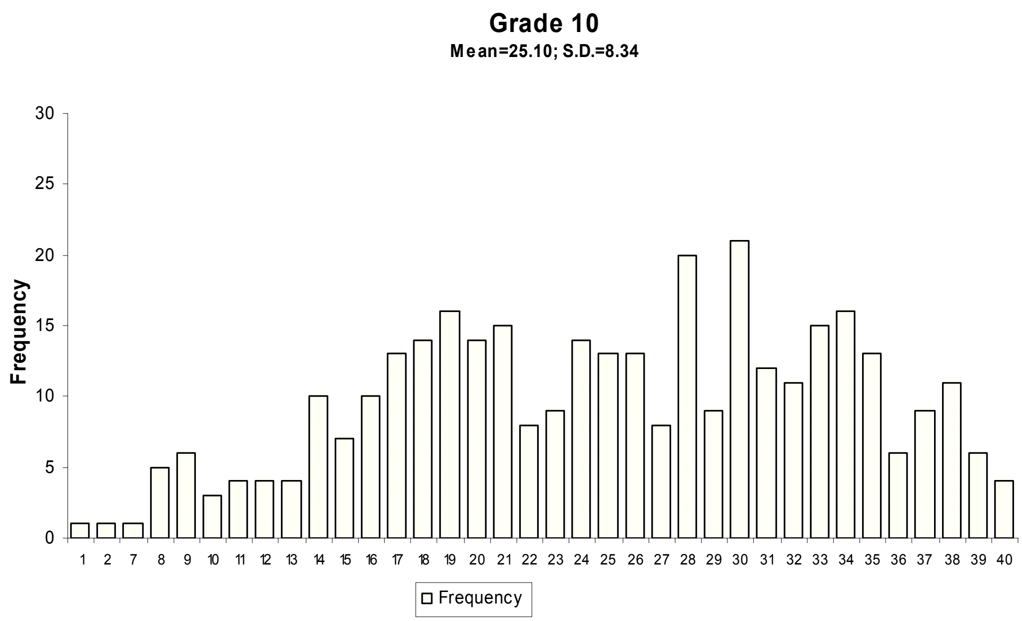
Overall Frequency Distribution by Total Score



Level Frequency Distribution Chart and Frequency Distribution

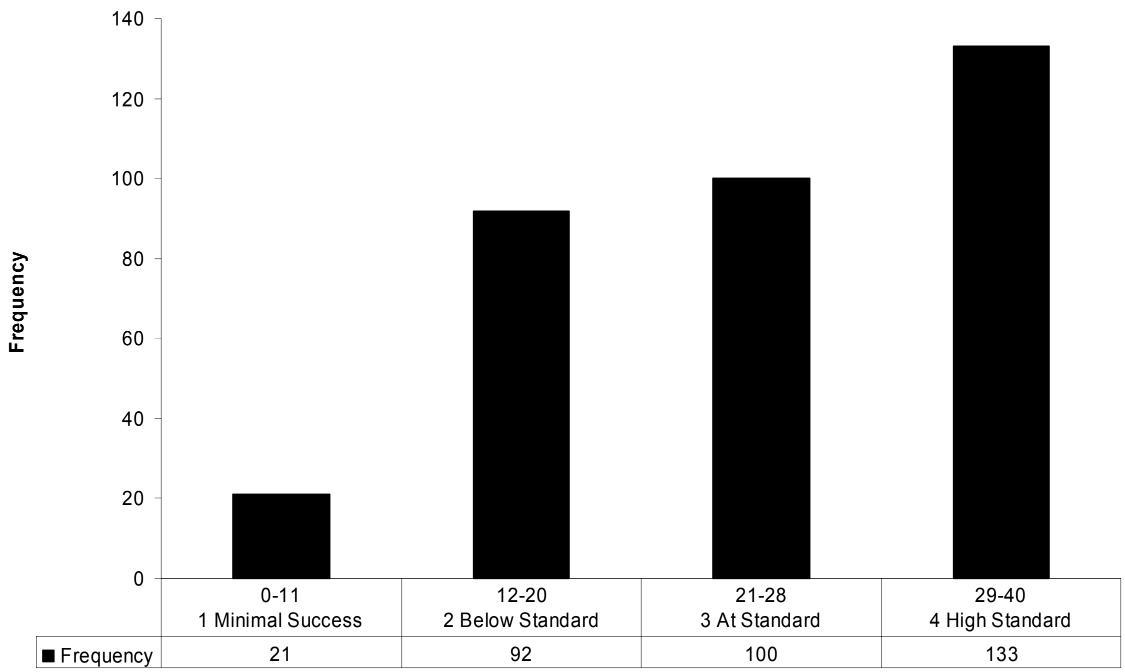
2004 - Numbers of students tested in Grade 10: 346

Grade 10 2000 - 2001

Level	% at ('00)	% at least ('00)	% at ('01)	% at least ('01)
1	33%	100%	32%	100%
2	45%	67%	52%	68%
3	18%	22%	14%	16%
4	4%	4%	2%	2%

Grade 10 2002 - 2004

Level	% at ('02)	% at least ('02)	% at ('03)	% at least ('03)	% at ('04)	% at least ('04)
1	44%	100%	25%	100%	6%	100%
2	40%	56%	39%	75%	27%	94%
3	9%	16%	23%	37%	29%	67%
4	6%	6%	14%	14%	38%	38%



Frequency	21	92	100	133

Course Two Task One Rectangles with Fixed Area

Student Task	Read and interpret tables about area, length, and height. Use table to plot graph and answer questions about non-linear functions and limits.
Core Idea 4: Geometry and Measurement	**Develop mathematical arguments about geometric relationships; and apply appropriate techniques, tools, and formulas to determine measurements.** • Understand and use formulas for area.
Core Idea 3: Algebraic Properties and Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** • Recognize and use equivalent graphical and algebraic representations of functions with their geometric characteristics such as intercepts.

Rectangles With Fixed Area

This problem gives you the chance to:
• explore the relationship between, length and height, in rectangles with a fixed area

Here we have a family of rectangles.

Each rectangle in this family has a fixed area of 24 square centimeters.

A few examples of this family of rectangles are shown in the table below:

1. Complete the last line of the table below.

Rectangle #	**Length (cm)**	**Height (cm)**	**Area (cm^2)**
Rectangle 1	48	0.5	24
Rectangle 2	24	1	24
Rectangle 3		2	24
Rectangle 4			24

2. Complete the table above by writing in the length and height of another rectangle with area 24 square centimeters.

3. Write a formula for the area A of the rectangle in terms of the length, l, and height, h.

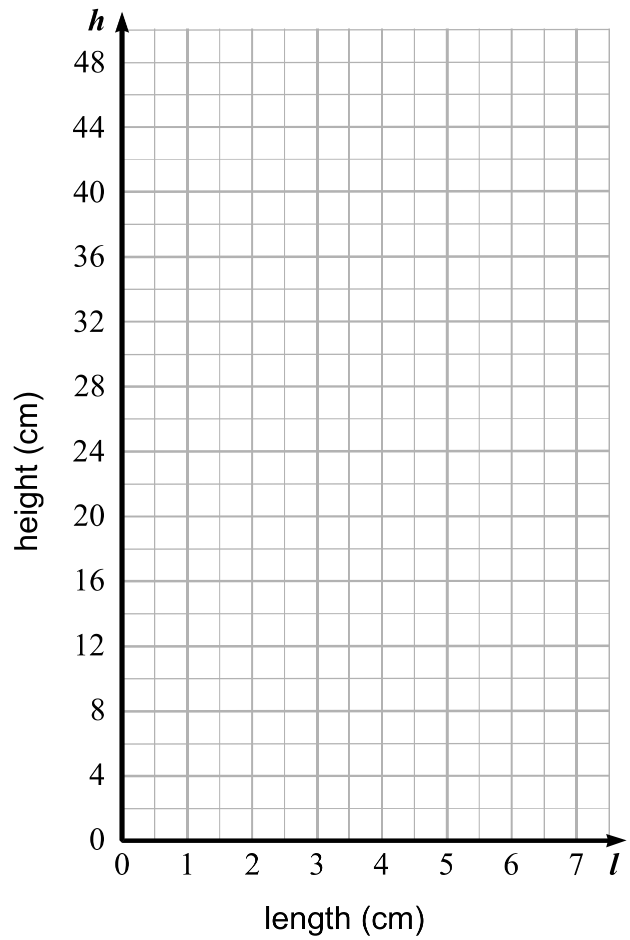
____

4. If the area of a rectangle is 24 square centimeters, then $h = \frac{24}{l}$.

Using the formula $h = \frac{24}{l}$, complete the table below:

l	0.5	1	1.5	2	3	4	5	6
h	48	24					4.8	

5. Use the above table of values to graph $h = \frac{24}{l}$.



6. How can you tell from the equation that it does ***not*** represent a linear function?

____

____

____

____

____

7. This graph does not cross the h-axis. Explain why this makes sense.

____

____

____

9

Rectangles With Fixed Area Test 10 Rubric		
The core elements of performance required by this task are: • explore the relationship between length and height, in rectangles with fixed area Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **12**	1	1
2. Gives correct answers: **6** and **4 or 8** and **3**	1	1
3. Gives correct answer: $A = lh$	1	1
4. Gives correct answers: **16, 12, 8, 6, 4** Four or five correct answers 2 points. *Partial credit* Two or three correct answers 1 point.	2 (1)	2
5. Plots a correct smooth curve. *Partial credit* Draws a graph with one error. **or** Points plotted but line not drawn.	2 (1)	2
6. Gives a correct explanation such as: The graph is not a straight line/is a curve. The equation is not of the form y = mx + c where m and c are constants.	1	1
7. Gives a correct explanation such as: If l = 0 then the rectangle becomes a straight line and the area = 0 **or** You cannot have a negative height/measurement.	1 **or** 1	1
Total Points		**9**

Looking at Student Work- Rectangles with Fixed Area

Many students did very well on this problem. Students could use the table and find missing lengths and heights necessary to keep the area the same. They could graph data from a table and make it into a smooth curve. Student A does a nice job of explaining the definition of a linear equation in part 6 and gives a contextual reason for why the curve does not cross the h-axis.

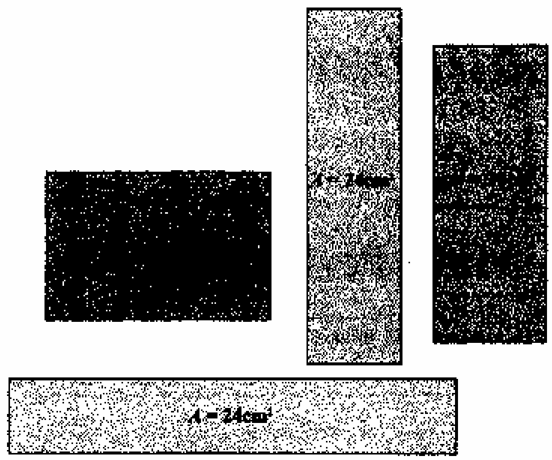
Student A

Rectangles With Fixed Area

This problem gives you the chance to:
• explore the relationship between, length and height, in rectangles with a fixed area

Here we have a family of rectangles.

Each rectangle in this family has a fixed area of 24 square centimeters.



A few examples of this family of rectangles are shown in the table below:

1. Complete the last line of the table below.

Rectangle #	Length (cm)	Height (cm)	Area (cm^2)
Rectangle 1	48	0.5	24
Rectangle 2	24	1	24
Rectangle 3	12 ✓	2	24
	✓		
Rectangle 4	6	4	24

2. Complete the table above by writing in the length and height of another rectangle with area 24 square centimeters.

3. Write a formula for the area A of the rectangle in terms of the length, l, and height, h.

$a = l \cdot h$ ✓

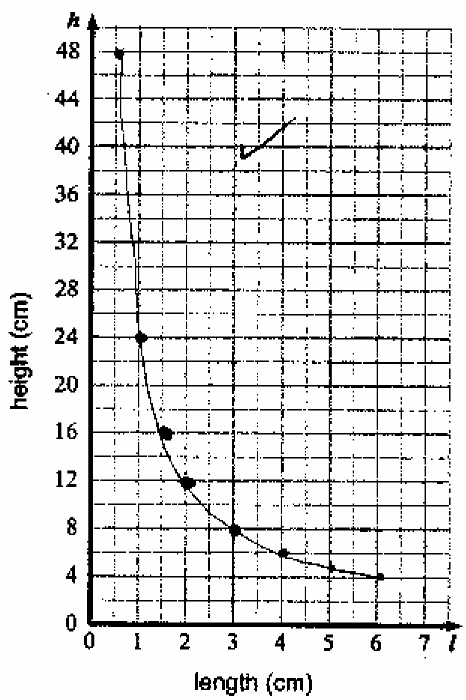
Student A

4. If the area of a rectangle is 24 square centimeters, then $h = \frac{24}{l}$.

Using the formula $h = \frac{24}{l}$, complete the table below:

l	0.5	1	1.5	2	3	4	5	6
h	48	24	16	12	8	6	4.8	4

5. Use the above table of values to graph $h = \frac{24}{l}$.



6. How can you tell from the equation that it does *not* represent a linear function?

If the equation $h = \frac{24}{l}$ were a linear equation, then the coordinates on the graph would be a line not a curve

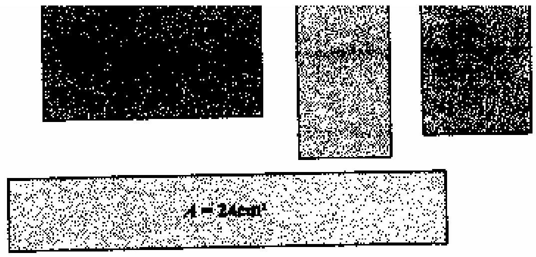
7. This graph does not cross the *h*-axis. Explain why this makes sense.

The graph does not cross the h axis because it is not possible for a side of a rectangle to have a negative length

Student B shows some of the thinking behind the answers in the table by listing the multiples of 24. Student B also lists two possibilities for Rectangle 4. Student B does not use the context to think about why the graph can't cross the h-axis. The student does not realize that graphs can curve and still cross an axis.

Student B

Multiples of 24:
1, 24
2, 12
3, 8
4, 6



rectangle 3:

$48 - 24 = 24$ $\quad$ $24 \div 2 = 12$

A few examples of this family of rectangles are shown in the table below:

1. Complete the last line of the table below.

Rectangle #	Length (cm)	Height (cm)	Area (cm^2)
Rectangle 1	48	0.5	24
Rectangle 2	24	1	24
Rectangle 3	12	2	24
Rectangle 4	8	3	24

2. Complete the table above by writing in the length and height of another rectangle with area 24 square centimeters.

	(length)	(height)	Area
rectangle 5	6	4	24

3. Write a formula for the area A of the rectangle in terms of the length, l, and height, h.

$A = lh$

Student B

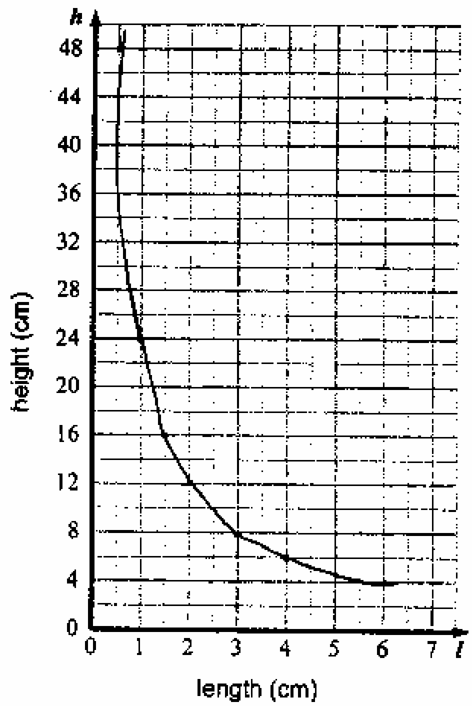
4. If the area of a rectangle is 24 square centimeters, then $h = \frac{24}{l}$.

Using the formula $h = \frac{24}{l}$, complete the table below:

l	0.5	1	1.5	2	3	4	5	6
h	48	24	16	12	8	6	4.8	4

2

5. Use the above table of values to graph $h = \frac{24}{l}$.



2

6. How can you tell from the equation that it does *not* represent a linear function?

The line is curved. linear means a straight line since the graph shows the line is curved, it cannot be linear

1

7. This graph does not cross the *h*-axis. Explain why this makes sense.

It's not a linear function. Therefore it doesn't have to intersect the h-axis. The graph shows the line is curved which automatically proves that it's not a linear function.

0

In general, students had difficulty with describing why the equation does not represent a linear function. Below are some typical responses:

- How can you tell from the equation that it does not represent a linear function: Because there is no relationship between them.
- It goes down.
- Because it is not linear to each other. There is no rectangle equation.
- I can tell you that from the equation that it doesn't represent a linear function because not all the functions are listed.
- I can tell because this equation has no y-value. A linear function has an x- and y- value.

Students also had difficulty explaining why the graph does not cross the h-axis. Some had trouble because they didn't think about the context or thinking about a limit. Some good answers included:

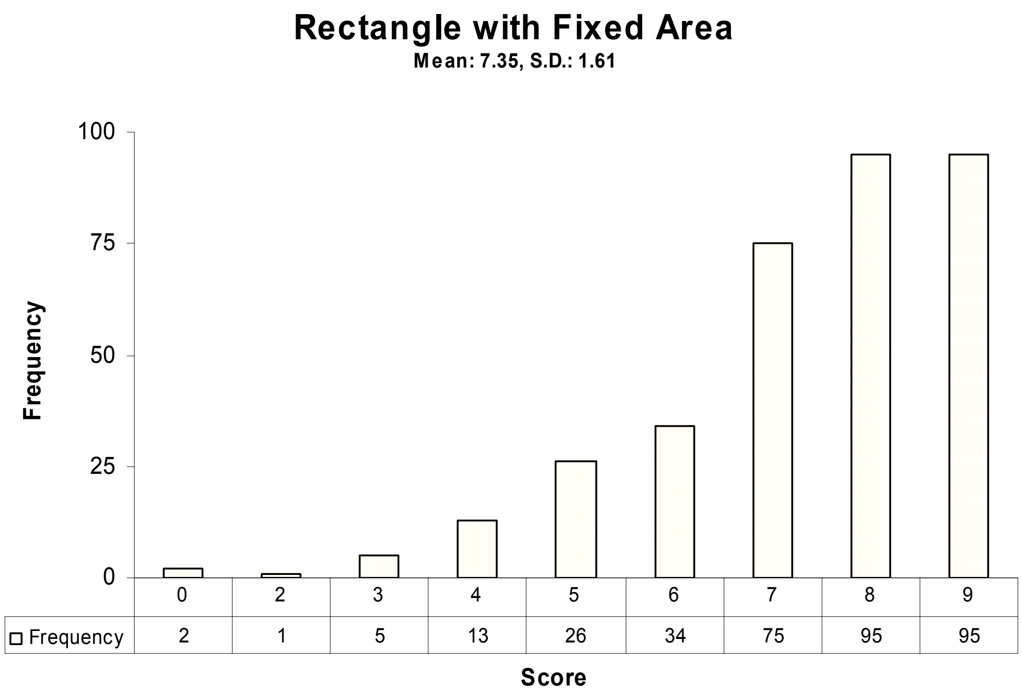
- A rectangle cannot have a negative area or height.
- Because in the equation you cannot have 0 as the denominator.

Some typical errors include:

- It is not a linear function.
- Because the chart gave us the answers to connect the dots.
- Because 48 is the highest it will go.
- Because 48 is double 24, so if it was going to cross the h axis the length would have to be very small.
- There is a down-slope arc, which will continue and result in not crossing the h-axis, also it began at a reasonable number.

Teacher notes:____

Frequency Distribution for each Task – Grade 10
Grade 10– Rectangle with Fixed Area



Score:	0	1	2	3	4	5	6	7	8
% < =	0.6%	0.9%	2.3%	6.1%	13.6%	23.4%	45.1%	72.5%	100.0%
% > =	100.0%	99.4%	99.1%	97.7%	93.9%	86.4%	76.6%	54.9%	27.5%

The maximum score available for this task is 9 points.
The cut score for a level 3 response, meeting standards, is 5 points.

86% of the students met standards on this task, they could fill in the table with correct values of length and height to make an area of 24 cm^2. Students with this score could also write a formula for area of a rectangle and fill in values in the table for height using the formula h=24/l. Many students, approximately 76% could also find plot the graph, but may have forgotten to connect the points to make a curve and could explain either why the graph was not linear or why the graph could not cross the h-axis. Almost 27% of the students could meet all the demands of the task. Less than 1% of the students scored no points on this task.

Rectangles with Fixed Area

Points	Understandings	Misconceptions
0	Less than 1 % of the students scored zero on this task.	
2	Students with this score could fill in lengths and heights on a table to make a rectangle with area of 24 cm^2.	The most common error on the table was to put the combination of 6 and 3 for rectangle 4. Students were trying to find a pattern within the table rather than making sense of the situation.
5	Students with this score could fill in a table about lengths and heights of a rectangle with an area of 24 cm^2. They could also use the formula h=24/l to fill in values in a second table. Students could also write the formula for finding area of a rectangle.	Some students forgot to answer the question about formula. Others confused area of a rectangle with the formula for circle or triangle giving answers like A^2=l x h, A= l x h^2, or A= 1/2 x h^2.
7	Students with this score could plot the values in the table and explain either why the graph was not linear or why the graph would not cross the h-axis.	About 25% of the students did not connect the points to make a curve. Almost 13% of the students graphed one incorrect point from the table.
8		Students with this score made a graphing error, couldn't explain a nonlinear function, or could not explain why the graph did not cross the h-axis. The errors were fairly evenly distributed between these three.
9	Students could find lengths and heights to make an area of 24 cm^2, use a formula to find missing heights, graph values from a table, and reason about why the function was not linear and about why the graph would not cross the h-axis. Their reasoning included ideas about not having negative areas or negative heights and lengths and/or not being able to divide by zero.	

Questions for Reflections on Rectangles with Fixed Area

- What opportunities have your students had to work on graphing linear and nonlinear functions? Do they have discussions about when it is appropriate or not appropriate to connect the points? Do they understand the meaning of what is represented by the line or curve between the points?
- What types of problems have students worked with that have limits? How do they attach meaning to why something does not cross a certain value? How can you help them develop an informal understanding of limits that will help them with later mathematical knowledge?
- Do students know the definition for linear function?

Teacher notes:____

Implications for Instruction:

Students need frequent opportunities to work with functions in context. At this grade level, the reasoning skills need to be expanded to consider how parts of equations relate to the shape of graphs and students should start to develop ideas about limits and why those limits might exist or make sense. Graphs need to be viewed from the perspective of conveying information. What does a curved line say about the situation? How is this different from the information conveyed by a straight line? Students need to continually be pushed to think about meaning as well as procedure, to view mathematics as a way of describing events, making predictions, and tools for making decisions.

Course Two Task 2 At the Gym

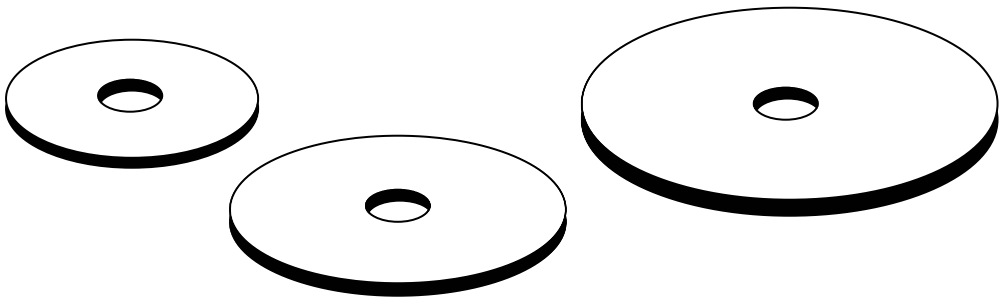
Student Task	Work with radius, area and volume of a set of solid circular discs. Apply and reason about volume in an unfamiliar setting.
Core Idea 4: Geometry and Measurement	**Apply appropriate techniques, tools, and formulas to determine measurements.** • Understand and use formulas for the area, surface area, and volume of geometric figures, including cylinders.

At the Gym

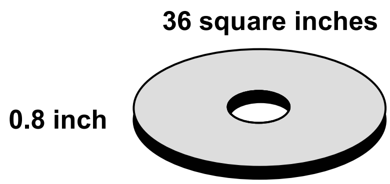
This problem gives you the chance to:
• work with radius, area and volume of a set of solid circular discs

At the gym, the weights class uses 3 different sizes of weights.

The weights are all circular with a hole in the center.



1. The area of the top surface of the smallest weight is 36 square inches and it is 0.8 inch thick.



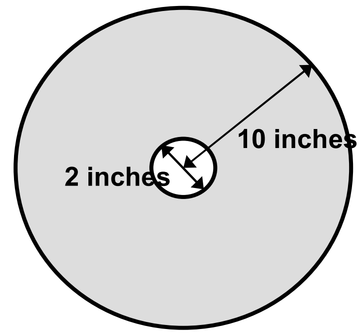
What is the volume of this weight? ____________________ cubic inches

Show your calculations.

2. This middle size weight has a radius of 10 inches.

   The hole in the middle has a diameter of 2 inches.

   What is the area of the top surface of this weight?



   ____________________ square inches

   Explain how you figured it out.

____

____

____

____

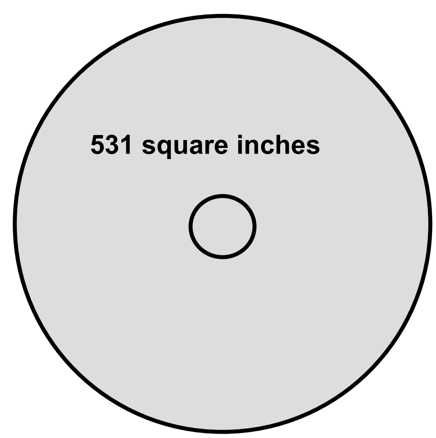
____

3. The area of the top surface of the largest weight is 531 square inches, ignoring the hole.

   Figure out the radius of the largest weight.

   ____________________ inches

   Show your work.



8

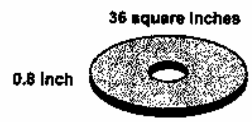
At the Gym		Test 10 Rubric
The core elements of performance required by this task are: • work with radius, area and volume of a set of solid circular discs Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives a correct answer: **28.8** cubic inches	1	
Shows work such as: 36 x 0.8	1	2
2. Gives a correct answer: **311** sq. ins. Accept 99π	1	
Gives explanation such as: Area of large circle: 314 sq. ins. Accept $10^2 x\pi$. Area of hole: 3 sq. ins. Accept $1^2 x\pi$. Area of weight = area large circle – area of hole	2	3
3. Gives correct answer: **13** inches	1	
Shows work such as: 531 / π	1	
and square root used.	1	3
Total Points		**8**

Looking at Student Work – At the Gym

About 16% of the students could meet all the demands of the task. Student A does a nice of showing formulas for calculations and describing in words how those calculations contributed to find the solution to the problem.

Student A

1. The area of the top surface of the smallest weight is 36 square inches and it is 0.8 inch thick.



What is the volume of this weight? 28.8 ✓ cubic inches

Show your calculations.

36 × 0.8 = 288, 000, 28.8 ✓

2. This middle size weight has a radius of 10 inches.

The hole in the middle has a diameter of 2 inches.

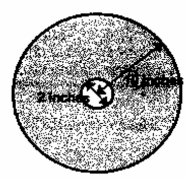
What is the area of the top surface of this weight?

310.86 ✓ square inches

Explain how you figured it out.

$a = \pi r^2$, $a = \pi(10)^2$, $a = 314$ ↑ entire

little. $a = \pi r^2$, $a = \pi(1)^2$, $a = 3.14$



The area of a circle is shown in the equation $a = \pi r^2$. Plugging in the number 10 as the radius for the entire circle the area of the entire circle would be 314 in². However, there is a hole in the middle, therefore we need to subtract the area of the little circle from the large one. Using the ✓ same equation the little circle in the middle has an area of 3.14 in² since the radius is 1. When subtracting 3.14 in² from 314 we get 310.86 in².

1

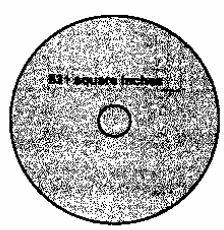
2

3. The area of the top surface of the largest weight is 531 square inches, ignoring the hole.

Figure out the radius of the largest weight.

≈ 13 ✓ inches

Show your work.



$a = \pi r^2$

$\frac{531}{\pi} = \frac{\pi r^2}{\pi}$

$\frac{531}{\pi} = r^2$ ✓

$\sqrt{\frac{531}{\pi}} = r$ ✓

$r \approx 13$ inches

1

1

1

8

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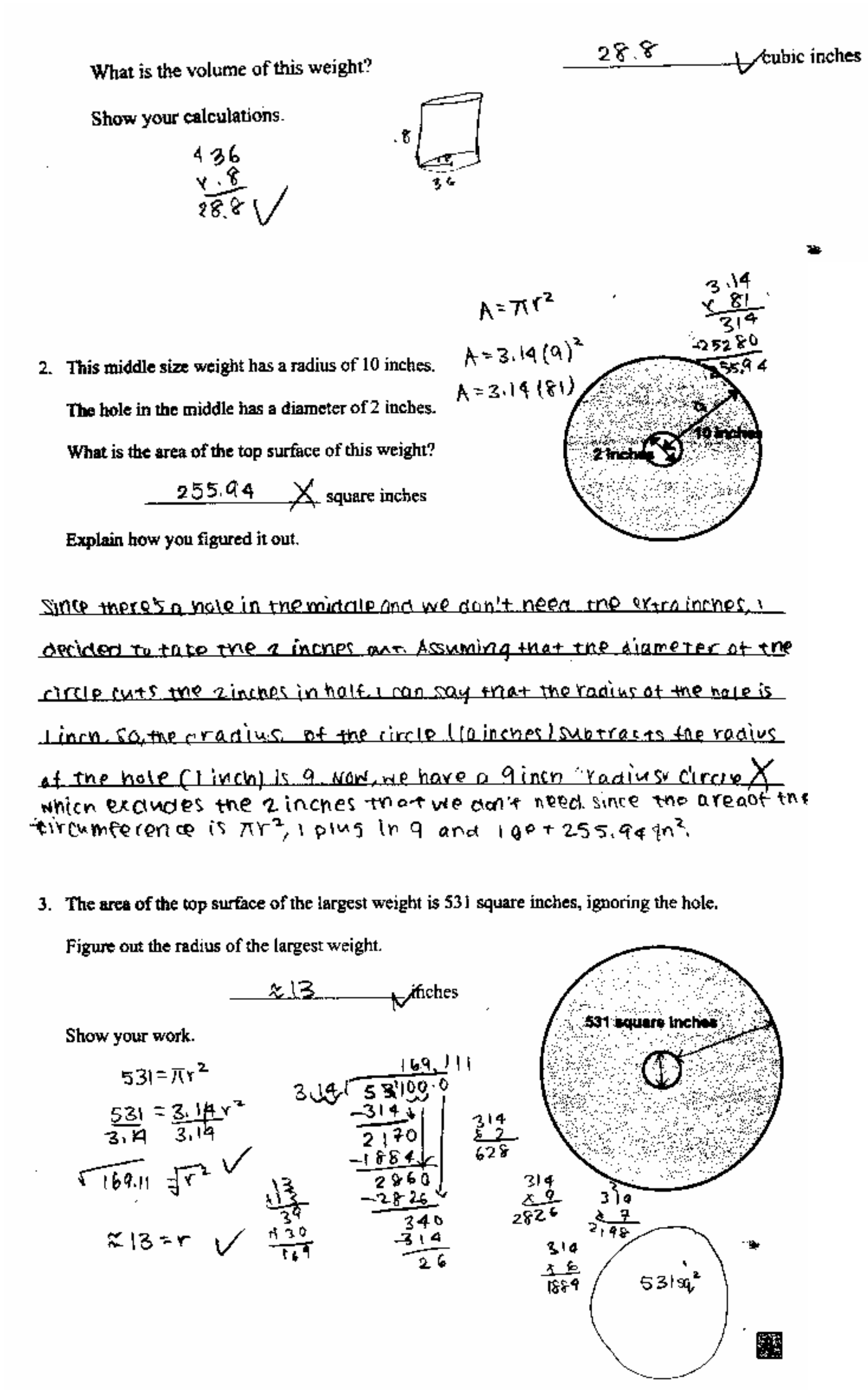
Student B shows evidence of using the diagram to help make sense of the task situation. The student knows that radius is important for finding area and divides the diameter by 2. However the student forgets to use this radius to calculate the area of the hole, subtracting instead the radius of the hole from the area of the outer circle.

Student B



What is the volume of this weight? 28.8 ✓ cubic inches

Show your calculations.

36 × .8 = 28.8 ✓

2. This middle size weight has a radius of 10 inches.

The hole in the middle has a diameter of 2 inches.

What is the area of the top surface of this weight?

255.94 ✗ square inches

Explain how you figured it out.

$A = \pi r^2$

$A = 3.14(9)^2$

$A = 3.14(81)$

Since there's a hole in the middle and we don't need the extra inches, I decided to take the 2 inches out. Assuming that the diameter of the circle cuts the 2 inches in half, I can say that the radius of the hole is 1 inch. So, the radius of the circle (10 inches) subtracts the radius of the hole (1 inch) is 9. Now, we have a 9 inch "radius" circle ✗ which excludes the 2 inches that we don't need. Since the area of the circumference is πr^2, I plug in 9 and I get 255.94 in².

3. The area of the top surface of the largest weight is 531 square inches, ignoring the hole.

Figure out the radius of the largest weight.

≈13 ✓ inches

Show your work.

$531 = \pi r^2$

$\frac{531}{3.14} = \frac{3.14 r^2}{3.14}$

$\sqrt{169.11} = \sqrt{r^2}$ ✓

$\approx 13 = r$ ✓

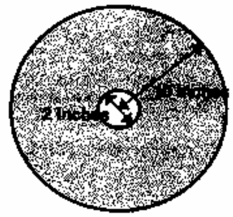
Many students had difficulty trying to figure out how the hole effected the area of the weight. Student C used the diameter instead of the radius to find area of the hole.

2. **This middle size weight has a radius of 10 inches.**

The hole in the middle has a diameter of 2 inches.

What is the area of the top surface of this weight?

301.6 X square inches



Explain how you figured it out.

first I got the entire area of the weight. I then got the area of the X hole. Finally I subtracted the area of the hole from the area of the entire weight.

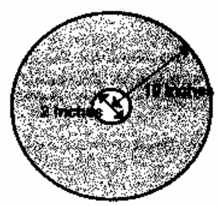
Other students had difficulty because they used the wrong formula. Student D uses the formula for circumference instead of the formula for area.

2. **This middle size weight has a radius of 10 inches.**

The hole in the middle has a diameter of 2 inches.

What is the area of the top surface of this weight?

about 56.549 / 18π X square inches



Explain how you figured it out.

First, the surface area of the weight without a hole would be 2πr (or πd). Substituting in the r (which is 10) into 2πr would result X in 20π. The area of the whole in the middle may be found by substituting 2 as d into πd, which results in 2π. 20π minus 2π is 18π, which is the area of the top surface of the weight. (≈ 56.549)

3. **The area of the top surface of the largest weight is 531 square inches, ignoring the hole.**

Figure out the radius of the largest weight.

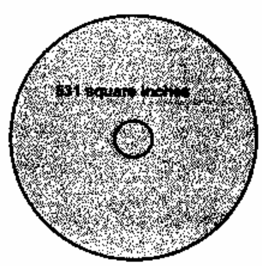
about 84.511 / X 265.5/π inches

Show your work.

$\frac{531}{2} = \frac{2\pi r}{2}$ X

$\frac{265.5}{\pi} = \frac{\pi r}{\pi}$ X

84.511 ≅ r



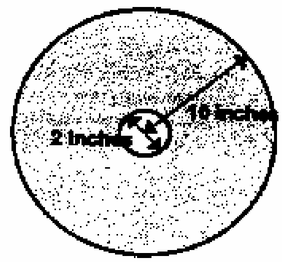
Student E forgets to use π in the formula for area. Student E also illustrates a common feature on this task of working one section of the task but makes no attempt on other sections of the task.

Student E

2. This middle size weight has a radius of 10 inches.

   The hole in the middle has a diameter of 2 inches.

   What is the area of the top surface of this weight?



   121 ✗ square inches

   Explain how you figured it out.

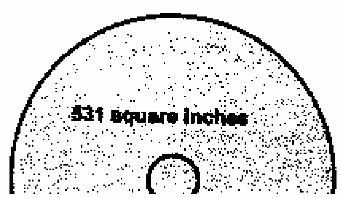
   To find the area of a circle, the formula is $A = r^2$. The complete radius of the circle is 11. Since the small circle's diameter is 2 half of that is 1. Half of that diameter is part of the radius. So the radius is 11, then you square 11 and get 121 for the answer.

3. The area of the top surface of the largest weight is 531 square inches, ignoring the hole.

   Figure out the radius of the largest weight.

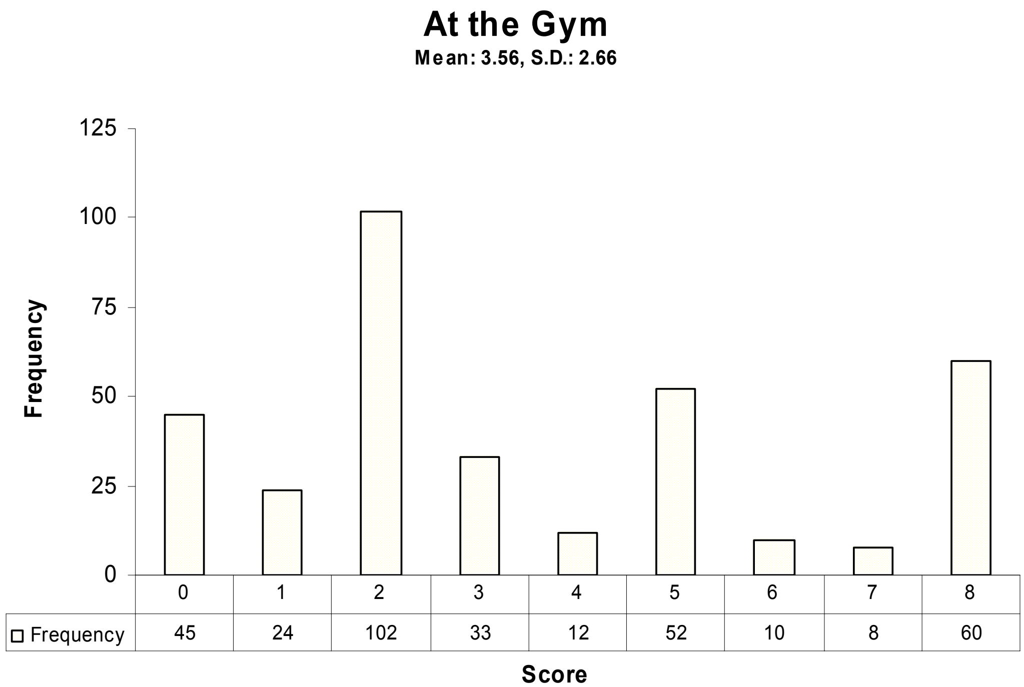
   ✗ inches

   Show your work.



Teacher Comments:

Frequency Distribution for each Task – Grade 10
Grade 10– At the Gym



Score:	0	1	2	3	4	5	6	7	8
% < =	13.0%	19.9%	49.4%	59.0%	62.4%	77.5%	80.3%	82.7%	100.0%
% > =	100.0%	87.0%	80.1%	50.6%	41.0%	37.6%	22.5%	19.7%	17.3%

The maximum score available for this task is 8 points.
The cut score for a level 3 response, meeting standards, is 4 points.

Most students, approximately 80%, could find the volume of the small weight given the area of the base and the height. About half the students could also do some of the steps for finding the radius of the large weight in part 3 of the task. Only 22% could do some successful work for finding the surface area of the middle weight when given a radius and a diameter. About 17% met all the demands of the task. About 13% of the students scored no points on the task. Most of those attempted the task.

At the Gym

Points	Understandings	Misconceptions
0	About 92% of the students with this score attempted the problem.	Students did not understand formulas for volume and area. Many students divided base by height or squared the base before multiplying by height.
2	Students could multiply base times height to find volume.	
3	About half the students with this score could find volume in part 1 and divided the surface area by π in part 3. The other half of the students with this score did not find the volume in part one, but could work backwards from the surface area to the radius in part 2.	About 8% of all students did not attempt part 2 of the task. About 15% of all students did not attempt part 3 of the task. In part 2, students were unsure about how to use the hole to find the surface area of the weight. They often did not notice that the two measurements were for different attributes, radius and diameter, so they substituted incorrect values in their formulas. The most common error was $100\pi - 4\pi$. The second most common error was to ignore formulas for area and just multiply the two numbers in the problem, 2 x 10=20.
4	Students could find volume and make a systematic attempt at solving for radius if given the surface area of a circle.	Students could not find the radius, made mistakes like taking the square root of 561 without thinking about π or dividing 561 by 2.
5	Students could find volume and radius of the weights.	The item preventing students from complete success on the task was finding the surface area of the middle weight.
8	Students knew the formulas for surface area and volume of a cylinder and could reason about them in a problem situation where the cylinder had a hole in the middle.	

Questions for Reflection on At the Gym

- When learning formulas, do students get opportunities to explore how they are put together? Do students discuss how part of the formula is "to find the area of the base" and that the other part is "to multiply by the height" or do you think they see the formula as a string of symbols? What is your evidence?

Look at you students' answers to part 2, finding the surface area of the base:

- Did students forget to divide the diameter of the small circle by two?
- Did they forget to use the area of the hole for subtraction? What other measurement did they use?
- Did students deal with circle formulas in an attempt to solve the problem or did they just use numbers in a set of random operations? Did students use π? Is there a squared term in their solution?

Here is a chart to help you track student thinking for part 2:

311 (99π)	301.44 (100π-4π)	255.34 $(10-1)^2\pi$	23.14 10*2+π	56.55 (10-2)π	40 2^2*10	20 2*10	No Ans.

How well were your students able to work backwards to find the radius of the weight in part 3?

- Did they attempt to use π?
- Did they know to use a square root? In what other ways did they misuse the exponent 2? What are the implications for instruction around squares and square roots?

Here is a chart to help you track student thinking for part 3:

13 531/3.14 sq.root of 169	**23.04** sq.root of 531	**84.5** (531/3.14) 169/2	**169** 531/3.14	**265.5** 531/2	**132.75** $531/2^2$	No ans.

About 16% of the students did not attempt part three of the task and about 11% did not attempt part two. Why do you think students were unwilling or unable to try these parts? How do we build in students a disposition for solving unfamiliar problems or a perseverance for problems that area bit challenging? What types of teacher moves or actions do you use to promote these mathematical habits of minds?

Teacher Comments: ____

Implications for Instruction:

Students need to learn formulas in a way of making sense of the world. Learning to generalize from specific formulas to a generalization that works for many shapes, eliminates some need for memorization and empowers students to solve problems for unfamiliar shapes. If students explore the idea of volume, it is important for them to understand the elements that compose that formula. In the case of volume, students should come to realize that volume is made of a base (which can take on many different looks depending on the shape) times a height. Students need to be preparing not just for the specific problems in the book, but also for the ability to think about mathematics when confronted with new shapes or problems.

Just as students should be able to decompose formulas, students should be able to decompose shapes into individual components and see how they relate to effect the whole. Students had difficulty thinking about the relationship of the hole to the weight. How does the hole effect the area of the base? The volume of the weight? Students need to learn to reflect on the relationships. At this grade level, composing and decomposing shapes should be a very natural part of their mathematical thinking.

Teacher Comments: ____

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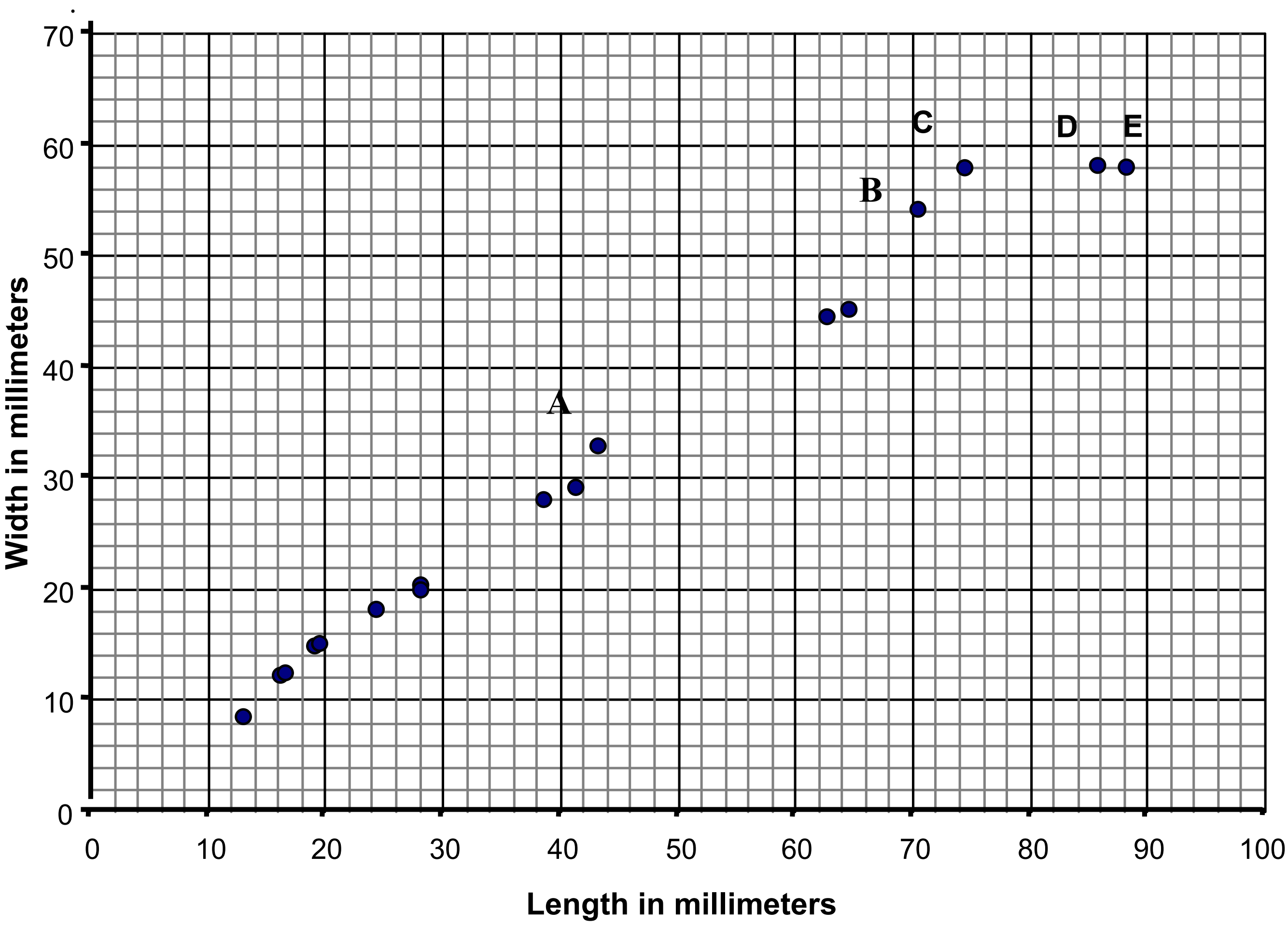
Course Two Task 3 Birds' Eggs

Student Task	Read and interpret scatterplot about length and width of birds' eggs. Understand, use and compare ratios of length to width of different eggs.
Core Idea 5: Data Analysis	For bivariate measurement data, be able to display a scatterplot and describe its shape.
Core Idea 3: Algebraic Properties and Representations	• Recognize and use equivalent graphical and algebraic representations of lines with their geometric characteristics, such as slope and intercepts. • Develop, analyze and explain methods for solving problems involving proportional reasoning, such as scaling and including equivalent ratios. • Approximate and interpret rates of change, from graphic and numeric data.

Birds' Eggs

This problem gives you the chance to:
• interpret a scatter diagram including comparing gradients

This scatter diagram shows the lengths and the widths of the eggs of some American birds.



1. Mallards' eggs have an average length of 57.8 millimeters and average width 41.6 millimeters.

   Use an **X** to mark a point that represents this on the scatter diagram.

2. What does the graph show about the connection between the lengths of birds' eggs and their widths?

3. Use the graph to estimate the width of the eggs with a length of 35 millimeters.

_______________ millimeters

4. Describe the differences in shape of the two eggs C and D.

5. Which of the eggs A, B, C, D, and E has the greatest ratio of length to width?
Explain how you decided.

7

Birds' Eggs		Test 10 Rubric
The core elements of performance required by this task are: • interpret a scatter diagram including comparing gradients Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Marks points correctly. Accept points within 1 square of correct position.	1	1
2. Gives a correct description such as: Generally, the greater the length of the egg, the greater is its width.	1	1
3. Gives correct answer: **25**mm approximately. Accept values between 22 and 28.	1	1
4. Gives a correct explanation such as: They have the same width but D is longer. **or** C is a shorter and fatter shape.	2 **or** 2	2
5. Gives a correct answer: **E** and Gives a correct explanation such as: The line joining E to the origin is the flattest of all the lines joining A, B, C, D, and E to the origin. **or** Gives all the ratios simplified for comparison.	1 1 **or** 1	2
Total Points		**7**

Looking at Student Work – Birds' Eggs

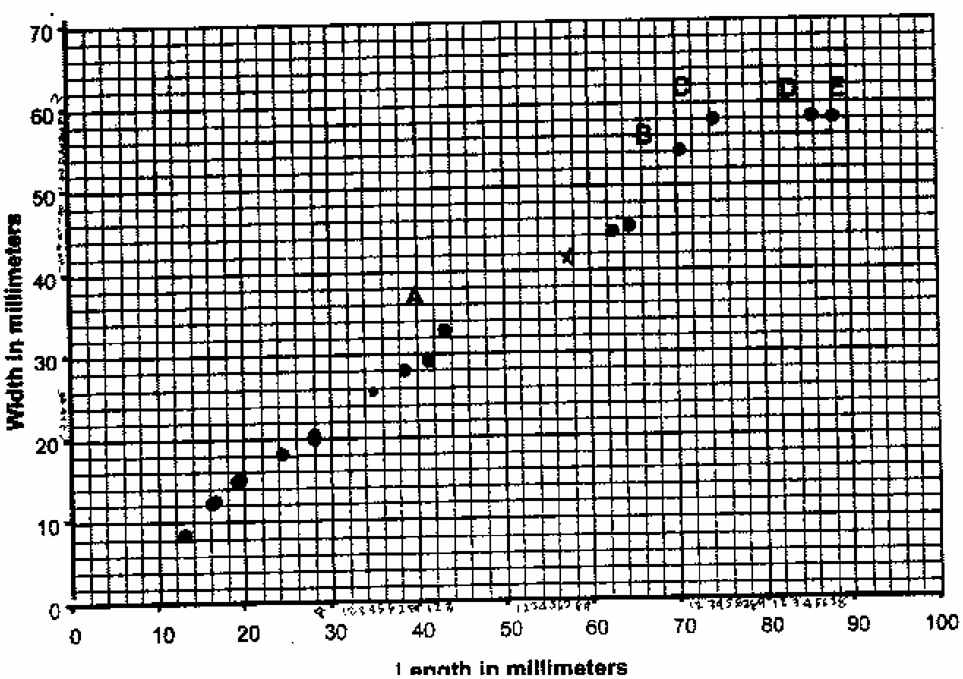
Many students did very well on this problem. Student A makes use of the graph by plotting the point for question 3. Student A also labels the axis in more detail to help think about the points and their values. In part 5, the student quantifies all the ratios and shows their conversion to decimals to make the comparison easier.

Student A

Birds' Eggs

This problem gives you the chance to:
- interpret a scatter diagram including comparing gradients

This scatter diagram shows the lengths and the widths of the eggs of some American birds.



2. What does the graph show about the connection between the lengths of birds' eggs and their widths?

As their lengths increases, the width increases too. However, when the egg has reached at least 72 millimeters for length, the width will stay the same as the length of the egg keeps growing. f. w.

3. Use the graph to estimate the width of the eggs with a length of 35 millimeters.

25.8 millimeters

4. Describe the differences in shape of the two eggs C and D.

Even though their widths are the same, their lengths are not. Egg D is 12 millimeter larger (in terms of lengths) than egg C.

5. Which of the eggs A, B, C, D, and E has the greatest ratio of length to width? Explain how you decided.

A = 43 to 33 B = 54 to 70 C = 74 to 58 D = 86 to 58 E = 88 to 58
Egg E has the greatest ratio of length to width. I decided to do division of all the ratios of the eggs and it turns out that egg E has the greatest ratio.

Student B makes a couple of common errors. In part 3, the student reverses the axes and reads from length to width, instead of width to length. In part 5 the student describes a procedure for comparing ratios, but does not quantify any of the points.

Student B

2. **What does the graph show about the connection between the lengths of birds' eggs and their widths?**

As a birds length increases so does its width. ✓

3. **Use the graph to estimate the width of the eggs with a length of 35 millimeters.**

46 ✗ millimeters

4. **Describe the differences in shape of the two eggs C and D.**

egg C & D have the same width, ✓ but egg D has a greater length. ✓

5. **Which of the eggs A, B, C, D, and E has the greatest ratio of length to width? Explain how you decided.**

egg E ✓ has the greatest ratio. I found this out by dividing the length by the width. 1

Student C uses pictures to help describe the differences in the shape of the two eggs. Student C has answered all questions through 4 correctly, but is not willing or able to deal with ratios in part 5.

Student C

2. What does the graph show about the connection between the lengths of birds' eggs and their widths?

The longer the birds' eggs are the ✓greater the width

3. Use the graph to estimate the width of the eggs with a length of 35 millimeters.

≈ 26 ✓ millimeters

4. Describe the differences in shape of the two eggs C and D.

eggs C and D are about the same size in ✓width but egg D has a greater✓ length. egg D is longer than egg C.

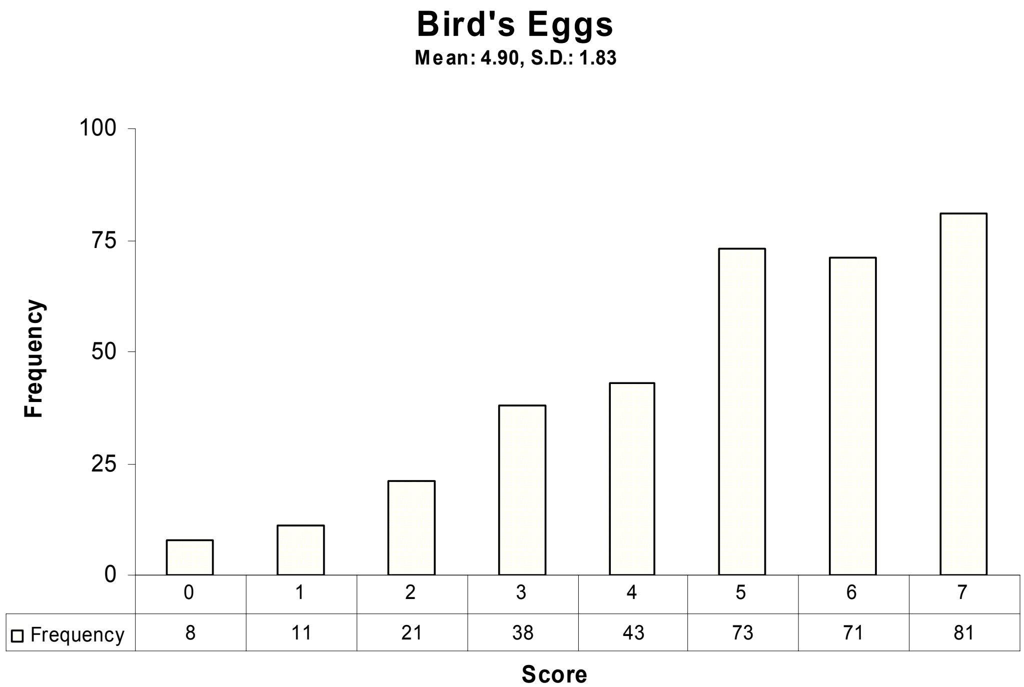
5. Which of the eggs A, B, C, D, and E has the greatest ratio of length to width? Explain how you decided.

The most difficult part of the task for students was trying to work with the ratios of length to width in part 5. Here are a few examples of their struggle:

- E has the greatest ration of length to width. This is simple to figure out due to the fact that E has the greatest length and has widths similar to C & D. B is not much of a factor to look at look at because it has closer to 1:1 ratio than E. A could have been a possibility, yet E has a greater ratio than A, leaving E as having the greatest ratio of length to width.
- I believe egg "E" because egg "e" has one of the highest widths yet has the longest length.
- Egg B because the width is 54mm and the length is approximately 71mm. For the other eggs, the width and length are about 10mm apart.
- I would say A because there aren't that many eggs that are super-long. There are more shorter ones. It is probably rare to get really long eggs.

Teacher Comments: ____

Frequency Distribution for each Task – Grade 10
Grade 10– Bird's Eggs



Score:	0	1	2	3	4	5	6	7
% < =	2.3%	5.5%	11.6%	22.5%	35.0%	56.1%	76.6%	100.0%
% > =	100.0%	97.7%	94.5%	88.4%	77.5%	65.0%	43.9%	23.4%

The maximum score available for this task is 7 points.
The cut score for a level 3 response, meeting standards, is 4 points.

Most students (89%) could plot a point on the scatter graph, give a description of the trend of the data, and either approximate the length of an egg for a given width or identify the egg with the greatest ratio of length to width. Many students (78%) could plot a point, find the length given the width, and describe the differences between egg D and C or describe the trend of the data and identify the egg with the greatest ratio. About 56% of the students could do the entire problem except find the ratios in part 5. 23% of the students met all the demands of the task. About 3% of the students scored no points on the task.

Birds' Eggs

Points	Understandings	Misconceptions
0		About half the students with this score attempted the problem. Students were not able to graph points accurately. In approximating the length for a given width, they often reversed the axis. In part 4 describing the differences between C and D, they often stated that D was just longer.
4	Students with this score could plot a point, approximate the length given a width, and either describe the difference between C and D or describe the trend in the data and identify the egg with the highest ratio of length to width.	Many students described a process for finding ratios, but did not give specific numbers. Others thought that E has the longest length therefore the largest ratio. This is only true because the widths of other eggs are similar, but is not a generalization that works for all cases. Most students did not use numbers in their reasoning.
5	Students could meet all the demands of the task except identify the egg with the largest ratio of length to width.	About 6% of the students did not attempt finding the ratios. A and D were the most common incorrect answers. Egg A is more in the middle, measurements are not as far apart. D is the longest width.
6	Students could identify the egg with the largest ratio, but could not explain why or quantify it.	
7	Students could plot and interpret data on a scatterplot, describe the correlation of the data, describe how length and width effect the look of the egg, and make numerical comparisons of the ratios of length to width for different eggs.	

Questions for Reflection on Birds' Eggs

- When analyzing various types of graphs, are students routinely asked to describe trends in the data?
- Why do you think students struggled with the issue of ratios?
- What work have students done with ratios this year?
- When making comparisons or justifications in class, what are the classroom norms for quantifying their thinking?
- Were calculators available on the test? Do you think lack of calculators may have affected students' willingness to work with ratios?
- When students picked eggs other than E, what kind of logic guided their choice?

Teacher Comments: ____

____

____

____

Implications for Instruction:

Students at this grade level need to view graphs as communication tools. When looking at graphs, a routine first question for students should be to think about what is the trend of the data, what is the author trying to convey. Students at this grade level frequently use graphs and do calculations around slope, but they don't often think about the meaning of the ratio or slope in terms of context and what information about the situation is given by this ratio. What types of experiences will help students see slope as a piece of sense-making information to describe some aspect of the situation? Students need to have the reasoning developed and pushed as they develop new mathematical procedures and skills. How is this procedure or skill useful? What is the purpose of slope or of ratios? Included in the reasoning students are developing, is the need for better levels of justifications or proofs for their choices or decisions. It should be a routine practice to use mathematical computations to back up their ideas.

Teacher Comments: ____

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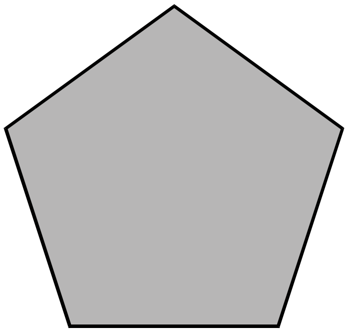
Course Two Task 4 Pentagons

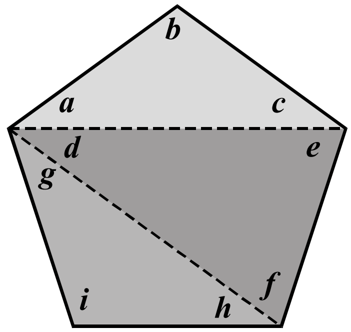
Student Task	Solve equations to find interior and exterior angles in pentagons and tiled pentagons.
Core Idea 4 Geometry and Measurement	**Analyze characteristics and properties of two-dimensional geometric shapes, develop mathematical arguments about geometric relationships, and apply appropriate techniques, tools, and formulas to determine measurements.**

Pentagons

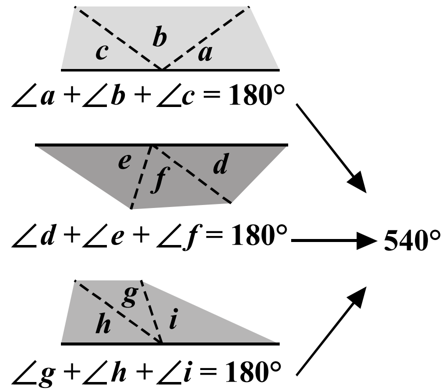
This problem gives you the chance to:
• solve angle equations to find interior and exterior angles in a pentagon



This is a pentagon.



Any pentagon can be divided into 3 triangles.

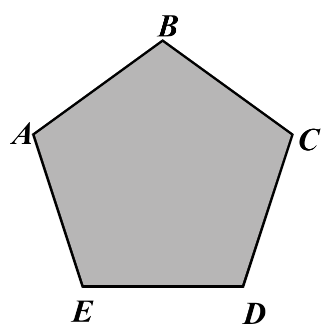


The sum of the angles in each triangle is 180°. So the sum of the interior angles of a pentagon is 540°.

1. ABCDE is a regular pentagon.
   This means that the edges are of equal length and the angle measures are equal.

   What is the measure of each interior angle of a regular pentagon?

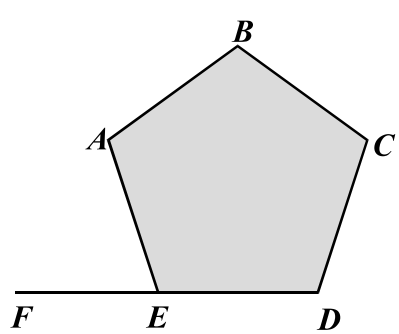
   ____



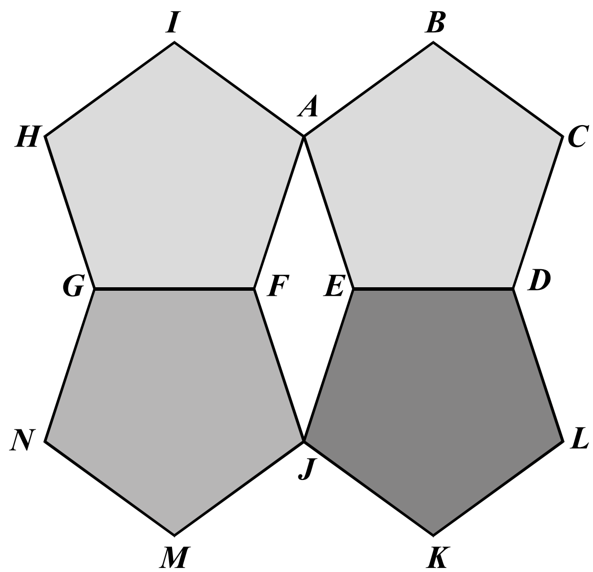
2. ABCDE is a regular pentagon. DEF lie on a line.

   What is the measure of the exterior angle AEF?

   ____



This diagram is made up of 4 regular pentagons that are the same size.



3. What is the measure of angle AEJ? ____________________

   Show your calculations.

4. What is the measure of angle EJF? ____________________

   Explain your reasons.

5. What is the measure of angle KJM? ____________________

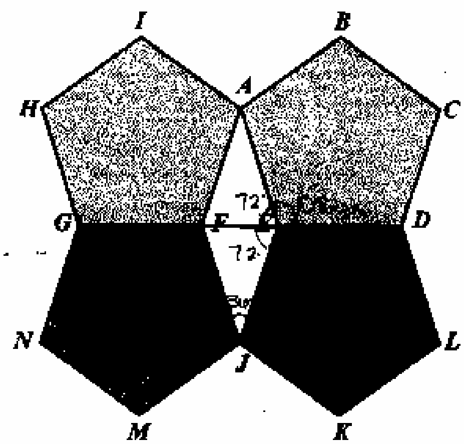
   Show how you figured it out.

Pentagons		Test 10 Rubric
The core elements of performance required by this task are: • solve angle equations to find interior and exterior angles in a pentagon Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **108°** c.a.o.	1	1
2. Gives correct answer: **72°**	1 ft	1
3. Gives correct answer: **144°** Allow 216° for exterior angle AEJ. Shows calculations such as: 360 – (108 + 108) or 2 x 108°.	1 ft 1 ft	2
4. Gives correct answer: **36°** Explains that the sum of the angles of quadrilateral AEJF is 360°. Using symmetry the angle EJF = 180° – 144°. Accept alternative valid explanations.	1 ft 1 ft	2
5. Gives correct answer: **108°** Shows work such as: 360 – (108 + 108 + 36)	1 ft 1 ft	2
Total Points		**8**

Looking at Student Work – Pentagons

Students seemed to either know quite a bit about reasoning with angles or almost nothing. The problem is interesting because students can use a variety of strategies to reason out the unknown measures. Student A subdivides AFJE into two triangles to reason out the size of AEJ. Student A does a good job of describing the idea that around a given point there are a total of 360°.

Student A



3. What is the measure of angle AEJ? 144° ✓

Show your calculations.

(lin. pr. with ∠AED) (lin pr. with ∠JED)
∠AED = 108° & ∠JED = 108° ∠AEF = 72° & ∠JEF = 72°
72° + 72° = 144° ✓

4. What is the measure of angle EJF? 36° ✓
Explain your reasons.

Since ∠FEJ is 72° & ∠EFJ = 72°, because they are lin prs to 108° angles, 72° + 72° = 144°. Since a triangle has a sum of 180° 180° − 144° = 36° ✓ so ∴ ∠EJF = 36°

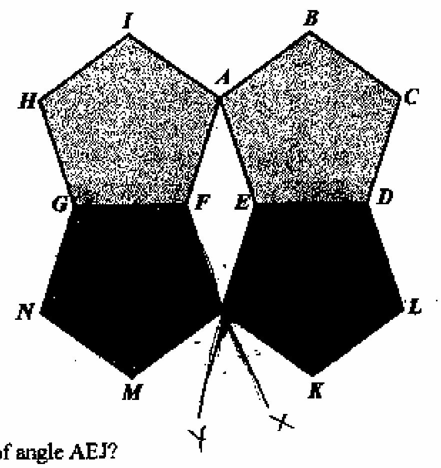
5. What is the measure of angle KJM? 108° ✓

Show how you figured it out.

If FJM = 108° & ∠EJK = 108° & ∠EJF = 36°, since there are 360° in a circle and ∠'s FJM + ∠EJF + ∠EJK + ∠KJM form a circle, 108 + 108 + 36 = 252°, 360 − 252 = 108° ✓

Student B sees angle AEJ as composed of two exterior angles and reasons about AFJE as a quadrilateral. Student B has a very unique way for solving part 5 by finding several angles and subtracting the overlap.

Student B



3. **What is the measure of angle AEJ?** 144 ✓ 1

Show your calculations.

72+72 = 144 ✓ 1

4. **What is the measure of angle EJF?** 36 ✓ 1

Explain your reasons.

AEJF is a parallelogram and since the sum of the interior angles of a quadrilateral is 360, and opposite angles of a parallelogram are congruent, I can subtract 288 from 360 and divide by 2 to get the angle measure of EJF. ✓ 1

5. **What is the measure of angle KJM?** 108 ✓ 1

Show how you figured it out.

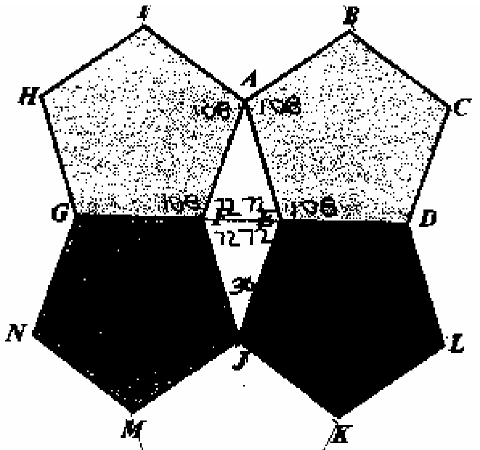
If I extend the EJ and FJ, then the exterior ∠ MJX = 72 and ∠KJY is 72. By adding them together I get 144. But they overlap at ∠ XJY, so I subtract 36 from it because ∠ EJF is 36 and by the vertical ∠ theorem XJY = 36 too. 1

Page 9 Pentagons Test 10

8

Student C makes a common assumption that a regular pentagon can be drawn at the vertex of angle MJK. While this can later be proved, the student does lay the groundwork to prove this assertion and therefore answers derived from this idea are not valid as a proof.

Student C



3. What is the measure of angle AEJ? 144° ✓

Show your calculations.

if each exterior angle is 72°, then 72 + 72 = 144°

4. What is the measure of angle EJF? 36° ✓

Explain your reasons.

i have permission to draw lines so i drew FE. that makes 2 triangles: △AFE, and △FEJ. i already know 2 angles are 72°, so 180 - 144 = 36°. ✓

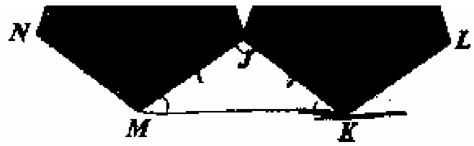
5. What is the measure of angle KJM? 108° ✓

Show how you figured it out.

its just another pentagon that goes there. so the angle is 108°. ∧

Student D is interesting because there is not quite enough information to follow the reasoning in part 5. Is the student thinking that the bottom angle in the triangle JMK is an exterior angle? Is the student looking at the straight line extending below point K as a straight line = to 180°? How can the student look at angle JMK and think that 36° is a reasonable answer?

Student D



3. **What is the measure of angle AEJ?** 144 ✓

Show your calculations.

<AED = 108 ✓

<AEF = 72 + 72 = <AEJ

4. **What is the measure of angle EJF?** 36 ✓
Explain your reasons.

Since all 4 pentagons are =, then FJ is = to EJ, and therefore it forms an exterior Δ. & <DEJ = 108, the < JEF and < JFE is 72. The <EJF = 36 from 180-72-72=36 ✓

5. **What is the measure of angle KJM?** 36 X

Show how you figured it out.

180-108= 72

180 -72- 72 = 36 X

Isoceles Δ Theorem ⇒ JM = JK

Student E again has an answer to part 5 that seems unreasonable by observation and estimation.

Student E

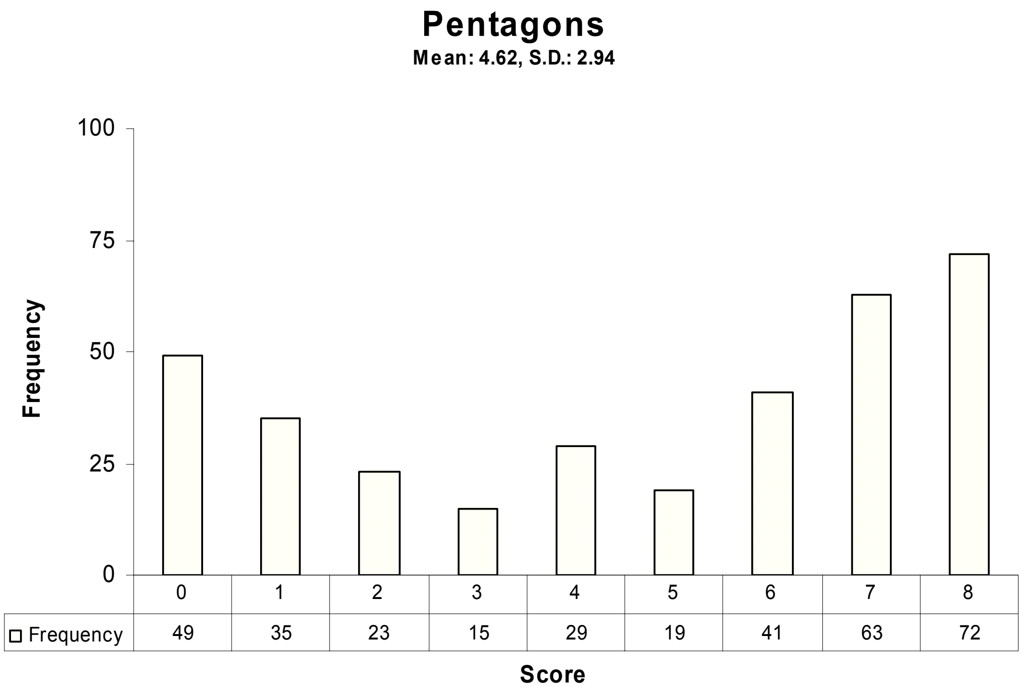
5. **What is the measure of angle KJM?** 288°

Show how you figured it out.

360° around

144 +144 = 288 X

Frequency Distribution for each Task – Grade 10
Grade 10– Pentagons



Score:	0	1	2	3	4	5	6	7	8
% < =	14.2%	24.3%	30.9%	35.3%	43.6%	49.1%	61.0%	79.2%	100.0%
% > =	100.0%	85.8%	75.7%	69.1%	64.7%	56.4%	50.9%	39.0%	39.0%

The maximum score available for this task is 8 points.
The cut score for a level 3 response, meeting standards, is 4 points.

Most students (86%) could use the diagram to reason the size of the angle for a regular pentagon. Many students (65%) could find the angle size for a regular pentagon, find the size of its exterior angle, and find the angle formed by two adjoining pentagons in part 3 of the task. 39% of the students could find all the missing angle measurements, but a little less than half of them could not explain how they found the final angle in part 5 of the task. 14% of the students scored no points on this task. Of that group, 85% attempted one or more parts of the task.

Pentagons

Points	Understandings	Misconceptions
0	85% of students with this score attempted the task.	The most common errors in part one of the task were 180°, 60°, or 540°.
1	Students with this score could generally find the size of the angle for a regular pentagon.	Many students struggled with the idea of exterior angle with almost 25% making some error. Very few of their errors were repeated by other students.
4	Students with this score could find the angle size for a regular hexagon and its exterior angle and find the angle formed by two adjoining pentagons in part 3 of the task.	The most common error for part 3 was to give no response, followed closely by students who thought angle AEJ was 180°. A large group also gave solutions larger than 180°. Students did not use estimation to check for reasonableness of their solutions.
6	Students with this score tended to get all parts one through 4 correct, or could get all measures correct but without correct explanations in part 4 and 5.	16% of all students did not attempt part 5. The three most common errors for part 5 were 36°, 180°, and 100°.
7		Students could not explain accurately how they found the measure of KJM. Some students made the assumption that another regular pentagon would fit in that position. Other students made assumptions about other angle sizes that they did not back up.
8	About 21% of the students could meet all the demands of the task, including detailed, logical strategies for finding the missing angles.	

Questions for Reflections on Pentagons

Look at you student work:

For part three, how many of your students:

- Reasoned about the exterior angles for AED and DEJ to find the measure of angle AEJ?__________
- Drew triangles AEF and EFJ to help find the measure of angle AEJ?_________
- Gave an angle of 180 or more degrees?_________
- Gave an angle of less than 90 degrees?_________

For part 4 and/or 5 how many of your students were unwilling to attempt these parts of the tasks? What conjectures can you make for this?

For part 5 of the task, see if you can sort student strategies. What were the strategies used by successful students?

How many students had strategies that could have led to the correct answer, but they maybe missed the final step because they lost track of what each calculation stood for?

What types of assumptions did students make that weren't backed up with mathematical calculations? What kinds of questions or experiences can you provide for students that will help them see their errors in logic?

Teacher Notes:

Implications for Instruction:

Students at this grade level should develop and be fluent with interior and exterior angles and their properties. They should be developing the reasoning power to use these relationships to find and quantify unknown angles. They should also be able to justify their angles. It is important that students not relay on the way things look in diagrams, but through deduction and calculation find the exact size of unknown angles.

Course Two   Task 5   Differences

Student Task	Use algebra to explore and explain a number sequence and its differences. Use algebra to solve equations for missing terms.
Core Idea 3 Algebraic Properties and Representations	**Represent and analyze mathematical situations and structures using algebraic symbols and solve equations.**
Core Idea 2 Mathematical Reasoning and Proofs	• Identify, formulate and confirm conjectures. • Explain the logic inherent in a solution process.

Differences

This problem gives you the chance to:
- use algebra to explore a sequence and its differences

Lee is working out sequences of numbers.

He starts to construct this table:

Table 1

Position	**1st term**		**2nd term**		**3rd term**		**4th term**		**5th term**		**6th term**
Sequence	1		7		19		37		61		91
1st differences		6		12		18					
2nd differences			6		6						

1. Fill in the gaps in the table.

2. Find the next two terms in the sequence, assuming that the pattern continues.

   7th term 8th term

Lee wants to find a formula for the n^{th} term of this sequence.

He knows that it must be of the form $an^2 + bn + c$ where a, b, and c are constants.
So he puts $n = 1, 2, 3$ and makes Table 2, which he will match with Table 1 to find a, b, and c.

Table 2

Position	**1st term**		**2nd term**		**3rd term**
Sequence	$a + b + c$		$4a + 2b + c$		
1st differences		$3a + b$			
2nd differences					

3. Explain why the 3rd term in Table 2 is $9a + 3b + c$.

4. Complete Table 2 to show the first three terms of the sequence and the first and second differences.

5. The formula for the n^{th} term of the sequence in Table 1 is $an^2 + bn + c$.

By comparing Table 1 and Table 2, Lee can now find a, b and c.

Write down the values of a, b and c.

a = ______________

b = ______________

c = ______________

8

Differences		Test 10 Rubric
The core elements of performance required by this task are: • use algebra to explore a sequence and its differences Based on these, credit for specific aspects of performance could be assigned as follows	points	section points
1. Gives correct answers: **1st differences are 24, 30** **2nd differences are 6, 6, 6**	1 1	2
2. Gives correct answer: **127, 169**	2 x 1	2
3. Gives correct explanation such as: I substituted n = 3 in the equation $an^2 + bn + c$.	1	1
4. Gives correct answers: $\mathbf{5a + b, 2a}$	1	1
5. Gives correct answers: $\mathbf{a = 3}$ $\mathbf{b = -3}$ $\mathbf{c = 1}$ All three values correct, 2 points. Two values correct, 1 point.	 2 (1)	2
Total Points		**8**

Looking at Student Work – Differences

"Differences" was quite a challenging task for students. Student A does a very nice job of explaining all thinking. In the table in part one, Student A not only fills in the missing values for 1st differences but also shows that they are multiples of 6. Student A then extends the table to find the 7th and 8th terms. In the second table Student A explains not only why 9a +3b =c is the 3rd term, but shows the calculations for finding all the missing values in the table. On page 2 of the task the student uses algebra to solve for missing values and then substitutes those values into other equations to check for accuracy.

Student A

...rences

...roblem gives you the chance to:
...algebra to explore a sequence and its differences

...working out sequences of numbers.

...rts to construct this table:

Table 1

	1st term		2nd term		3rd term		4th term		5th term		6th term				
...e	1		7		19		37		61		91		127		169
...ences		6x1= 6		6x2= 12		6x3= 18		6x4= 24		6x5= 30		6x6= 36		42	
...rences			6		6		6		6		6		6		6

...ll in the gaps in the table. 2

...nd the next two terms in the sequence, assuming that the pattern continues. 2

...th term 127 ✓   8th term 169 ✓

...ants to find a formula for the n^{th} term of this sequence.

...ows that it must be of the form $an^2 + bn + c$ where a, b, and c are constants.
...puts $n = 1, 2, 3$ and makes Table 2, which he will match with Table 1 to find a, b, and c.

Table 2

Position	1st term		2nd term		3rd term
Sequence	a + b + c		4a + 2b + c		9a+3b+c
1st differences		3a + b		5a+b ✓	
2nd differences			2a		2a ✓

9a+3b+c − 4a+2b+c = 5a+b

5a+b − 3a+b = 2a

1

...plain why the 3rd term in Table 2 is $9a + 3b + c$.

Because the 1st term is $1^2a+1b+c$ and the 2nd term is $2^2a+2b+c$, so the third term must be $3^2a+3b+c$. n increases by 1. ✓

5. **The formula for the n^{th} term of the sequence in Table 1 is $an^2 + bn + c$.**

By comparing Table 1 and Table 2, Lee can now find a, b and c.

Write down the values of a, b and c.

$a =$ 3

$b =$ -3 ✓

$c =$ 1

$1 = a + b + c$

$7 = 4a + 2b + c$

$\frac{6}{2} = \frac{2a}{2}$

$3 = a$

$6 = 3a + b$

$6 = 3(3) + b$

$6 = 9 + b$

$-3 = b$

$a + b + c = 1$

$(3) + (-3) + c = 1$

$c = 1$

Check: $4a + 2b + c \stackrel{?}{=} 7$

$4(3) + 2(-3) + 1 \stackrel{?}{=} 7$

$12 - 6 + 1 \stackrel{?}{=} 7$

$6 + 1 \stackrel{?}{=} 7$

$7 \stackrel{\checkmark}{=} 7$

Student B uses differences to think about how the pattern is growing above the table. It is not clear at what point the student notices these are the same values as those in the 1st difference row. Student B also uses algebra to solve for the values in part 5.

Student B

Lee is working out sequences of numbers.

He starts to construct this table:

+6 +12 + 18 + 24 + 30 36 42

Table 1

Position	1st term		2nd term		3rd term		4th term		5th term		6th term
Sequence	1		7		19		37		61		91
1st differences		6		12		18		24		30	
2nd differences			6		6		6		6		6

1. Fill in the gaps in the table.

2. Find the next two terms in the sequence, assuming that the pattern continues.

7th term 127 ✓ 91 + 36 = 127 8th term 169 ✓ 127 + 42 = 169 2

Lee wants to find a formula for the n^{th} term of this sequence.

He knows that it must be of the form $an^2 + bn + c$ where a, b, and c are constants.
So he puts n = 1, 2, 3 and makes Table 2, which he will match with Table 1 to find a, b, and c.

Table 2

Position	1st term		2nd term		3rd term	
Sequence	$a + b + c$		$4a + 2b + c$		$9a+3b+c$	$16a+4b+c$
1st differences		$3a + b$		$5a+b$ ✓		$7a+b$
2nd differences			$2a$ ✓		$2a$	

3. Explain why the 3rd term in Table 2 is $9a + 3b + c$.

Substituting $n=3$ in an^2+bn+c results in $a3^2+b3+c$, which equals $9a+3b+c$. ✓ 1

Student B

5. The formula for the n^{th} term of the sequence in Table 1 is $an^2 + bn + c$.

By comparing Table 1 and Table 2, Lee can now find *a*, *b* and *c*.

Write down the values of *a*, *b* and *c*.

$a =$ 3 ✓

$b =$ −3 ✓

$c =$ 1 ✓

$2a = 6 \quad a = 3$

$3a + b = 6 \quad 3(3) + b = 6 \quad b = -3$

$a + b + c = 1$

$3 - 3 + c = 1 \quad c = 1$

Student C does not make a connection between the formula an^2+bn+c and the table in part 2. Student C uses the answer for the 3rd term to work backwards to find the other missing values in the table. The student does not attempt part 5.

Student C

Table 2

Position	1st term		2nd term		3rd term
Sequence	$a + b + c$		$4a + 2b + c$		$9a + 3b + c$
1st differences		$3a + b$		$5a + b$ ✓	
2nd differences			$2a$ ✓		$2a$

3. Explain why the 3rd term in Table 2 is $9a + 3b + c$.

✗

Because of table 1's pattern of increments were all equal, table 2's must also be equal. If you take 9a + 3b + c & subtract 4a + 2b + c you get 5a + b. Now if you take 5a + b & subtract 3a + b you get 2a.

Student D has a partial understanding of the formula an^2+bn+c, but can only explain the first term in 9a +3b+c. The student does not seem to understand how the second differences are derived. This limits the student to only using the whole formula in part 5. The student attempts to substitute a value of 7 for n, but the numbers are too unwieldy for guess and check.

Student D

Table 2

Position	1st term		2nd term		3rd term
Sequence	$a + b + c$		$4a + 2b + c$		$9a+3b+C$
1st differences		$3a + b$		$5a+b$	
2nd differences			9a X		6a

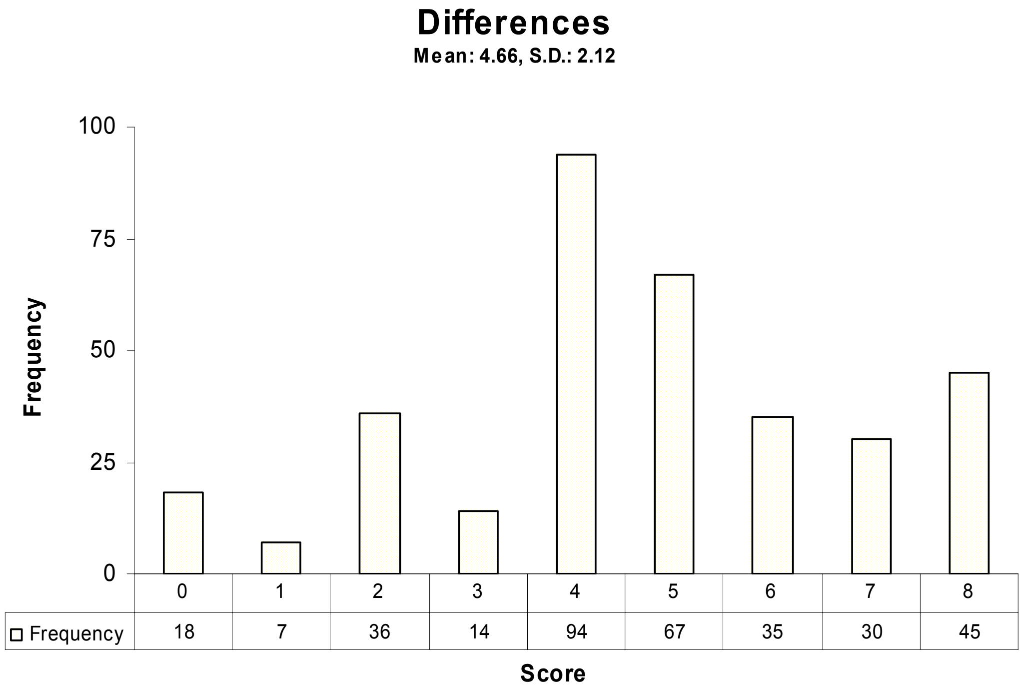
3. Explain why the 3rd term in Table 2 is $9a + 3b + c$.

3 is after 2 and 9 is 3 to the 2nd power like 4 is 2 to the 2nd power 1.

$a(7)^2 + b(7) + C$

$49a + 7b + C$

Frequency Distribution for each Task – Grade 10
Grade 10– Differences



Score:	0	1	2	3	4	5	6	7	8
% < =	5.2%	7.2%	17.6%	21.7%	48.8%	68.2%	78.3%	87.0%	100.0%
% > =	100.0%	94.8%	92.8%	82.4%	78.3%	51.2%	51.2%	21.7%	13.0%

The maximum score available for this task was 8 points.
The cut score for a level 3 response, meets standards, was 4 points.

Most students (93%) could fill in the values for the first differences in the first table. Many students (79%) could fill in both the 1st and 2nd differences in the table. A little more than half could also fill in the values using algebraic notation in part 4. 22% could fill in both tables and find the values for a,b, and c in the formula an^2+bn+c. 13% of the students met all the demands of the task. 5% of the students scored no points on this task. About half of this group attempted the task.

Differences

Points	Understandings	Misconceptions
0	Only about half the students with this score attempted the problem.	Was time a factor in student success?
2	Most students could fill in the numerical table of differences for question 1.	There is some evidence that students worked this pattern, using contextual clues other than finding differences. Students may have just counted by sixes or seen that the bottom row was all the same.
4	Students could fill in the table of differences and extend it out to the 7th and 8th terms.	25% of the students did not attempt to explain why 9a+3b+c is the third term in the algebraic sequence. Almost 15% of the students used the pattern of differences from the table to find the expression, rather than using the formula an^2+bn+c. The next most common error was to think the *a* term just increased by 2 each time.
5		Students could not explain how to find the 3rd term in the algebraic sequence even though they could fill out the table. By not seeing the significance of the formula, they could not find the solution for a, b, and c in part five. 34% of the students did not attempt part 5. Other students attempted to use an equation with a squared value and therefore could not use normal methods for multiple equations to solve for the unknowns.
7	Students could fill in numeric and algebraic tables of differences, extend the pattern, and find the values for a,b, and c.	Students could not articulate why the 3rd term was 9a+3b+c.
8	Students could fill in numeric and algebraic tables of differences, extend the pattern, and find the values for a,b, and c. Students could pick out useful segments of the second table and use algebra to solve for a,b, and c. They could	

	also use substitution into the formula to justify the 3rd term of the sequence.	

Questions for Reflection – Differences

- Were your students willing to attempt question 3? Did they use the formula to solve for the third term, did they use the pattern from the table to find the 3rd term, or did they make some other argument?
- What types of opportunities do students have to make justifications in class? Do they have problems that help them apply their algebraic skills to make proofs or justifications?
- Could students use the information from the second table to set up equations to solve for a,b, and c? How many students are still relying on guess on check? Did you students try to use an equation with a squared value to find the missing values?
- What other problems could you incorporate into your program to help students maintain and practice algebraic skills while furthering their ability to make good mathematical justifications?

Teacher Notes:

Implications for Instruction:

Students need to see the usefulness of using their algebraic skills as tools for solving problems. Students need to have explicit experiences that help them transition from guess and check to using equations. They need to have discussions about when and why equations are more efficient than using guess and check as a strategy. Students should frequently have problems that challenge their level of thinking and push them to make mathematical justifications. Students need to see themselves as capable of tackling unknown situations, as having a variety of tools to help them figure out things they have not previously encountered. Students should be given frequent opportunities to solve rich tasks that allow for a variety of solutions and to justify their solutions. At this grade level, this self-image of being able to meet challenges is critical to their interest in mathematics and in their choices about pursuing further mathematics.

Teacher Notes: