

Agree or Disagree

Student Task	Determine whether or not two quantities are equal. Explain the relationship using words, numbers, or pictures.
Core Idea 3 Algebra, Patterns and Functions	Understand patterns and use mathematical models to represent and to understand qualitative and quantitative relationships. <ul style="list-style-type: none">• Compare principles and properties of operations, such as commutativity between addition and subtraction.• Use concrete, pictorial, and verbal representations to develop an understanding of symbolic notations.

Agree or Disagree?

1. Look at this number sentence: $8 + 5 = 5 + 8$. Do you agree or disagree? _____

Use words, numbers, or pictures to explain your answer.

2. Look at this number sentence: $6 - 4 = 4 - 6$. Do you agree or disagree? _____

Use words, numbers, or pictures to explain your answer.

Mathematics Assessment Collaborative Performance Assessment Rubric Grade 2

	Agree or Disagree?: Grade 2:	Points	Section Points
	<p>The core elements of the performance required by this task are:</p> <ul style="list-style-type: none"> • Compare principles and properties of operations, such as commutativity, between addition and subtraction * Use concrete, pictorial, and verbal representations to develop an understanding of symbolic notations. • Communicate reasoning using words, numbers or pictures. <p>Based on these credit for specific aspects of performance should be assigned as follows</p>		
1	<p>Agree</p> <p>Uses a convincing argument such as:</p> <p>$13 = 13$</p>	<p>1</p> <p>2</p>	3
2	<p>Disagree</p> <p>Uses a convincing argument such as:</p> <p>$2 \neq 4 - 6$</p> <p>or a story problem such as “ I have 6 cookies and I ate 4 cookies is different because if I have 4 cookies, I cannot eat 6 cookies.”</p> <p>For an answer such as: “Because you cannot subtract a larger number from a smaller number”</p>	<p>1</p> <p>2</p> <p>(1)</p>	3
	Total Points		6

Looking at Student Work – Agree or Disagree?

13% of our students met all the demands of this task. In order to do so, the students needed a justification as to why $6-4$ was not equal to $4-6$. There was a great deal of variety in how they explained their answer. It was acceptable to say “they are not the same” as we read in the work of Student A.

Student A

Agree or Disagree?

1. Look at this number sentence: $8 + 5 = 5 + 8$. Do you agree or disagree? agree ✓

1

Use words, numbers, or pictures to explain your answer.

$$\begin{array}{ccc} 8+5=13 & & 5+8=13 \\ & \swarrow \quad \searrow & \\ & 13=13 & \\ & \text{same} & \end{array}$$

2

2. Look at this number sentence: $6 - 4 = 4 - 6$. Do you agree or disagree? disagree ✓

1

Use words, numbers, or pictures to explain your answer.

$$\begin{array}{ccc} \begin{array}{r} 6 \\ -4 \\ \hline 2 \end{array} & & \begin{array}{r} 4 \\ -6 \\ \hline \end{array} \\ & \swarrow \quad \searrow & \\ & \text{not same} & \end{array}$$

2

Students B & C realized that the two expressions had different answers and justified this through pictures in the case of Student B and by setting the expressions in a context in the case of Student C.

Student B

Agree or Disagree?


1. Look at this number sentence: $8 + 5 = 5 + 8$. Do you agree or disagree? agree ✓

Use words, numbers, or pictures to explain your answer.

so $8 + 5 = 13$
 $5 + 8 = 13$
 I agree because $8 + 5 = 13$ so
 $5 + 8 = 13$ because it has the
 same answer.

2. Look at this number sentence: $6 - 4 = 4 - 6$. Do you agree or disagree? disagree ✓

Use words, numbers, or pictures to explain your answer.

 I disagree because
 I cannot $-4-6$ because
 4 is a smaller number
 than 6.
 two left

Student C

Agree or Disagree?

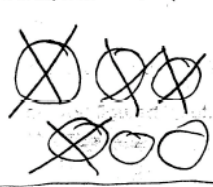
1. Look at this number sentence: $8 + 5 = 5 + 8$. Do you agree or disagree? agree ✓

Use words, numbers, or pictures to explain your answer.

$8 + 5 = 13$ | $5 + 8 = 13$
 13 | 13

2. Look at this number sentence: $6 - 4 = 4 - 6$. Do you agree or disagree? disagree ✓

Use words, numbers, or pictures to explain your answer.

 where are the others to
 be eaten?
 4 can be eaten | only 4 can be eaten (6)

50% of those students who had some level of justification on question 2 were comfortable using negative numbers. Students D-F show three variations of explanations. Student D shows us simply that the first expression is equal to 2 and the second expression is equal to negative 2 and that 2 is not equal to 2 - (sic).

Student D

1. Look at this number sentence: $8 + 5 = 5 + 8$. Do you agree or disagree? agree ✓

Use words, numbers, or pictures to explain your answer.

$$\begin{array}{c} 8+5=5+8 \\ \swarrow \quad \searrow \\ 13=13 \end{array}$$

2

2. Look at this number sentence: $6 - 4 = 4 - 6$. Do you agree or disagree? disagree ✓

Use words, numbers, or pictures to explain your answer.

$$\begin{array}{c} \text{6-4=4-6} \\ \swarrow \quad \searrow \\ 2 \neq 2 \end{array}$$

2

Student E reminds us that 2 is not equal to negative 2. He knows that 2 is greater than negative 2 but has heard that we are not allowed to use these numbers!!

Student E

2. Look at this number sentence: $6 - 4 = 4 - 6$. Do you agree or disagree? disagree ✓

Use words, numbers, or pictures to explain your answer.

$$\begin{array}{c} 6-4=2 \quad 4-6=-2 \\ 2 > -2 \end{array}$$

2

And remember your
not aloud to use
negitaves!!

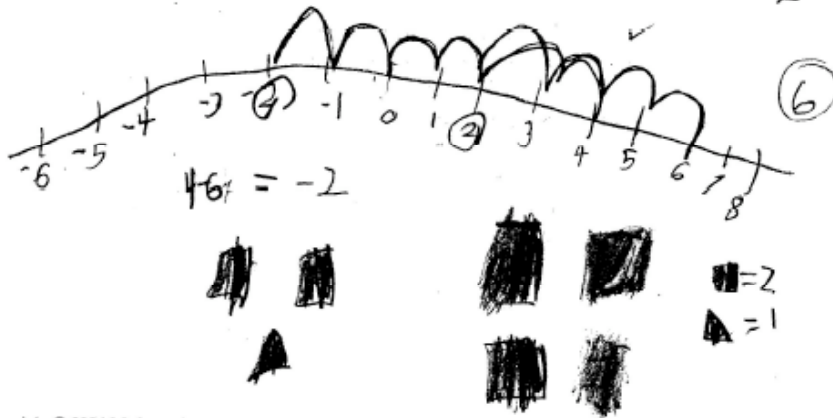
(6)

Student F shows both subtractions on a number line and we can see that one answer is 2 while the other is negative 2.

Student F

2. Look at this number sentence: $6 - 4 = 4 - 6$. Do you agree or disagree? disagree ✓ 1

Use words, numbers, or pictures to explain your answer. 2



In many instances, the inequality in question number 2 was explained as “you can’t take 4 from 6” as seen in Student G’s paper. Students also explained that one had to put the bigger number first when you subtract as seen in Student H’s paper. Student H statement regarding addition shows clarity in his/her understanding of commutativity over addition.

Student G

2. Look at this number sentence: $6 - 4 = 4 - 6$. Do you agree or disagree? disagree ✓ 1

Use words, numbers, or pictures to explain your answer.
I counted And (1)
the answer was 2
You can't take away
4 from 6 (5)

Student H

1. Look at this number sentence: $8 + 5 = 5 + 8$. Do you agree or disagree? yes ✓

Use words, numbers, or pictures to explain your answer.

I looked at the numbers and all they did was swick the numbers. ✓ 2

2. Look at this number sentence: $6 - 4 = 4 - 6$. Do you agree or disagree? no ✓

Use words, numbers, or pictures to explain your answer.

I think no because when you - you have to put the bigger number first. (1)

Second graders are constructing meaning in these areas and as such their explanations can be incomplete and their reasoning unsophisticated – especially with regard to order in subtraction. Student I's paper shows partial understanding. It is clear that she knows that the two expressions are not equal but gives an incorrect answer for 4-6. *What happens when we get to zero and need to go further? How can I explain that in numbers?*

Student I

1. Look at this number sentence: $8 + 5 = 5 + 8$. Do you agree or disagree? yes I agree ✓

Use words, numbers, or pictures to explain your answer.

$$8 + 5 = 13 \quad 5 + 8 = 13 \quad \checkmark$$

2. Look at this number sentence: $6 - 4 = 4 - 6$. Do you agree or disagree? I disagree ✓

Use words, numbers, or pictures to explain your answer.

$$6 - 4 = 2 \quad 4 - 6 = 0$$

Student J's paper reflects an understanding of the commutative property over addition and an erroneous explanation that it also holds over subtraction. 17% of respondents agreed that both of the subtraction expressions were equal to 2.

Student J

Agree or Disagree?

1. Look at this number sentence: $8 + 5 = 5 + 8$. Do you agree or disagree? agree ✓ 1

Use words, numbers, or pictures to explain your answer.

I added $8 + 5 = 13$, and I added $5 + 8$ and it equals 13 too, ✓ 2

2. Look at this number sentence: $6 - 4 = 4 - 6$. Do you agree or disagree? agree ✗ 0

Use words, numbers, or pictures to explain your answer.

I subtracted $6 - 4 = 2$, and I subtracted $4 - 6 = 2$ ✗ 0

Slightly over 30% of our students did not have access to working on these 2 problems. They thought that they were both incorrect as “ $8+5=13$ not 5” and because “ $6+4=2$ not 4”. Research shows that this type of answer reflects the thinking or belief that the equal sign tells us to provide the answer and not that it is representing an equality relationship. Student K’s paper reflects this thinking.

Student K

1. Look at this number sentence: $8 + 5 = 5 + 8$. Do you agree or disagree? disagree X

Use words, numbers, or pictures to explain your answer.

because $8+5=13$ not 5. X

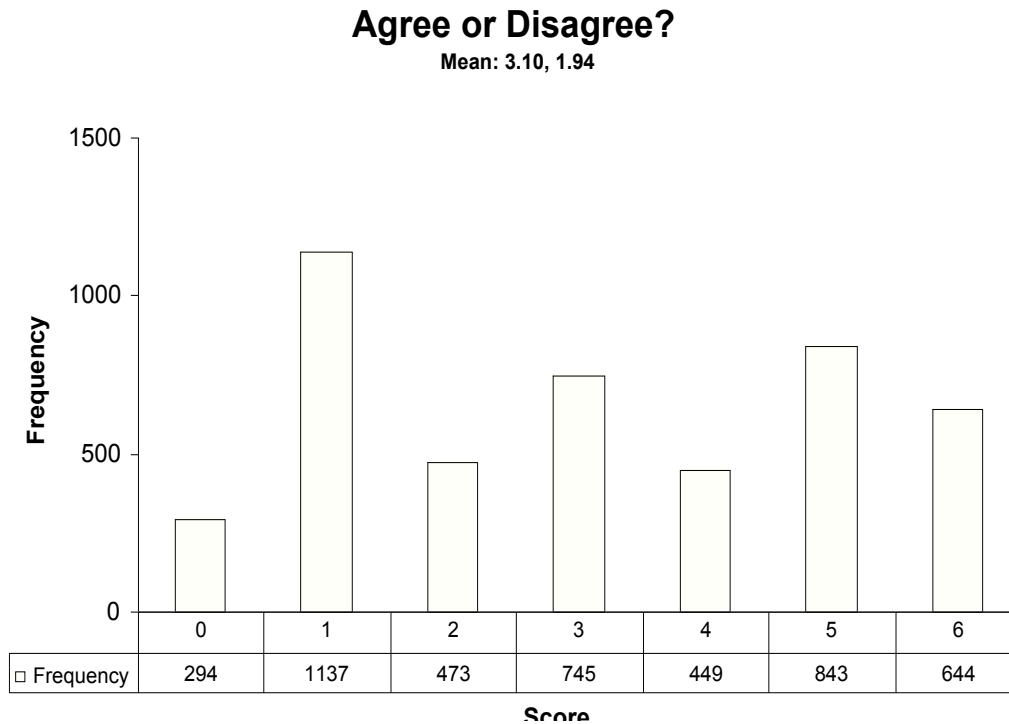
2. Look at this number sentence: $6 - 4 = 4 - 6$. Do you agree or disagree? disagree ✓

Use words, numbers, or pictures to explain your answer.

because $6-4=2$ not 4.

Teacher Notes:

Frequency Distribution for each task – Agree or Disagree?



Score:	0	1	2	3	4	5	6
% < =	6.4%	31.2%	41.5%	57.8%	67.6%	86.0%	100.0%
% > =	100.0%	93.6%	68.8%	58.5%	42.2%	32.4%	14.0%

The maximum score available for this task is 6 points.
The cut score for a level 3 response is 3 points.

58.5% of the students could fully explain that the addition expressions were equal. 32% of the students understood the addition equality and could describe, give examples for or justify why the subtraction expressions were unequal. Just over 6% of the students scored zero on this task. Approximately 1% of the students did not attempt either question.

Agree or Disagree?

Points	Understandings	Misunderstandings
0		1% of the students did not attempt this problem.
1-2	In general, these students did understand the language around agree and disagree. In 90% of these papers, students gave an explanation to support this agreement or disagreement.	Students scoring in this range think of the equal sign as a signal for “and the answer is...”. For many, it was “just not right” to have two expressions with one on each side of the equal sign.
3-4	These students explained and justified problem number one.	Could not explain why the subtraction expressions were not equal to one another. Approximately 15% of the students thought that the subtraction expressions were equal or did not attempt this part of the task.
5	These students knew that the addition expressions were equal and the subtraction expressions were unequal.	Struggled to fully explain why the subtraction expressions were not equal to one another.
6	13% of all respondents could justify why the addition expressions were equal as well as why the subtraction expressions were not equal. Students used a variety of methods to justify their answers including number lines, pictures, story problems, number sentences and modeling of addition and subtraction.	

Based on teacher observations, this is what 2nd graders seemed to know and be able to do:

- the commutative property over addition
- $8+5=5+8$ because of fact families
- could justify their response to question number one
- had a partial understanding that commutativity does not work under subtraction
- could see each side of the equation as its own equation
- seemed ready to understand negative numbers

Areas of difficulty for second graders, these students struggled with:

- being able to subtract four from six
- not familiar with the vocabulary of “agree” and “disagree”
- negative numbers
- giving a complete explanation for the inequality in the subtraction expressions
- $8+5 = 5+8$: “ $8+5$ is not equal to 5 ”
- how to explain why $6-4$ was not equal to $4-6$

Questions for Reflection – Agree or Disagree?

- What opportunities do your students have to discuss and formalize their understanding of the properties of commutativity? Associativity? Equality?
- When working with symbolic notation, do they tie that notation to other more concrete representation (pictures, drawings, manipulatives)?
- How often do your students encounter number sentences such as these in this task?
- How often are your students able to discuss the equal sign as expressing the relationship between two quantities?
- Do your students have the opportunity to solve problems on a regular basis? Do your students have the opportunity to make up story problems given number sentences?
- What responses would you expect from your students to:
 - $4+5 = \underline{\quad} + 6$?
 - $4+5 = 9+0$?
 - $9 = 9$?

Teacher Notes:

Instructional Implications:

The teachers who scored these papers stated that central to success with this task is an understanding of the equal sign and its meaning. A limited conception of what the equal sign means is one of the major stumbling blocks in learning algebra. Virtually all manipulations on equations require understanding that the equal sign represents a relationship. It is important that young children begin to encounter and understand equality as indicative of a relationship between two quantities. Through symbolic representation and manipulation of objects, young children can be exposed to this concept as well as those of commutativity and associativity. Children need many opportunities to generalize from observations about numbers and operations that will then help to form a basis for algebraic thinking.

Unfortunately, most examples of number sentences that children see begin with an operation to the left of the equal sign and the answer on the right. They over-generalize from these examples and actually need many, many exposures to relationship examples, over time, before it can fully develop. Number talks provide a forum for discussions in all of these areas. As students share strategies for solving problems the issues around number properties and equality can be linked to the strategies students are currently using.

From the data we have collected here, as well as from current research on the subject, 2nd grade students can begin to understand negative numbers. A foray in this area requires a number line with positive and negative numbers as well as many experiences with getting, giving, and owing money (among other experiences). Start slow and use small numbers should you choose to do so. More important, though, is their discussions around equality and inequality, commutativity and associativity.

Resources: [Lesson for Algebraic Thinking](#) (von Rotz and Burns), [Thinking Mathematically](#) (Carpenter, Franke, Levi), [Young Mathematicians At Work: Constructing Addition and Subtraction](#) (Fosnot and Dolk), [Navigating Through Algebra](#) (NCTM Publication – Pre-k – 2nd Grade)

Teacher Notes:
