack of understanding about the length of measuring diagonal lines? Did this seem to be a problem for any of your students?
- Many students did not interpret the cord as a radius. What holes in their mathematical experiences might have led to this misunderstanding or lack of transference? If you gave this problem in the future, what questions might you ask to push their thinking and help them discover this relationship? Would you make string available when you posed the problem originally?

Teacher Notes:

Implications for Instruction:

Students need more experience with measurement and using appropriate mathematical tools. Students need to have practice making sense of diagrams and making their own diagrams as an aid to problem solving. Students need more experience applying mathematics in problem-solving contexts or in contexts different from the problem sets in standard textbooks. This allows teachers to check for deeper understanding and the ability of students to transfer knowledge.

Teacher Notes:

Snakes

Student Task	Read and interpret scatter plots. Locate points on a scatter plots to identify which scatter plot best fits the coordinates or values given.
Core Idea 5 Data Analysis	• Understand the relationship between two sets of data, display such data in a scatterplot, and describe trends and shape of the plot including correlations and lines of best fit. • Make inferences based on the data and evaluate the validity of conclusions drawn.
Core Idea 2 Mathematical Reasoning	• Show mathematical reasoning in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams and models. • Draw reasonable conclusions about a situation being modeled.

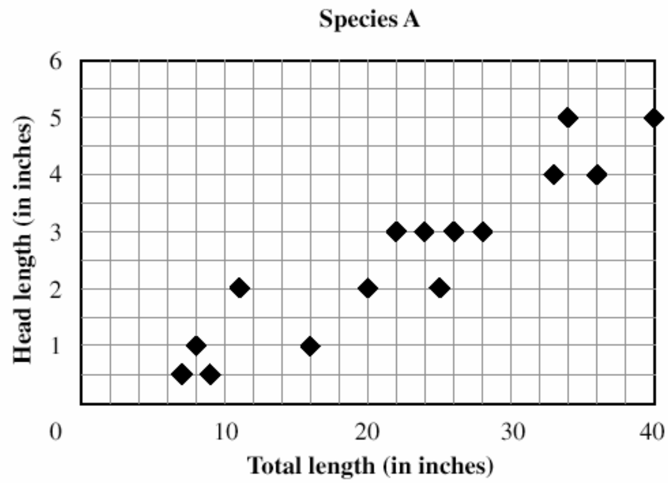
Snakes

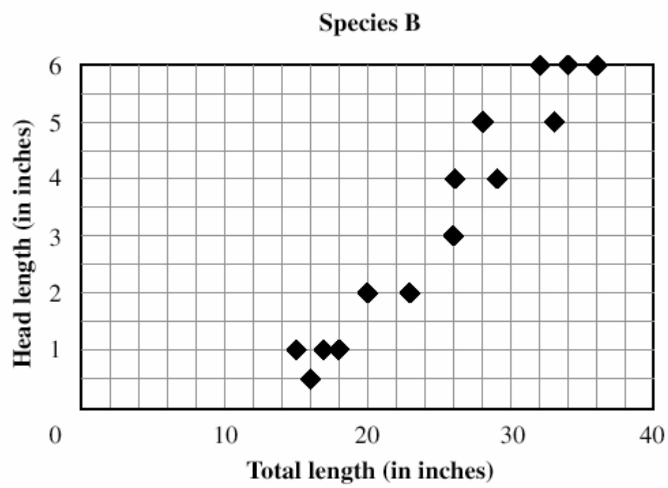
This problem gives you the chance to:
- interpret scatter plots and identify values that fit

Rita is a zoologist. She is studying two species of snake.

Rita measures the head length and the total length of some snakes of each species.

She records the measurements on two scatter plots, as shown below.





Rita catches 5 more snakes.

She wants to know whether they belong to species A or to species B.

The measurements of these snakes are shown in the table below.

Snake	1	2	3	4	5
Total length (in inches)	36	39	9	16	18
Head length (in inches)	6	5	1	0.5	1

Use the scatter plots to decide whether these snakes belong to species A or species B.

Record your answers in the table below.

Snake	1	2	3	4	5
Species					

5

Snakes

Test 9 Form A Rubric

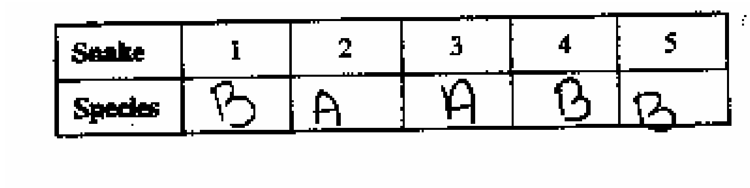
The core elements of performance required by this task are: • interpret scatter plots and identify values that fit Based on these, credit for specific aspects of performance should be assigned as follows:	Points	Section Points
Correctly completes the table:	5 × 1	5
Total Points		**5**

Snake	1	2	3	4	5
Species	**B**	**A**	**A**	**B**	**B**

Looking at Student Work – Snakes

Most students did very well on this task with almost 53% scoring all the points on the task. Their work however provides no insights into their thinking process, as most of them show no marks on their papers except filling in the letters in the table as shown by the work of Student A below.

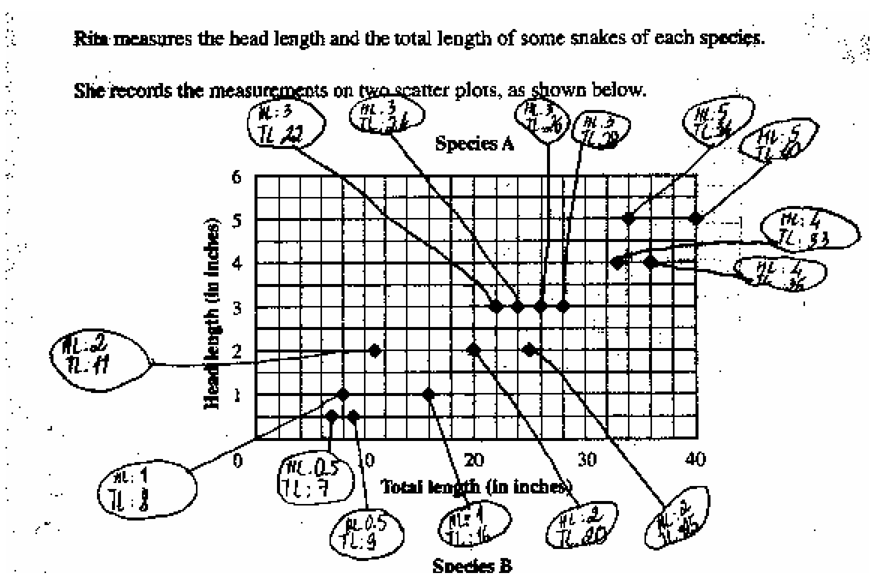
Student A



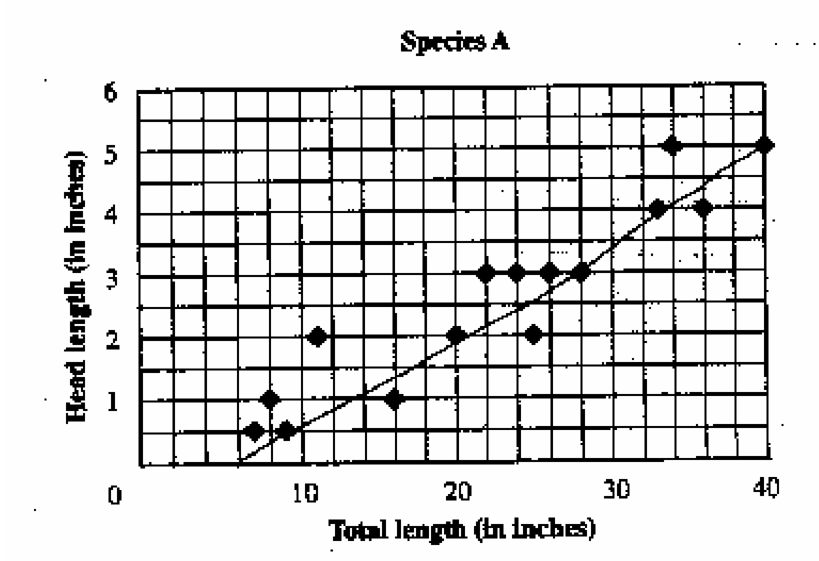
Snake	1	2	3	4	5
Species	B	A	A	B	B

In thinking about this task, students might have located the point on both graphs to look for reasonableness, labeled points already on graph to use for comparison (see work of Student B), or attempted a line of best fit to use for comparison (see work of student C). Only 3% of the students showed any marks on their paper beyond responses written in the table.

Student B



Student C



Approximately 8% of the students filled in the table with numbers instead of letters. See the work of Students D and E.

Student D

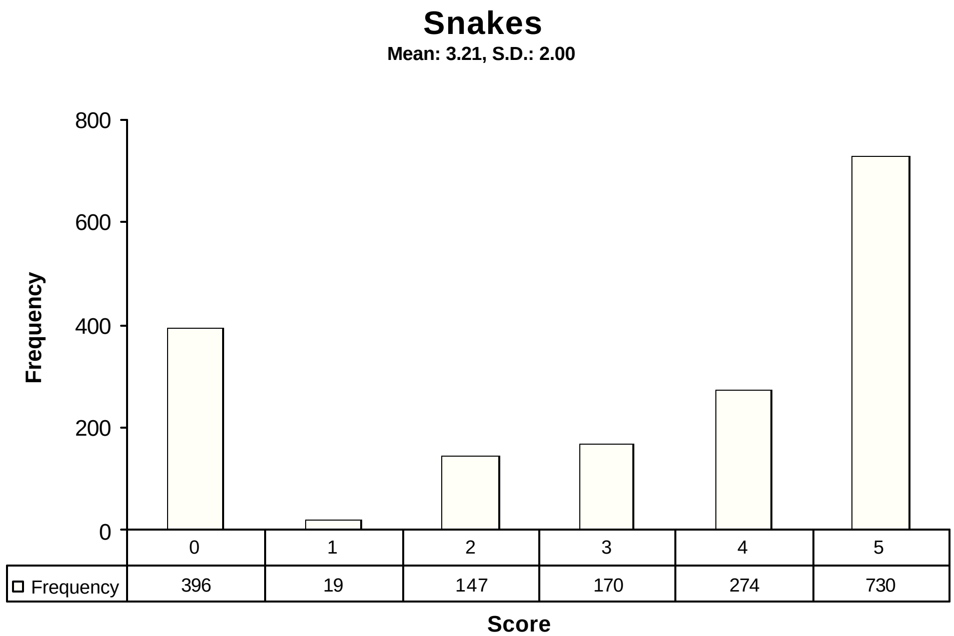
¡ respuestas en la tabla siguiente.

Serpiente	1	2	3	4	5
Especie	6	7	8	9	10

Student E

Snake	1	2	3	4	5
Species	5	5	3	4	4

Teacher Notes:



Score:	0	1	2	3	4	5
% < =	22.8%	23.9%	32.4%	42.2%	57.9%	100.0%
% > =	100.0%	77.2%	76.1%	67.6%	57.8%	42.1%

The maximum score available on this task is 5 points.
The cut score for a level 3 response is 3 points.

Most students (approximately 78%) could successfully identify the species of at least one of the snakes, usually 1 or 5. More than half the students could identify the species of 4 or more of the snakes. 42% of the students met all the demands of the task. Almost 23% of the students scored no points on this task. 52% of the students with scores of zero did not attempt the problem.

Snakes

Points	Understandings	Misunderstandings
0	Almost 30% of the students scored no points on this test. 48% of those students attempted the problem.	8% of all students used numbers instead of letters to fill in the table. Only 3% of the students showed any of their thinking by making marks on the graphs. Therefore evidence for analyzing error patterns or reasoning is lacking.
1	Students with this score could identify the correct graph for Snake 5.	
2	While there were a variety of correct combinations for this score, the most common pattern was to identify snake 1 and 3.	
3	Students could identify 3 snakes. They were more likely to identify combinations including 1 and 3 and less likely to have a combination including either 2,4 or 5.	
4	Students with this score could identify the species for 4 out of the 5 snakes.	The most common error was snake 4, followed by snake 5 or 2.
5	Students could read, interpret, and compare scatter plots to use data from a table to identify the species of a snake.	

Teacher Notes:

Based on teacher observations, this is what AlgebraOone students seemed to know and be able to do:

- Read and compare scatter plots.
- Correctly locate points on a grid.
- Interpret graph of best fit for information from a table.

Areas of difficulty for Algebra One students, Algebra One students struggled with:

- Attempting problems for which they were unsure of the solution.
- Interpreting relationships and fits for values that did not exactly fit the points on the grids.

Questions for Reflection on Snakes:

- What experiences have students had this year with interpreting scatter plots?
- When looking at data on graph, do you frequently ask students to describe trends or to look for correlations? Give some examples students have had this year in problem solving with data.
- What are your classroom norms for showing thinking when doing problem solving? Do you have any conjectures about why more students didn't use the grid to help them make the comparisons?
- What percentage of your students did not attempt the problem? What are some of the factors that may have contributed to this?

Teacher Notes:

Implications for Instruction:

Students at this grade level should be familiar not only with plotting points and making scatter plots, but should see the pur pose and usefulness of scatter plots for looking at trends, making predictions, modeling situations, and solving problems. Students at this grade level also need to feel that their thinking is important and feel comfortable about showing their work and ha ve the confidence in their mathematical thinking to attempt unfamiliar problems.

Teacher Notes:

Crisscross Numbers – MAC Core Ideas

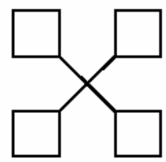
Student Task	Investigate number patterns on a hundreds chart. Describe rules or patterns in words or symbols. Use algebra to prove why the rules hold true for all cases.
Core Idea 3 Algebraic Properties and Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** Use symbolic algebra to represent and explain mathematical relationships. Judge the meaning, utility, and reasonableness of results of symbolic manipulations. Use symbolic expressions to represent relationships arising from various contexts.

Crisscross Numbers

This problem gives you the chance to:
• use algebra to explain number patterns in a number square

Pauline is investigating relationships between numbers in a 10-by-10 number square. She is using a crisscross pattern with four numbers. An example is shown on the number square below.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

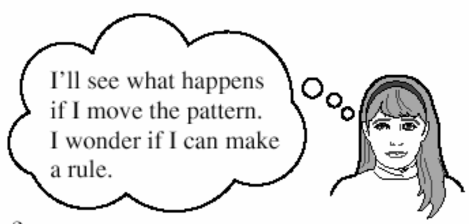


Pauline adds the pairs of numbers in the diagonally opposite corners of the pattern.

1. What do you think she notices?

____

____



2. What rule could Pauline make?

____

____

3. Use algebra to show why the rule always works.

Leon multiplies the pairs of numbers in the diagonally opposite corners of the pattern.

$24 \times 46 = 1104$ $\quad\quad$ $26 \times 44 = 1144$

He finds the difference between his two answers.

$1144 - 1104 = 40$

4. Investigate what happens when he moves the pattern to a different position.

5. What rule can Leon make?

____

____

____

____

6. Use algebra to show why the rule always works.

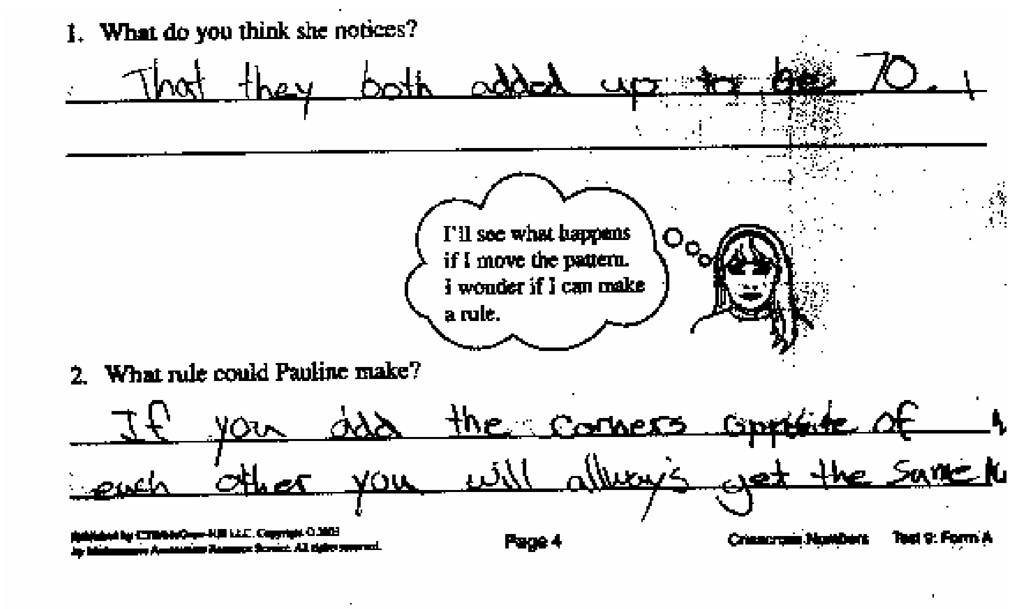
10

Crisscross Numbers		Test 9 Form A Rubric
The core elements of performance required by this task are: • use algebra to explain number patterns in a number square Based on these, credit for specific aspects of performance should be assigned as follows:	Points	Section Points
1. Shows that: $24 + 46 = 26 + 44 = 70$ **or** States that both answers are the same.	1 **or** 1	1
2. Gives a correct answer such as: The sum of the numbers in the diagonally opposite corners are always the same, twice the number in the center of the cross. *Accept alternative correct rules.* *Partial credit:* Gives a correct but incomplete explanation.	2 (1)	2
3. Shows work that supports student's rule such as: $x + (x + 22) = (x + 2) + (x + 20) = 2x + 22 = 2(x + 11)$ *Accept alternative correct proofs.*	2	2
4. Gives correct answer as: For at least one different position, shows that the difference between the two products is always 40.	1	1
5. Gives correct answer as: The difference between the two products of the numbers in the diagonally opposite corners is always 40. *Partial credit:* Gives a correct but incomplete explanation.	2 (1)	2
6. Shows work such as: $(x + 2)(x + 20) - x(x + 22) = x^2 + 22x + 40 - x^2 - 22x = 40$ *Accept alternative correct proofs.* *Allow partial credit for a correct equation.*	2 (1)	2
Total Points		**10**

Looking at Student Work – Crisscross Numbers

Students struggled with Crisscross Numbers. In the 150 sample papers examined for the toolkit, there were no scores above 6. Student A shows work of a typical 6. The student correctly identifies the patterns, but does not use algebra to show why the patterns work. Most students attempted to prove their conjectures by giving numerical examples.

Student A



1. What do you think she notices?

That they both added up to be 70.

I'll see what happens if I move the pattern. I wonder if I can make a rule.

2. What rule could Pauline make?

If you add the corners oppisite of each other you will allway's get the same n

Page 4 Crisscross Numbers Test 9: Form A

Student A

$$26 + 44 = 70 \qquad 24 + 46 = 70$$

Leon multiplies the pairs of numbers in the diagonally opposite corners of the pattern.

$24 \times 46 = 1104$ $\qquad$ $26 \times 44 = 1144$

He finds the difference between his two answers.

$1144 - 1104 = 40$

4. Investigate what happens when he moves the pattern to a different position.

$10 \times 28 = 280 \qquad 30 \times 8 = 240$

$280 - 240 = 40$

5. What rule can Leon make?

What ever Multipley and then find the difference it will always be 40.

6. Use algebra to show why the rule always works.

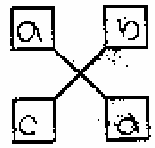
$$10 \times 28 = 80 + 200 = 280 \qquad 30 \times 8 = 240$$

$$280 - 240 = 40$$

Student B is another example of the work with a score of 6. This student uses letters to help clarify the parts of the crisscross and make the rule crisp. Student B's proof is using numerical substitution to prove a specific case. Algebra is not used to justify the general rule. 70% of the students with scores higher than zero used proof by numeric example.

Student B

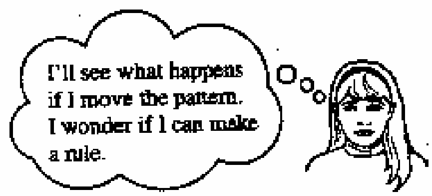
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Pauline adds the pairs of numbers in the diagonally opposite corners of the pattern.

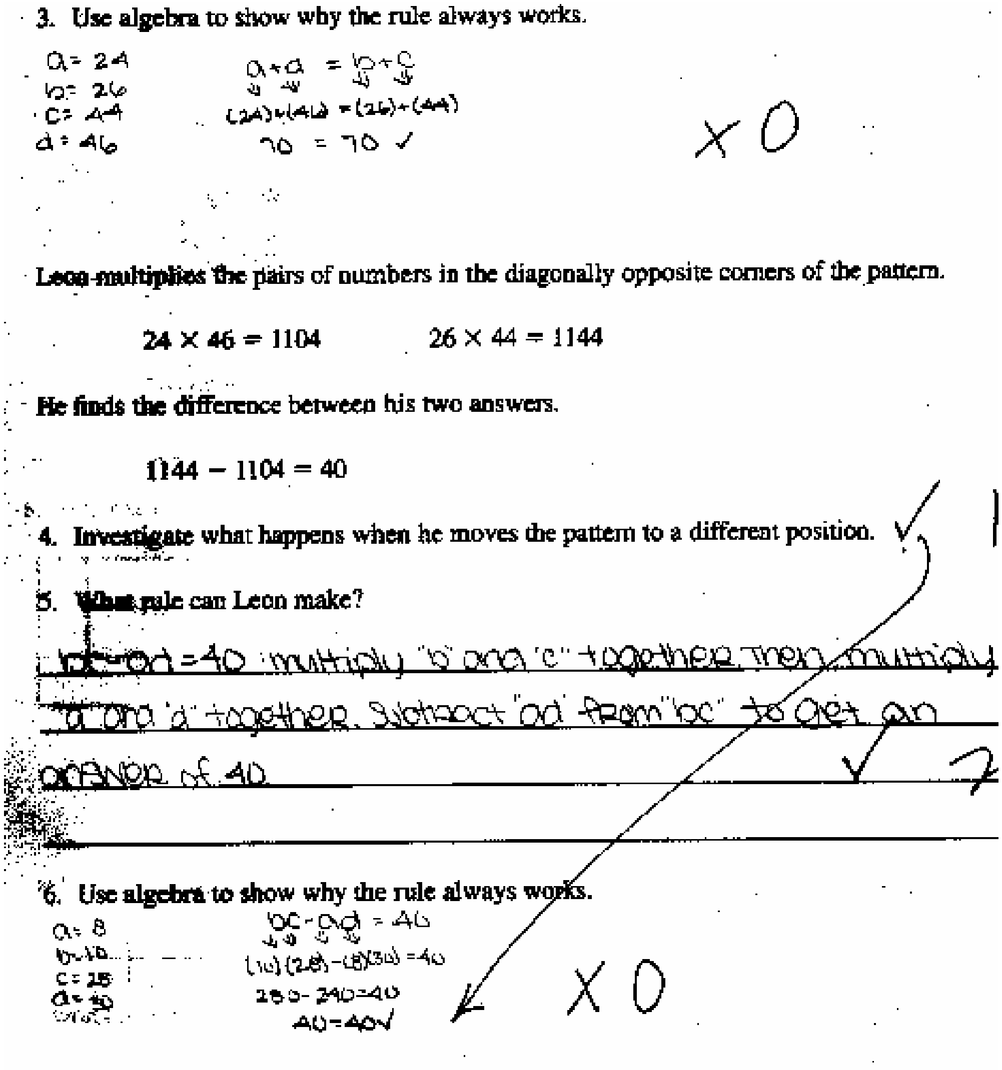
1. What do you think she notices?

That when you add the pairs of numbers diagonally they are both equal to each other. (24+46)(26+44) (70=70) ✓



2. What rule could Pauline make?

a+d=b+c : when you add "a" and "d" together they are equal to when you add "b" and "c" together. ✓ 2



3. Use algebra to show why the rule always works.

a = 24
b = 26
c = 44
d = 46

a + d = b + c
(24) + (46) = (26) + (44)
70 = 70 ✓

× 0

Leon multiplies the pairs of numbers in the diagonally opposite corners of the pattern.

24 × 46 = 1104   26 × 44 = 1144

He finds the difference between his two answers.

1144 − 1104 = 40

4. Investigate what happens when he moves the pattern to a different position. ✓ 1

5. What rule can Leon make?

bc − ad = 40 multiply "b" and "c" together then multiply "a" and "d" together. subtract "ad" from "bc" to get an answer of 40 ✓ 2

6. Use algebra to show why the rule always works.

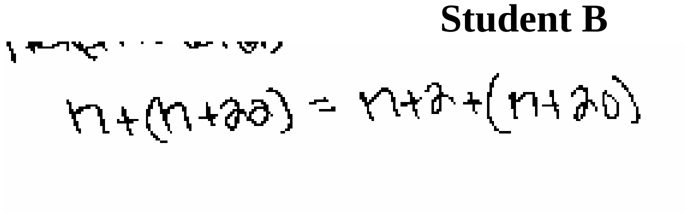
a = 8
b = 10
c = 28
d = 30

bc − ad = 40
(10)(28) − (8)(30) = 40
280 − 240 = 40
40 = 40 ✓

× 0

Very few students attempted to use algebra to solve the problem (less than 5%). Those who did tended to use more than 2 variables. They did not use a single variable to help describe the relationships between the numbers in the crisscross. Student B is one of the few able to quantify the relationship of the four corners.

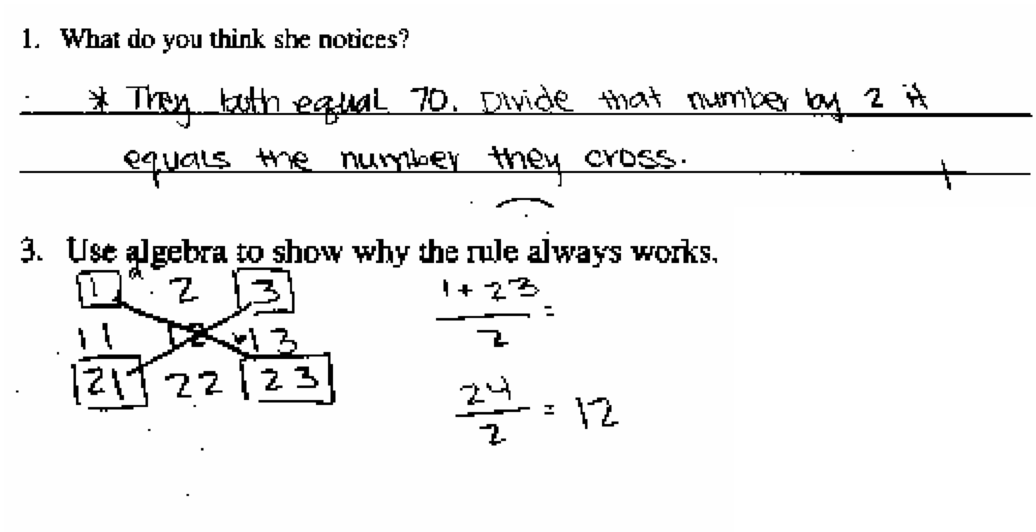
Student B



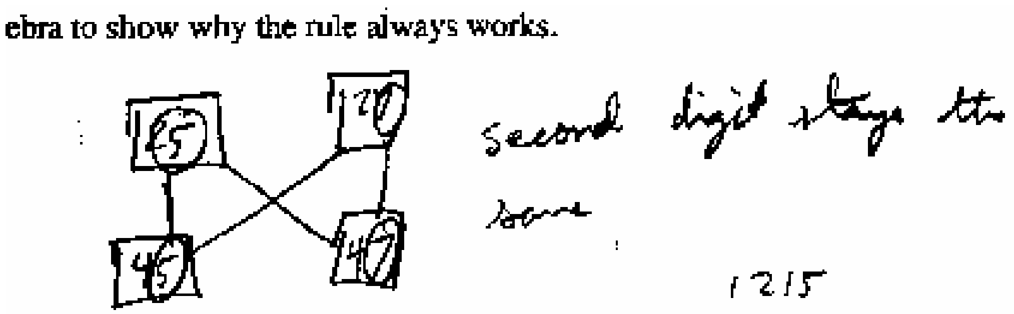
$n + (n+22) = n+2+(n+20)$

Some students found alternate patterns or noticed unrelated attributes. Student C notices that the center number is half the sum of either diagonal. The student uses a numerical example instead of algebra to prove the rule. Student D notices that the ones' digits in any column are the same. Student E makes the common observation of noting the sums are even. Other alternate patterns are to note the differences between diagonal corners are 18 and 22.

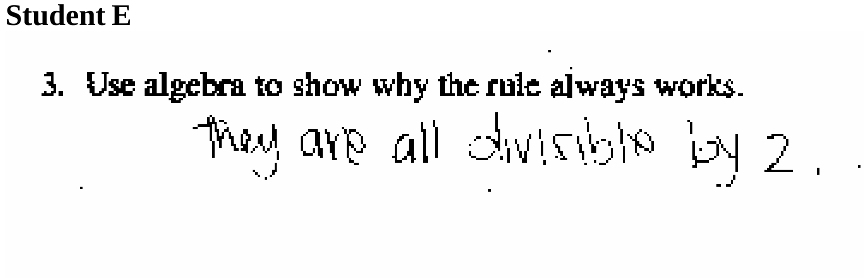
Student C



Student D

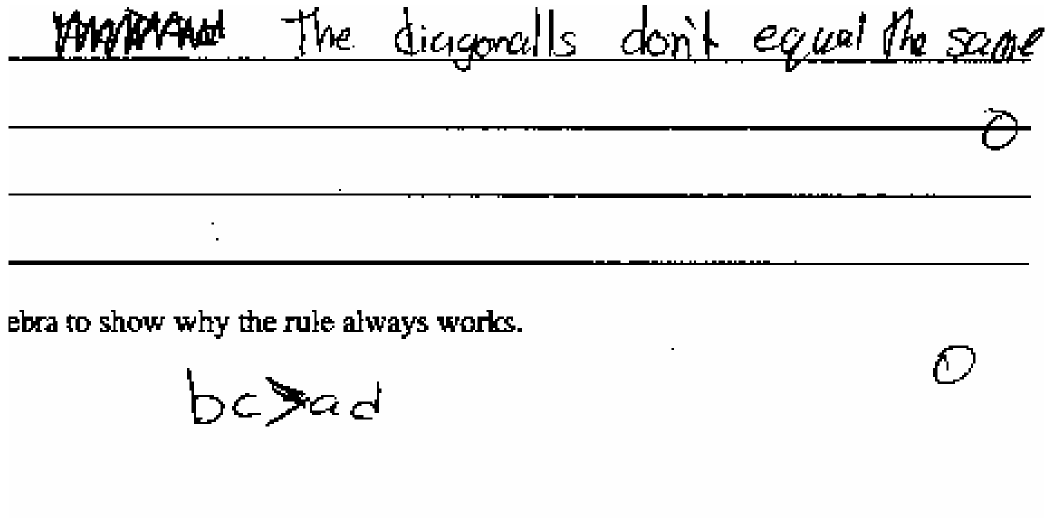


Student E



Alternate observations in the multiplication pattern are to note that the products are different without an attempt to quantify that difference (see Student E) or have computational errors prevent finding the pattern (see student F).

Student E


The diagonalls don't equal the same

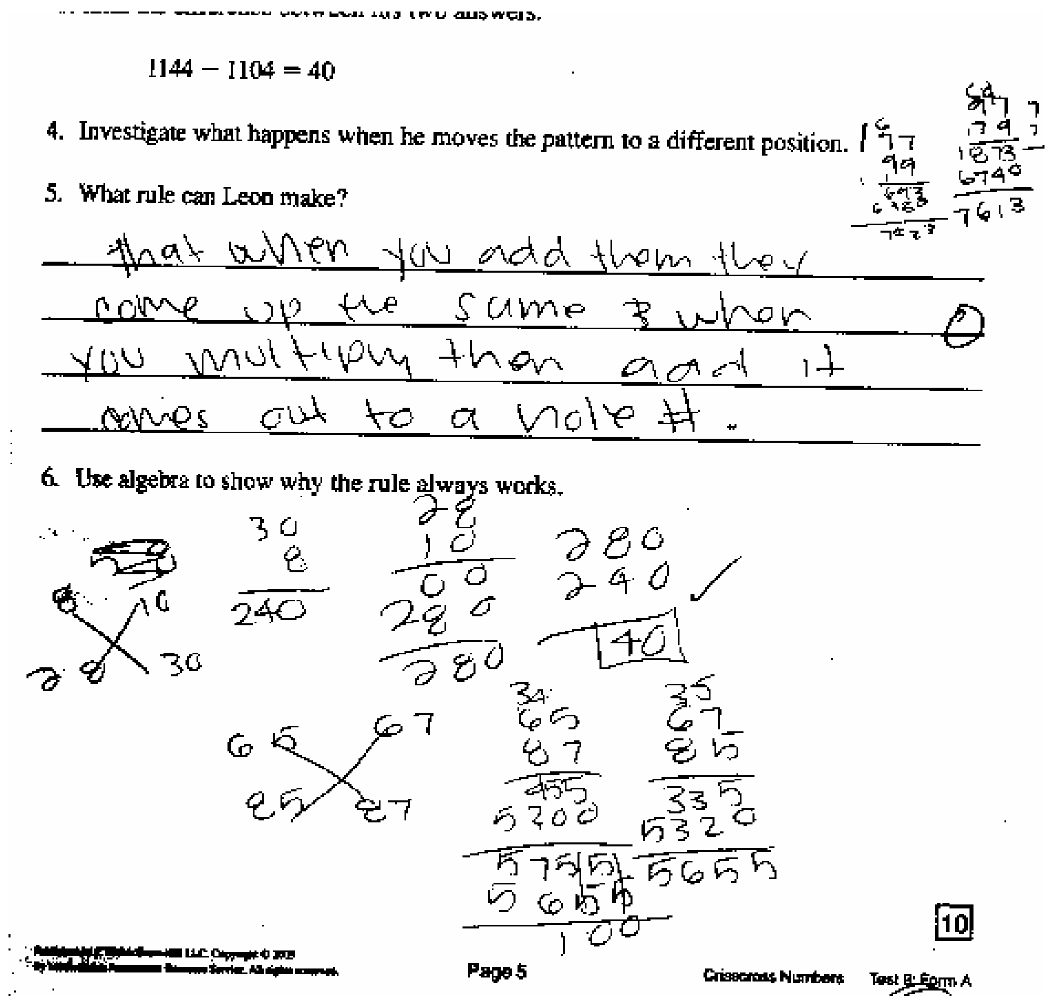
0

ebra to show why the rule always works.

0

bc>ad

Student F



... the difference between his two answers.

$1144 - 1104 = 40$

4. Investigate what happens when he moves the pattern to a different position.

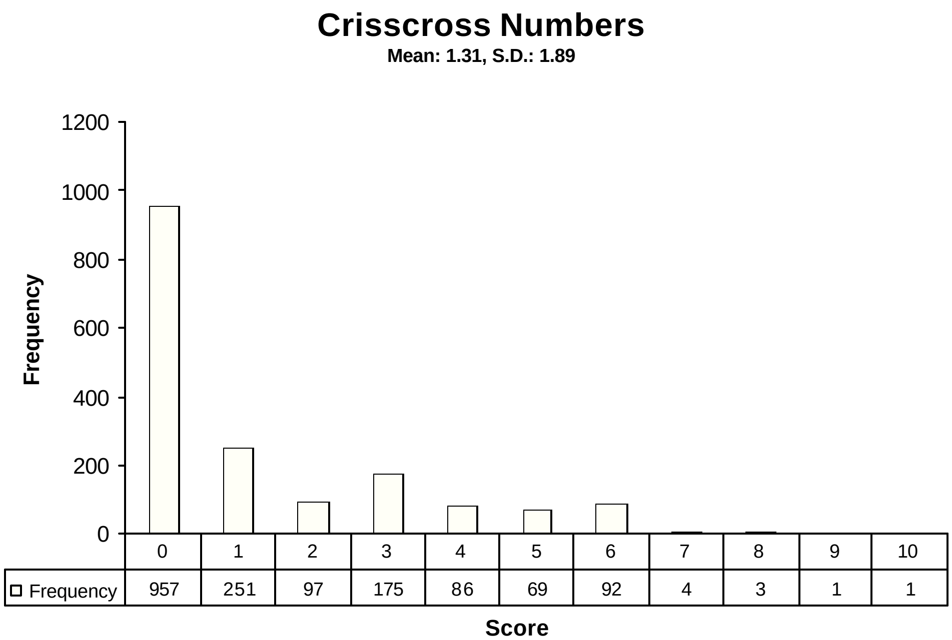
5. What rule can Leon make?

that when you add them they come up the same & when you multiply them and it comes out to a whole #.

6. Use algebra to show why the rule always works.

Page 5 Crisscross Numbers Test B: Form A

Grade 9 – Crisscross Numbers



Score:	0	1	2	3	4	5	6	7	8	9	10
% < =	55.1%	69.6%	75.2%	85.3%	90.2%	94.2%	99.5%	99.7%	99.9%	99.9%	100.0%
% > =	100.0%	44.9%	30.4%	24.8%	14.7%	9.8%	5.8%	0.5%	0.3%	0.1%	0.1%

The maximum score available on this task is 10 points.
The cut score for a level 3 response is 5 points.

Less than half the students (approximately 45%) could identify that adding the numbers on the diagonals of the crisscross gave you a sum of 70 or the same sum. About 25% of the students could make both of those observations. Less than 1% of the students used algebra appropriately to describe the relationships of the numbers in the crisscross or to prove why the rules held true. 55% of the students scored no points on this task. Of those students with a score of zero, approximately 59% made some attempt at solving the problem. Of the remaining 41% who didn't try this task, 33% made attempts at one or more of the later tasks on the test.

Crisscross Numbers

Points	Understandings	Misunderstandings
0	Approximately 55% of the students scored no points. A little more than half of those students attempted the task.	About 80% of the students who didn't try the problem, did work on some further section of the test. For students who attempted the task, errors included noting that the final digits (11%) were the same or that the sums were even (23%). Others might state that there was a pattern, but not identify what it was.
1	Students with this score were able to do question one. They knew that in the given example, both sums were 70.	Students may not have explored other crisscrosses and thought all sums would be 70. They may have made computational errors.
3	Students with this score knew that the sums of the diagonals in the given example were 70 and that for any crisscross the sums would be equal.	Students used numeric examples to prove their rule instead of using algebra as suggested in the prompt.
5	Students with this score could identify that in the sample crisscross the sum of the diagonals was both 70, that when the design is moved the sums will always be equal, and that the difference between the products is always 40.	Students did not show evidence that they checked other crisscross patterns to see if the products always had a difference of 40. They don't understand that one example is not enough proof to make a general rule.
6	Students in this category showed evidence of testing the difference of the products on at least one other case.	Students did not use algebra to prove why they rule works. If they used letters they were multiple and did not express the relation of the numbers to each other. They don't understand the importance of using one to quantify relationships between different parts of a problem.
7-10	Students could make some appropriate use of algebra and variables to quantify relationships between numbers within the crisscross or prove why one or both rules would work for all locations of the crisscross.	

Based on teacher observations, this is what Algebra One students seemed to know and be able to do:

- Calculate sums of diagonal numbers on a grid.
- Recognize that the sums would remain the same if the design was repeated elsewhere on the grid.

Areas of difficulty for Algebra One students, Algebra One students struggled with:

- Understanding that specific numeric examples are not a proof or justification for a rule.
- Recognizing that the numbers in the crisscross could be described using one variable.
- Using algebra to prove why a rule works.

Questions for Reflection on Crisscross Numbers:

- Students start identifying odd or evenness at primary grades. What experiences or problems have your students had this year that help them identify patterns? What kinds of questions do you ask to push them to looking for more complex patterns and relationships? What level of proof or evidence do you expect from students to justify their rules or relationships? Do you students get enough variety of pattern problems to be on the look out for counterexamples?
- What do you think your students understand about variables? From reading through these problems, do they think it solely stands for a missing number or do some of them seem to understand that it can be used to describe relationships between parts of a problem? Do your students ever get the opportunity to discuss whether a problem can be solved with only one variable and what the advantages might be for doing that?
- Do students get opportunities to use their skills at symbol manipulation to prove statements or conjectures? What are some of the problems that students have done this year that might help them build skills in using algebra to make proofs? Where might be good places in the curriculum to interject this types of problem-solving experiences?

Teacher Notes:

Implications for Instruction:

Part of learning algebra is gaining the appreciation of symbols to quantify relationships and using symbols and knowledge of the number system to prove conjectures. Students need to gain a deeper understanding of variables and their use to solve problems. Students need to be pushed to use their ability to manipulate symbols to solve problems. Students need more experiences with problem solving, looking for patterns, and making proofs. Discussions about how many variables are needed for given problems give students the opportunity to think more deeply about the purpose of variables and see their usefulness.

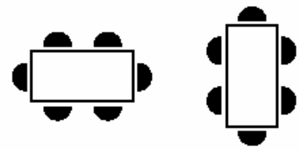
Conference Tables – MAC Core Ideas

Student Task	Find and extend patterns in a geometric context. Use inverse relationships to solve problems. Describe a rule or write a formula to explain how to find any number in the pattern.
Core Idea 1 Functions and Relations	**Understand patterns, relations, and functions.** • Generalize patterns using explicitly defined functions. • Understand relations and functions and select, convert flexibly among, and use various representations for them.
Core Idea 3 Algebraic Properties and Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** • Use symbolic algebra to represent and explain mathematical relationships. • Use symbolic expressions to represent relationships arising from various contexts.

Conference Tables

This problem gives you the chance to:
- find and extend a number pattern in a practical geometric context
- find and use a rule or formula

The Conference Company supplies tables for conferences.

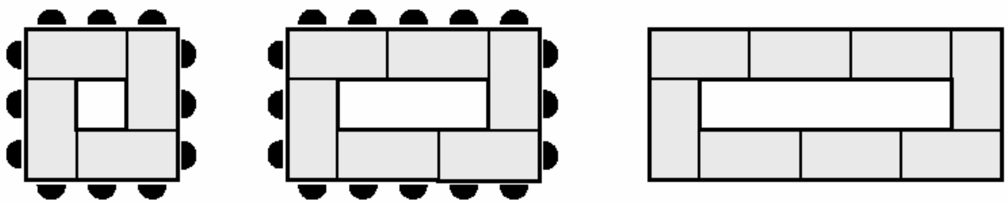


Each table is a rectangle.

Each table can seat one person at its short edge and two people at its long edge.

The diagrams below show how these tables can be arranged for different numbers of people.

No one sits inside the arrangements.



Size 1
4 tables
12 people

Size 2
6 tables
16 people

Size 3

1. How many tables are in a Size 3 arrangement? ________________ tables

2. How many tables are in a Size 7 arrangement? ________________ tables

3. Write down a rule or formula for working out how many tables there are when you know the size number.

____

____

____

4. Find the number of tables needed for a Size 13 arrangement. ________________ tables

5. How many people can sit at a Size 3 arrangement? ________________ people
You may find it helpful to use the diagram on the first page of this problem.

6. How many people can sit around a Size 6 arrangement? ________________ people

7. Write a rule or formula for finding the number of people who can sit at a Size *s* arrangement.

____

____

____

8. What size arrangement is needed for 72 people? ________________

10

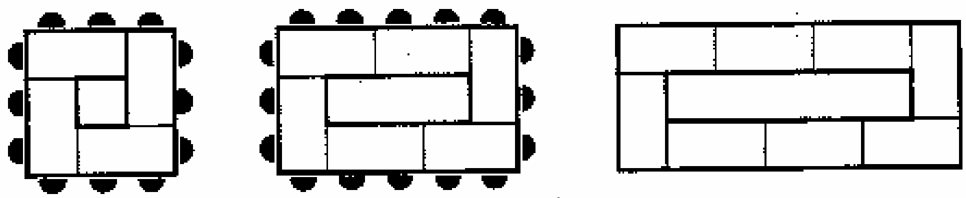
Conference Tables		**Test 9 Form A Rubric**
The core elements of performance required by this task are: • find and extend a number pattern in a real life geometric context • find and use a rule or formula Based on these, credit for specific aspects of performance should be assigned as follows:	Points	Section Points
1. Gives correct answer as: **8** tables	1	1
2. Gives correct answer as: **16** tables	1	1
3. Gives correct answer as: $\mathbf{T = 2}s\mathbf{ + 2}$ (or equivalent) **or** The number of tables is twice the Size number plus 2.	2 **or** 2	2
4. Gives correct answer as: **28** tables	1	1
5. Gives correct answer as: **20** people	1	1
6. Gives correct answer as: **32** people	1	1
7. Gives correct answer as: $\mathbf{P = 4}s\mathbf{ + 8}$ (or equivalent) **or** The number of people is four times the Size number plus 8. *Accept: The number of people is two times the number of tables plus 4.*	2 **or** 2	2
8. Gives correct answer as: Size **16**	1	1
Total Points		**10**

09mr_A03BAM•••2/20/03•••final cx•••Arista

Looking at Student Work – Conference Tables

Students seemed to have good access to this task. It gives students the opportunity to solve a problem using a variety of strategies. Student A met all the demands of the task. Student A appears to use the first number in each sequence and as constant and then use the rate of change of change as a factor to multiply by a variable expression using size number. The student may be just testing variable expressions to fit the numbers in the chart or may be using a visual model to derive the variable expression.

Student A



Size 1
4 tables
12 people

Size 2
6 tables
16 people

Size 3

1 1. How many tables are in a Size 3 arrangement? 8 tables

1 2. How many tables are in a Size 7 arrangement? 16 tables

3. Write down a rule or formula for working out how many tables there are when you know the size number.

2 We know that Size 1 have 4 tables, we call size : x ; tables for size : y

We have y = 4 + 2(x − 1) [each size next different 2 tables]

4. ~~Find the~~ number of tables needed for a Size 13 arrangement. 28 tables 1

5. How many people can sit at a Size 3 arrangement? 20 people 1

You may find it helpful to use the diagram on the first page of this problem.

6. How many people can sit around a Size 6 arrangement? 32 people 1

7. Write a rule or formula for finding the number of people who can sit at a Size *s* arrangement.

We know size 1 have 12 people, we call size: x, people in each size: y

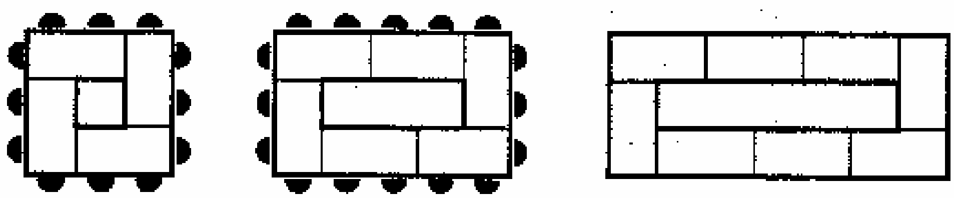
In each size different the next size 4 people.

We have $y = 12 + 4(x-1)$ 2

8. What size arrangement is needed for 72 people? 16 1

Looking carefully at the work by the table drawings. Student B appears to be using the numbers in the informal table to test for a rule. This student is most likely looking at the number patterns. Student B does applies the algebraic rule to solve for the inverse relationship in part 6 of the task.

Student B



Size 1 · 4+8	Size 2 · 4 + 8	Size 3 · 4 + 8
4 tables	6 tables	8 tables
12 people	16 people	20 people

1. How many tables are in a Size 3 arrangement? 8 tables 1

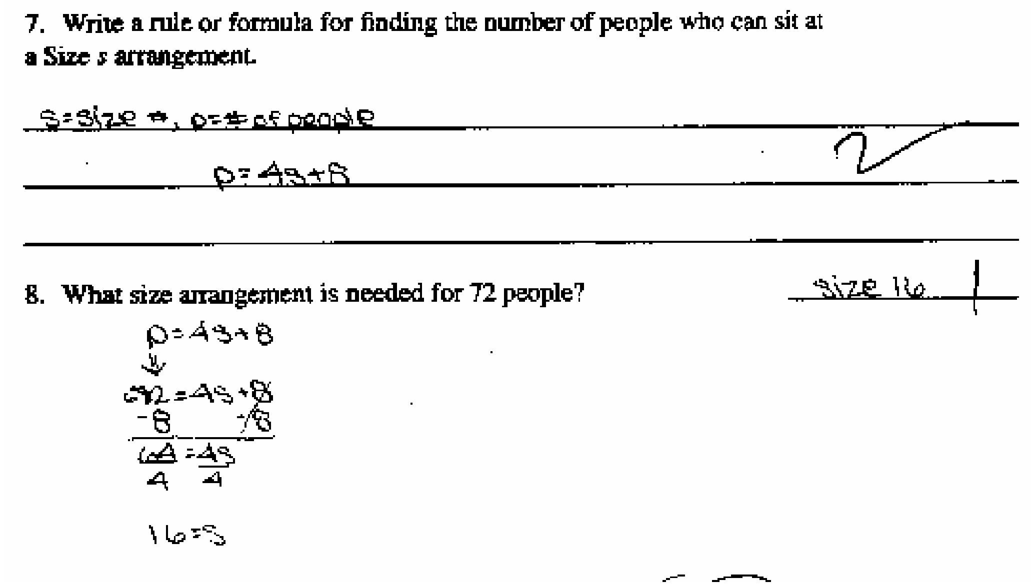
2. How many tables are in a Size 7 arrangement? 16 tables 1

3. Write down a rule or formula for working out how many tables there are when you know the size number.

S=size #, t=tables

$t = 2s+2$ 2✓

Student B


7. Write a rule or formula for finding the number of people who can sit at a Size *s* arrangement.

s=size #, p=# of people

p=4s+8

8. What size arrangement is needed for 72 people? size 16

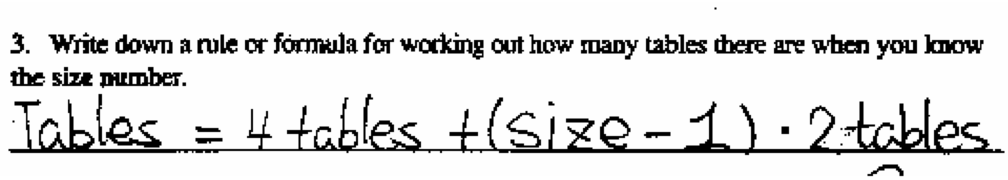
p=4s+8

72=4s+8
-8 -8
64=4s
4 4

16=s

Student C has a formula like the one of Student A, but the labels provided give clues that the student most likely used a visual strategy. The student probably saw that each arrangement has four tables in the corners. Then the middle tables are equal to the side number minus one. There are two rows of middle tables. Getting students to be able to verbalize what they are seeing, what attributes they are paying attention to, helps them to use symbolic notation to make generalizations in problem situations.

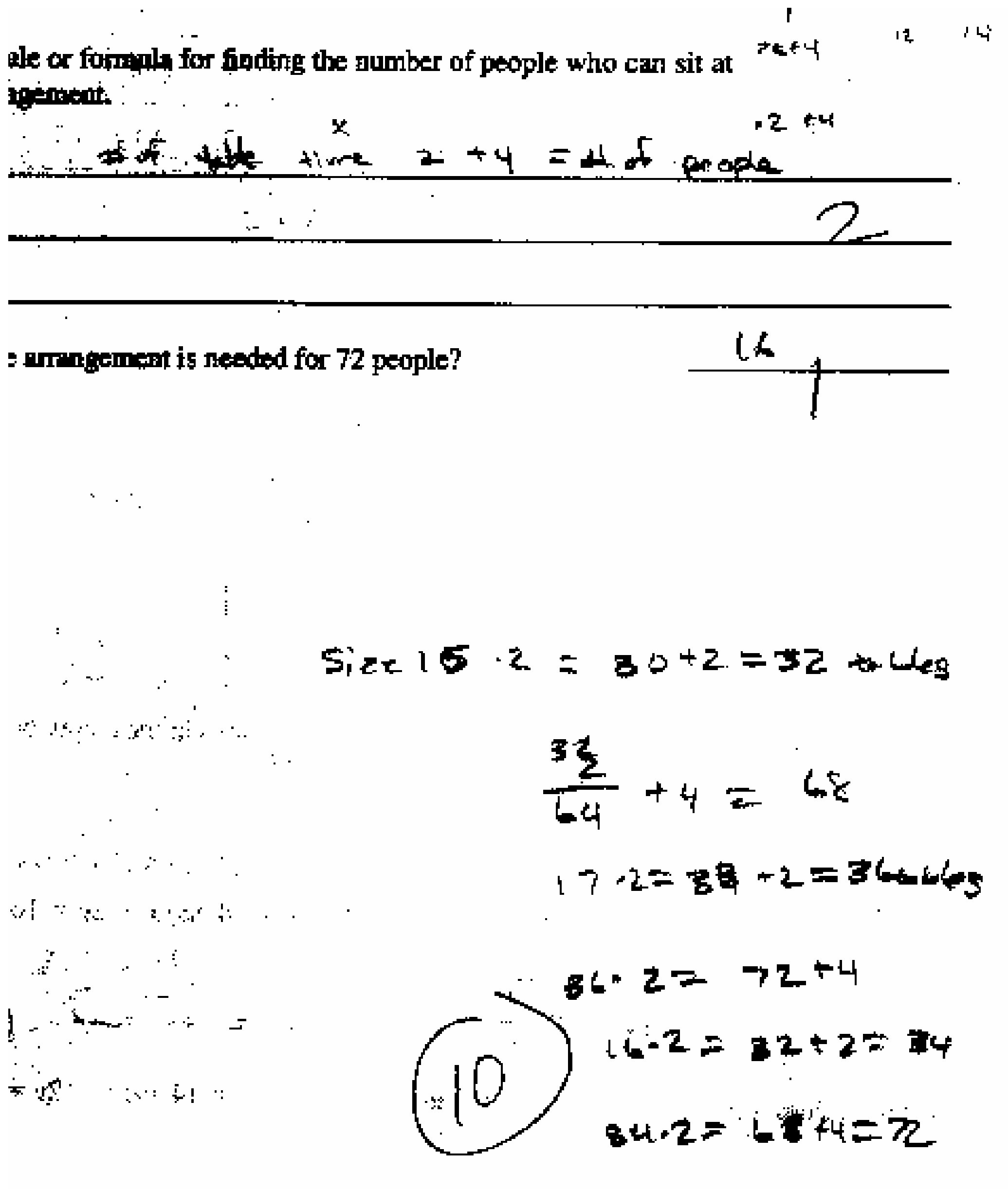
Student C


3. Write down a rule or formula for working out how many tables there are when you know the size number.

Tables = 4 tables + (size − 1) · 2 tables

Some students used a two-step rule for finding the number of people in part 7. The awkwardness of these rules made students resort to a guess and check strategy (see Student D) or make errors and arrive at unreasonable answers (see work of Student E).

Student D


ule or formula for finding the number of people who can sit at
ngement.

\# of table size × 2 + 4 = # of people

e arrangement is needed for 72 people? 16

Size 15 · 2 = 30 + 2 = 32 table leg

32/2 = 64 + 4 = 68

17 · 2 = 38 + 2 = 36 table leg

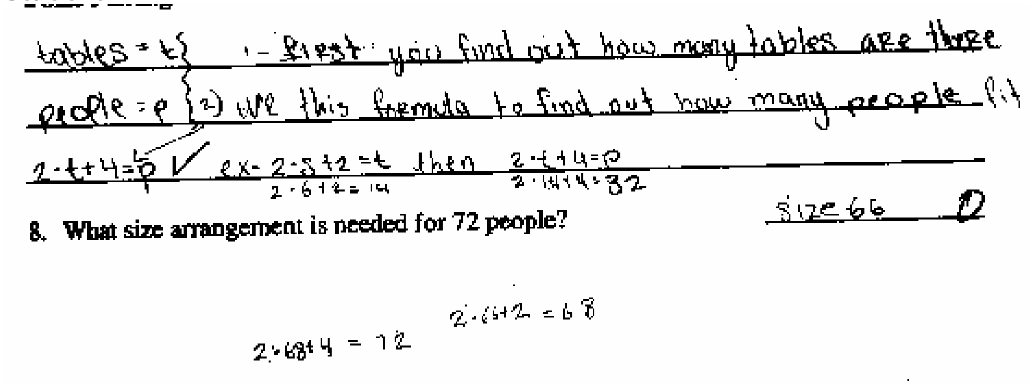
36 · 2 = 72 + 4

16 · 2 = 32 + 2 = 34

34 · 2 = 68 + 4 = 72

10

Student E



tables = t } 1 - first: you find out how many tables are there.

people = p } 2) use this formula to find out how many people fit:

2·t+4=p ✓ ex- 2·5+2=t then 2·t+4=p

2·6+2=14 2·14+4=32

8. What size arrangement is needed for 72 people? size 66 D

2·65+2 = 68

2·68+4 = 72

Most students who attempted a rule wrote a recursive expression. Using this approach students needed to continue a table (see Student F) or draw a picture and count to solve part 8 (see Student G). These strategies are cumbersome and often lead to errors.

Student F

7. Write a rule or formula for finding the number of people who can sit at a Size *s* arrangement.

~~Size 1~~ 12 people sit at the table Size 2 16 people sit at the table and Size 3 20 people sit at the table so it goes by ~~four~~ more people sit at the table? Like in Size 6 there are 14 tables and 32 people.

8. What size arrangement is needed for 72 people? 6

Size 3 = 8 tables = 20 people
Size 4 = 10 tables = 24 people
~~Size~~ Size 5 = 12 tables = 28 people
Size 6 = 14 tables = 32 people
Size 7 = 16 tables = 36 people
Size 8 = 18 tables = 40 people
Size 9 = 20 tables = 44 people
Size 10 = 22 tables = 48 people
Size 11 = 24 tables = 52 people
Size 12 = 26 tables = 56 people
Size 13 = 28 tables = 60 people
Size 14 = 30 tables = 64 people

(5)

10

Page 12 Conference Tables Test 9: Form A

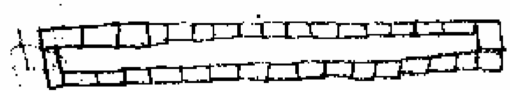
Student G

[illegible] many people can sit around a Size [illegible] arrangement?

7. Write a rule or formula for finding the number of people who can sit at a Size n arrangement.

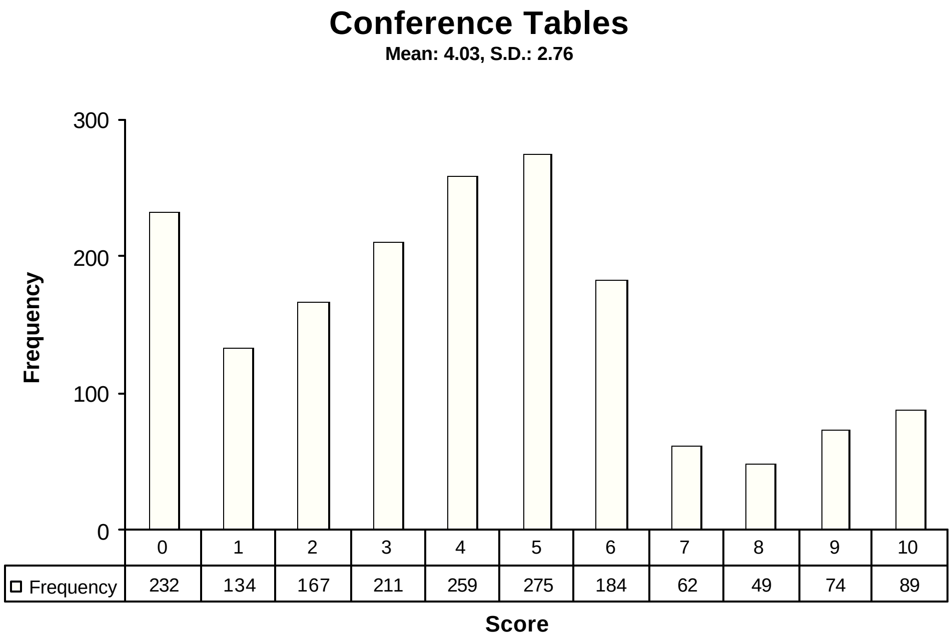
You [illegible] adding four to how ever many people there are. C

8. What size arrangement is needed for 72 people? 20 tables (



Teacher Notes:

Grade 9 – Conference Tables



Score:	0	1	2	3	4	5	6	7	8	9	10
% < =	13.4%	21.1%	30.7%	42.9%	57.8%	73.6%	84.2%	87.8%	90.6%	94.9%	100.0%
% > =	100.0%	86.6%	78.9%	69.3%	57.1%	78.9%	26.4%	15.8%	12.2%	9.4%	5.1%

The maximum score available for this task is 10 points.
The cut score for a level 3 response is 6 points.

Most students (87%) could count the number of tables in the diagram. Many students (about 79 %)could count the number of tables and use the diagram to find the number of people in the diagram for size 3 or count the number of tables in size three and extend the pattern for tables to size 7. More than half the students could find and extend the pattern for tables for size 7 and 13 and find the number of people in size three in the diagram. 16% of the students could correctly identify one or both rules for the geometric patterns. About 13% of the students scored zero points on this task. More than 80% of those students with a score of zero did not attempt the task.

Conference Tables

Points	Understandings	Misunderstandings
0	About 14% of the students scored zero points. 2/3 of these are students who didn't attempt anything else on the test.	Time might have been an issue on this task. Only 20% of the students with this score attempted the task.
2	Students with this score could find the number of tables in a size 3 and either extend the tables to size 7 or find the number of people in a size 3 arrangement.	The most common errors for question 2 were 14, 18,36, and 12. Each of these occurred with about equal frequency. The most common errors for question 6 were no response or 28. These were in a ratio of 2/1.
4	Students with this score could find the number of tables for size 3 and 7 and the number of people for size 3 and 6.	Students didn't have strategies to help them extend the table pattern to size 13 or to work backwards from 72 people to the size number. Less than 10% of the students thought to use drawings or extend a table to help them solve the problem. The most common error for number of tables in size 13 was 26.
5	Students could find the number of tables for various sizes and the number of people for various sizes.	They could not use the inverse relationships to find the size number when given the number of people. They are not comfortable with doing and undoing or working backwards.
6	Students could work with all the numeric relationships including working backwards from people to size numbers.	Students could not generate a verbal rule or formula to express either the pattern for tables or the pattern for people. More than 35% of all students wrote recursive rules for number of tables and number of people. They did not look for functional relationships.
8	Students could find one or both rules, but may have missed some part of extending the patterns. About 24% were able to come up with at least one rule. Most of those used symbolic notation.	Students with this score may have had difficulty find the rule for people. If they found a rule for people, it was often a two-step rule requiring them to find the number of tables first. In this case many had difficulty using their rule to help them solve for size number when given the number of people.
10	Students could find and extend patterns. Use inverse relationships to work backwards. They could generate a rule or formula to describe the patterns.	

Based on teacher observations, this is what Algebra One students seemed to know and be able to do?

- Count tables and people in a diagram.
- Find recursive rules to describe a pattern.
- Extend a pattern.

Areas of difficulty for Algebra One students, Algebra One students struggled with:

- Working backwards or using inverse relationships.
- Generating functional rules to describe patterns.

Teacher Notes:

Questions for Reflection on Conference Tables:

Look carefully at your student work. In attempting to generate rules, how many of your students made:

Recursive Rule (+2, +4)	Verbal Rule	Symbolic rule	Alternate Rule for People using # of tables instead of size #	No response

- What types of opportunities have students had to look at patterns and generate rules or formulas? What are some of your favorite problems for developing this skill? What kinds of questions can you ask to help students see the usefulness of functional rules over recursive rules?
- When testing for understanding, students need to be asked questions given various pieces of information. Do you give students opportunities to work backwards or use inverse operations? What types of discussions do you have to help students understand the mathematical principles involved in inverse operations?

When students make errors, there is often so common (but faulty) logic at work. Sometimes understanding that logic gives insight into ways to help further their mathematical progress.

- Look at problem #2. What might be the thinking that would lead a student to come up with an answer of 14? 18? Or 12?
- Look at problem #4. Why might a student give an answer of 26?
- Look at problem #6. Why might a student give an answer of 14 or 28?

- Answers for question 8 varied immensely, with a range of 3 to 936. What logic lies behind an answer of 18 or 288? Are there any other frequent answers in you student papers?
- How does thinking about this logic make you think about questions to ask in your classroom to help students uncover their faulty logic? What do you think the big ideas are, which underlie their error patterns? What further experiences do they need?
- What are your classroom norms for showing or explaining your thinking? As you looked at your student papers, could you tell who used visual thinking to generate rules, who looked at the numeric patterns, and who had a procedure for building rules from tables? Did you see evidence of students checking their rules with known numbers to verify that they were correct?

Instructional Implications:

Students at this grade level need to be able to go beyond counting and add-on strategies. They should be looking for and generating functional rules when looking at pattern problems. Students need frequent opportunities to generate their own algebraic expressions and use them to solve problems, including inverse operations. Students should be comfortable with use of symbolic notation and the ability to use appropriate variables in generating their rules.

Teacher Notes:

Number Towers – MAC Core Ideas

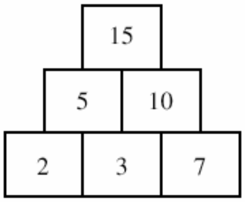
Student Task	Combine numbers and variables using addition or multiplication to fill in blanks in a number tower. Use symbol manipulation to prove why expressions from the number tower are equivalent to given expressions. Find values of unknowns in equations.
Core Idea 3 Algebraic Properties and Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** Understand the meaning of equivalent forms of expressions, equations, inequalities, or relations. Write equivalent forms of equations, inequalities and systems of equations and solve them. Use symbolic algebra to represent and explain mathematical relationships.
Core Idea 2 Mathematical Reasoning	**Employ forms of mathematical reasoning and proof appropriate to the solution of the problem.**

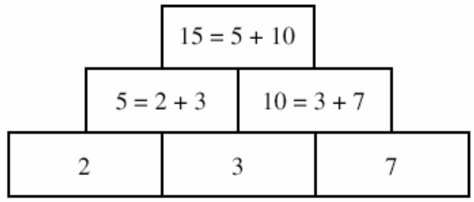
Number Towers

This problem gives you the chance to:
- form and solve equations obtained from a number pattern

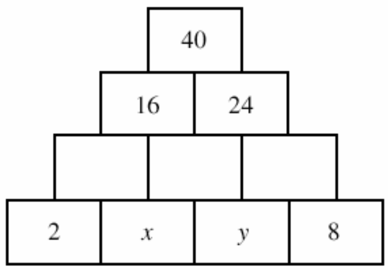
An addition number tower is shown below.

In this tower, each number is the sum of the two numbers just below it.





Marcie makes a bigger number tower:

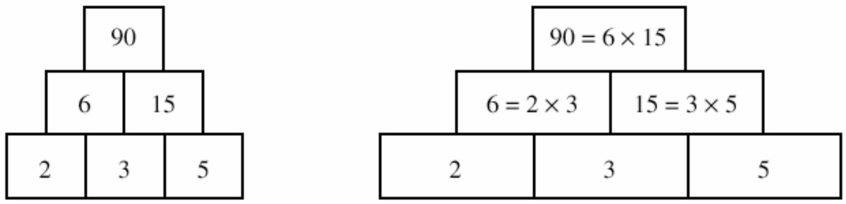


1. Complete the number tower by writing algebraic expressions in the empty boxes.

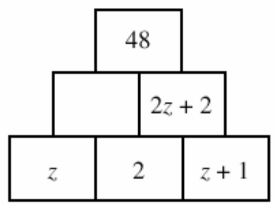
2. Use the completed number tower to show that $2x + y = 14$ and $x + 2y = 16$.

3. Find values of x and y that satisfy both equations: $2x + y = 14$ and $x + 2y = 16$.

Marcie also designs a **multiplication** number tower.



Look at the multiplication tower below.



4. Complete the multiplication tower by writing an algebraic expression in the empty box.

5. Use the completed multiplication tower to show that $4z^2 + 4z = 48$.

6. Find the possible values of z in $4z^2 + 4z = 48$.

10

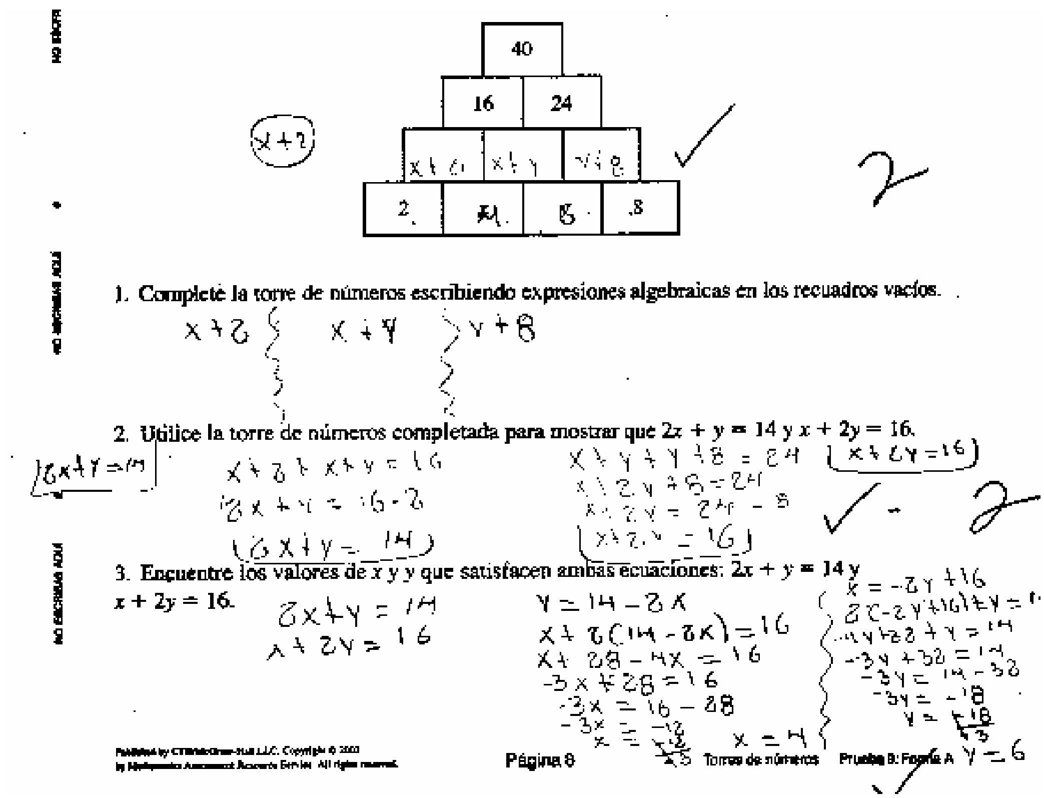
Number Towers		**Test 9 Form A Rubric**
The core elements of performance required by this task are: • form and solve equations obtained from a number pattern Based on these, credit for specific aspects of performance should be assigned as follows:	**Points**	**Section Points**
1. Gives correct answers as: **$2 + x$, $x + y$, $y + 8$** All three correct answers: 2 points *Partial credit:* Two correct answers: 1 point	2 (1)	2
2. Gives correct answers as: **$2 + 2x + y = 16$** therefore $2x + y = 14$ **$x + 2y + 8 = 24$** therefore $x + 2y = 16$	1 1	2
3. Gives correct answers as: $x = $ **4** $y = $ **6**	1 1	2
4. Gives correct answer as: **$2z$**	1	1
5. Gives correct answer as: **$2z(2z + 2) = 48$** therefore $4z^2 + 4z = 48$	1	1
6. Gives correct answers as: $z^2 + z - 12 = 0$ $(z + 4)(z - 3) = 0$ $z = $ **3 or $^-4$** *Partial credit for only one solution.*	 2 (1)	2
Total Points		**10**

Looking at Student Work – Number Towers

Students did not do well on Number Towers. Of the 150 papers reviewed, 56 did not attempt the problem and an additional 51 scored no points on the task.

Student A shows evidence of understanding the patterns within the number towers and being able to represent each space symbolically. This is shown clearly in the logic structure used for the proofs in part 2. This student understands the tools of algebra and can solve simultaneous equations with two unknowns in part 3. The student also uses the quadratic equation to solve for the missing roots in part 6.

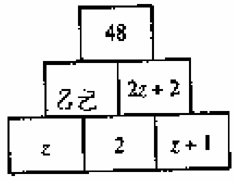
Student A


1. Complete la torre de números escribiendo expresiones algebraicas en los recuadros vacíos.

2. Utilice la torre de números completada para mostrar que $2x + y = 14$ y $x + 2y = 16$.

3. Encuentre los valores de x y y que satisfacen ambas ecuaciones: $2x + y = 14$ y $x + 2y = 16$.

Página 8 Torres de números Prueba 8: Forma A



4. Complete la torre de multiplicación escribiendo una expresión algebraica en el recuadro vacío.

5. Utilice la torre de multiplicación to show that $4z^2 + 4z = 48$.

$2z(2z+2) = 48$

$4z^2 + 4z = 48$

6. Encuentre los valores posibles de z en $4z^2 + 4z = 48$.

$4z^2 + 4z - 48 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{(-4) \pm \sqrt{(4)^2 - 4(4)(-48)}}{2(4)}$

$x = \frac{(-4) \pm \sqrt{16 + 768}}{8}$

$x = \frac{(-4) \pm \sqrt{784}}{8}$

$x = \frac{-4 \pm 28}{8}$

$x = \frac{-4 + 28}{8} = 3$

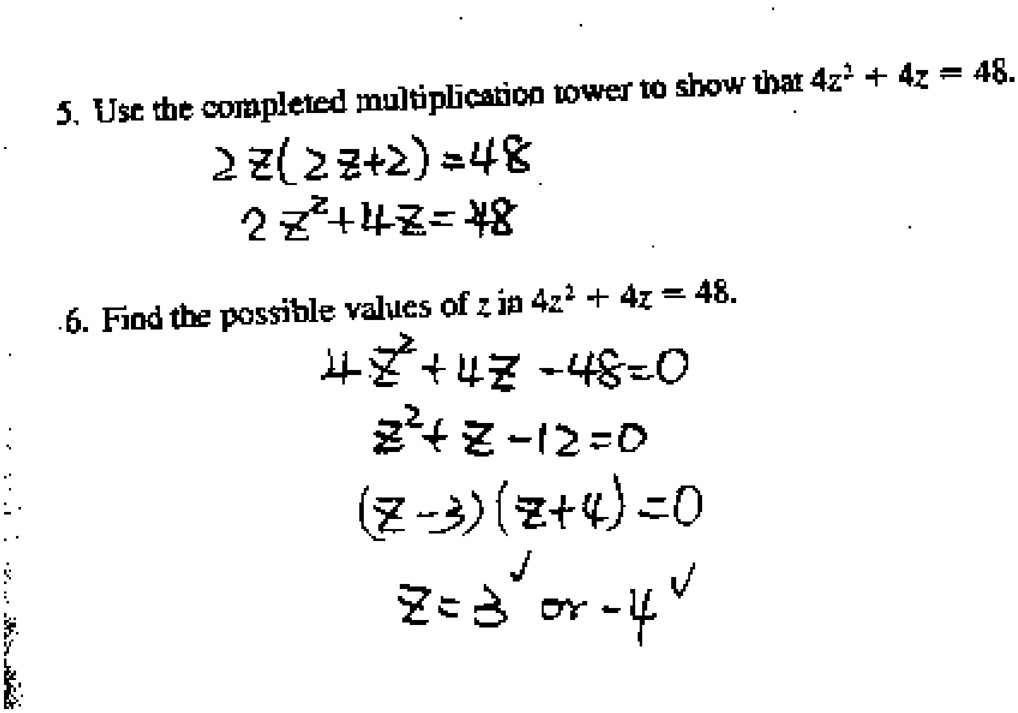
$x = \frac{-4 - 28}{8}$

x = 3
x = -4

10

Student B demonstrates the same understandings, but chooses to use factoring to solve for the roots in part 6 of the task.

Student B


5. Use the completed multiplication tower to show that $4z^2 + 4z = 48$.

$2z(2z+2)=48$

$2z^2+4z=48$

6. Find the possible values of z in $4z^2 + 4z = 48$.

$4z^2+4z-48=0$

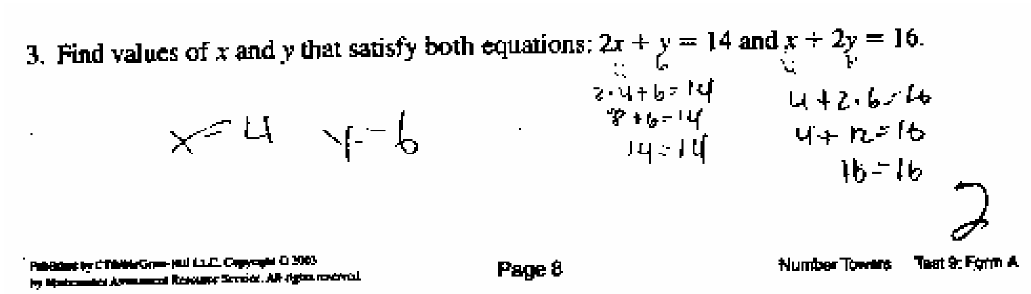
$z^2+z-12=0$

$(z-3)(z+4)=0$

$z=3$ ✓ or -4 ✓

The most challenging demand of the task was to use algebra to prove why certain algebraic expressions were true. Students did not understand how to do that piece particularly for part 2. However many students with weaker skills good solve pa rt 3 and 5 using guess and check or substitution. See the work of Student C.

Student C


3. Find values of x and y that satisfy both equations: $2x + y = 14$ and $x + 2y = 16$.

$x=4$ $y=6$

$2\cdot 4+6=14$

$8+6=14$

$14=14$

$4+2\cdot 6=16$

$4+12=16$

$16=16$

2

Page 8 Number Towers Test 9: Form A

5. Use the completed multiplication tower to show that $4z^2 + 4z = 48$.

$2z(2z+2)$

$4z^2 + 4z = 48$

$4(3)^2 + 4(3) = 48$

$4(9) + 12 = 48$

$36 + 12 = 48$

$48 = 48$

6. Find the possible values of z in $4z^2 + 4z = 48$.

$4(3)^2 + 4(3) = 48$

$4(9) + 12 = 48$

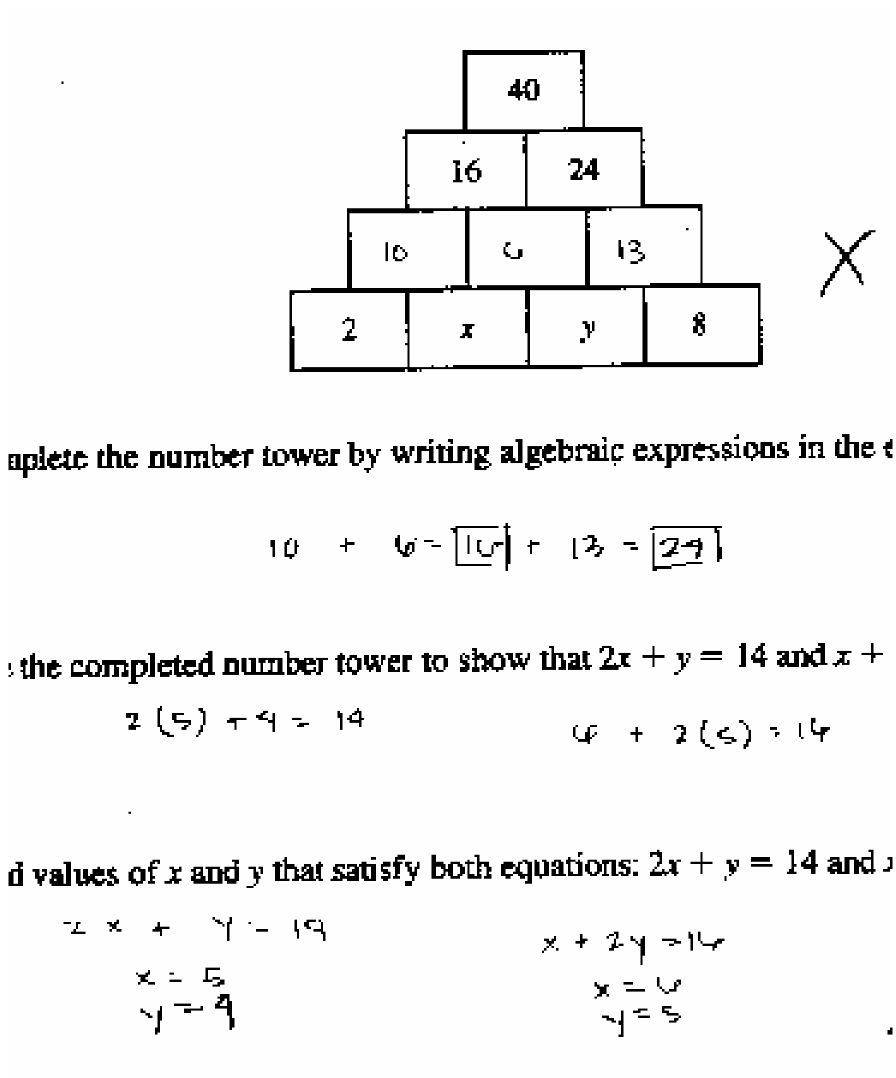
$36 + 12 = 48$

$48 = 48$

$z = 3$

More than half the students who responded to Number Towers put numbers into the blank boxes in the number towers instead of algebraic expressions. Student D picks numbers, which will add to 16, but then puts in an incorrect number for the third box. In doing guess and check in part 3, the student does not understand the concept that the two equations represent conditions that must both be true. The student just generates different values for x and y to satisfy each equation. This is a very fundamental issue in working with algebra.

Student D



40

16 24

10 6 13

2 x y 8

mplete the number tower by writing algebraic expressions in the e

10 + 6 = 16 + 13 = 29

the completed number tower to show that $2x + y = 14$ and $x +$

2 (5) + 4 = 14 6 + 2 (5) = 14

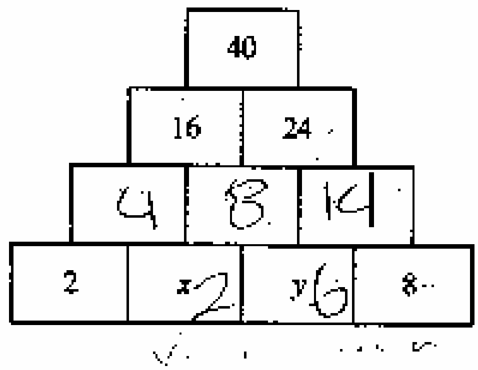
d values of x and y that satisfy both equations: $2x + y = 14$ and

2x + y = 14
x = 5
y = 4

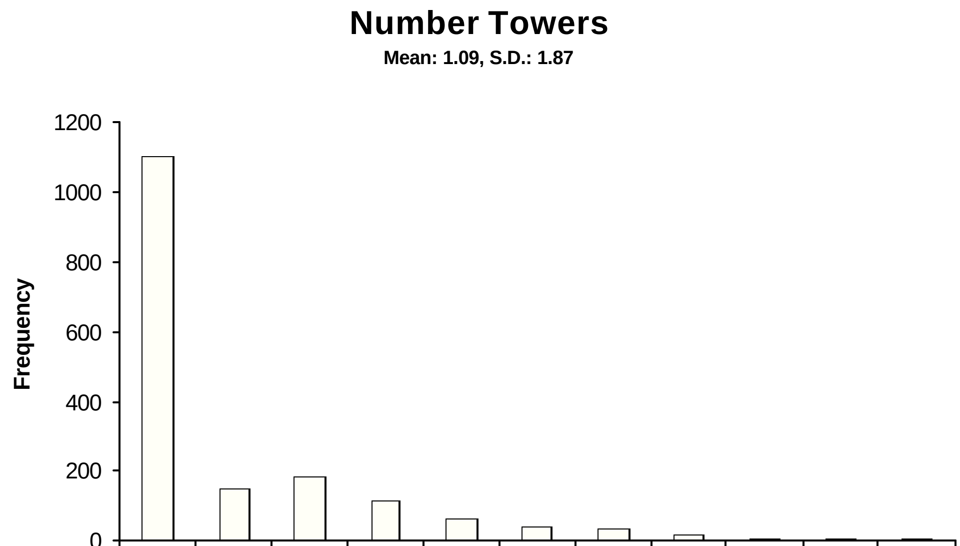
x + 2y = 14
x = 6
y = 5

Student E generate possible values for x and y in the bottom row of the tower, then uses the addition rule to fill in the missing numbers in the second row. The student does not appear to notice that the numbers in the second row will not produce the sums given for the third. Student E does not respond to any further parts of the task.

Student E



Teacher Notes:



	0	1	2	3	4	5	6	7	8	9	10
Frequency	1105	148	184	115	66	39	34	21	9	8	7

Score

Score:	0	1	2	3	4	5	6	7	8	9	10
% < =	63.7%	72.2%	82.8%	89.4%	93.2%	95.4%	97.4%	98.6%	99.1%	99.6%	100.0%
% > =	100.0%	36.3%	27.8%	17.2%	10.6%	6.8%	4.6%	17.2%	1.4%	0.9%	0.4%

The maximum score available for this task is 10 points.
The cut score for a level 3 response is 5 points.

Some students (about 36%) could find one correct solution to satisfy the equations in part 3 or could fill in the correct symbolic expression for the multiplication tower. Less than 10% of the students met standard on this task. Some of these students managed to do this use guess and check strategies to solve for unknowns. More than 60% of the students scored zero on this task. About half of the students who scored zero attempted the task.

Number Towers

Points	Understandings	Misunderstandings
0	More than 60% of the students scored no points on this task. About half of them attempted the task.	Students who attempted the task did not understand how to use symbolic notation to continue the pattern in the towers. They used numbers instead of expressions to fill in the blank boxes. While some of the numbers may have fit the pattern, there were still flaws in the solution.
2	Students with this score could usually use a guess and test or substitution strategy to find the correct values for the equations in part 3. A few students could write symbolic expressions in the number towers.	
3	Students could fill in the symbolic expressions for both number towers.	
5	Students could use substitution to find the unknowns in 3 and the positive root in 6. They could fill in the multiplication tower and use the distributive property to make the proof for part 6.	Only 11 of the 150 students used algebra to solve part 3. Another 2 students attempted to use algebra in part 3.
7	Students with this score could not find the negative root for part 6 and made one other error in some other part of the task.	2 students used the quadratic equation successfully in part 6, another 2 attempted to use the quadratic formula. 2 students used factoring successfully in part 6 and 7 attempted to use factoring.
10	Students could use symbolic notation and convert among equivalent expressions to form a proof. Students could solve two equations for two unknowns using guess and check or algebra. Students could find roots for a quadratic equation using factoring or the quadratic formula.	

Based on teacher observations, this is what Algebra One students seemed to know and be able to do:

- Use guess and check or substitution to solve for two unknowns in two equations.
- Write symbolic multiplication expressions.
- Simplify a multiplication expression using the distributive property.

Areas of difficulty for Algebra One students, Algebra One students struggled with:

- Writing symbolic addition expressions
- Checking their solutions to see if they fit all the conditions or parameters of the task
- Using symbolic expressions to form equivalent expressions
- Solving two equations for two unknowns
- Using factoring or the quadratic formula to find missing roots

Questions for Reflection on Number Towers

- Did your students use numbers or symbolic expressions to fill in the number towers? If they used numbers, did the numbers fit all parameters of the tower, e.g. did the numbers lead to the correct final solution? What are your classroom norms around checking having students check their work? What do you think they didn't understand about the pattern?
- What opportunities have your students had to use algebra to do a proof? What types of experiences or problems might help them develop these skills? What would you have liked to seen in a complete and thorough explanation for part 2? What skills would be needed for that solution?
- While guess and check or substitution is a useful tool for solving problems, at this grade level students need to have their skills expanded to use more efficient strategies. What types of questions can help students see the application of the algebra for solving problems? How do can you provide learning experiences to help students transition from guess and check to using algebra?

Looking at student work, how many of your students in part 3:

Did not attempt this part?	Used guess and check or substitution?	Used algebra to solve?

In part 6, how many of your students:

Did not attempt this part?	Used the quadratic formula?	Used factoring?

- Do you think students would have done better on this problem if they had more time? Where might this problem fit into your curriculum to test for these understandings?

Teacher Notes:

Instructional Implications:

Students at this grade level need frequent opportunities to describe situations in symbolic notation. It is important that they not just use expressions provided for them for symbol manipulation. Students also need to develop their logic and have opportunities to use algebra to prove conjectures or rectify different solutions or formulae. Students should know that given two equations, the common variables stand for the same values. Students should be able to find positive and negative solutions for quadratic equations and have a variety of tools to help them.

Teacher Notes: