

2nd grade

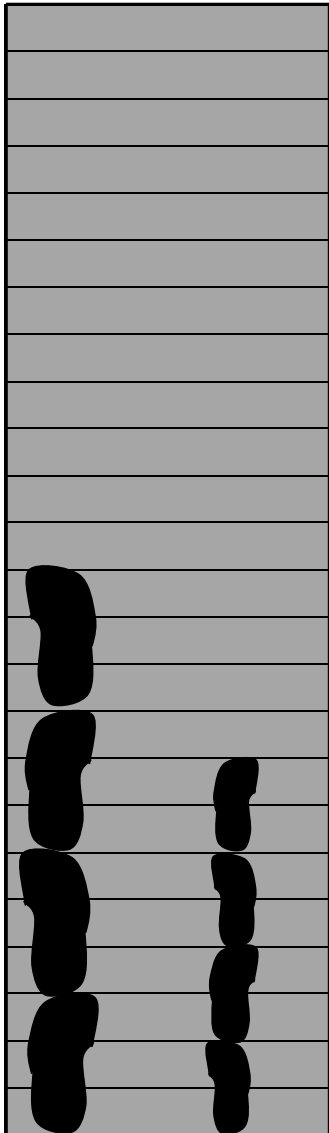
Task 2

Footsteps On The Rug

Student Task	Find the number of footsteps taken by a father and son who walk the same distance. Show how you found the number of footsteps.
Core Idea 4 Geometry and Measurement	Apply appropriate techniques to determine measurements. <ul style="list-style-type: none">• Understand how to measure using non-standard and standard units.• Select an appropriate unit and tool for the attribute being measured (length)

Footsteps On The Rug

Look at the footsteps on the rug. Dad measures the rug by counting his steps. Aaron measures the rug by counting his steps.



Dad

Aaron

1. Who has to take more steps to walk across the whole rug? _____

2. If Dad walks across the whole rug, how many steps will he take in all?

Show how you figured it out.

3. If Aaron walks across the whole rug, how many steps will he take in all?

Show how you figured it out.

4. If Aaron and his dad each took 6 steps, who would walk farther?

Mathematics Assessment Collaborative Performance Assessment Rubric Grade 2

	Footsteps On The Rug: Grade 2:	Points	Section Points
	<p>The core elements of the performance required by this task are:</p> <ul style="list-style-type: none"> • Understanding how to measure using non-standard units • Describes a quantitative relationship • Communicates process using words, numbers, or pictures. <p>Based on these, credit for specific aspects of performance should be assigned as follows</p>		
1	Aaron	1	1
2	<p>8 steps</p> <p>Shows calculations such as: Drawn on rug, numbered steps, or explains how the answer is known.</p>	<p>1</p> <p>1</p>	2
3	<p>12 steps</p> <p>Shows calculations such as: Drawn on the rug, numbered steps, or explains how the answer is known.</p>	<p>1</p> <p>1</p>	2
4	Dad	1	
	Total points		6

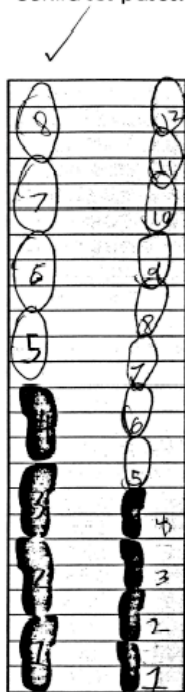
Looking At Student Work – Footsteps on the Rug

“Footsteps on the Rug” has given us a great deal of information of student thinking around measurement. Student A has shown understanding that units are of equal size (drawn more or less equally), that units are iterated with no gaps nor overlays and has shown through drawing and numbers how to arrive at the given answer.

Student A

Huellas en la Alfombra

Mira las huellas en la alfombra. Papá mide la alfombra por contra sus pasos. Aaron mide la alfombra por contar sus pasos.



Papá Aaron

1. ¿Quién tiene que dar más pasos para caminar por toda la alfombra?

Aaron

2. Si Papá camina por toda la alfombra, ¿cuántos pasos dará en total?

8 PASOS

Demuestra cómo llegaste a la respuesta.



3. Si Aaron camina por toda la alfombra, ¿cuántos pasos dará en total?

12 PASOS

Demuestra cómo llegaste a la respuesta.



4. Si Aaron y su papa dan 6 pasos cada uno, ¿quién caminará más lejos?

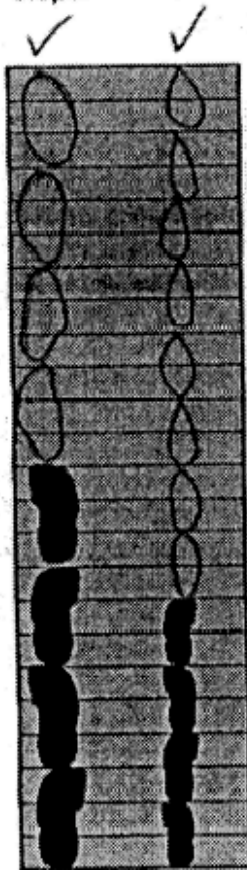
la papa de Aaron

Students B, C, D, and E also have strong understandings of the principles of measurement. It is interesting to see the ways in which they go about proving their answers.

Student B used drawings on the rug and a dialogue about the steps taken as well as a comment on comparing the size of the two footsteps. Without the drawing, the explanation would have been incomplete.

Student B

Look at the footsteps on the rug. Dad measures the rug by counting his steps. Aaron measures the rug by counting his steps.



Dad Aaron

1. Who has to take more steps to walk across the whole rug? Aaron ✓

2. If Dad walks across the whole rug, how many steps will he take in all? 6 footsteps ✓

Show how you figured it out.
I put the rest of his steps and it came out as 6.

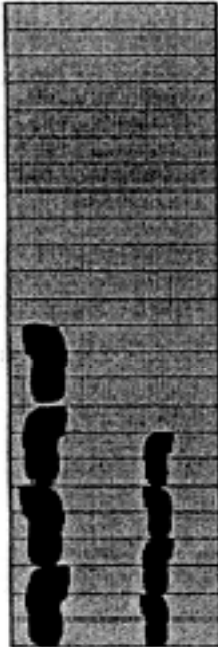
3. If Aaron walks across the whole rug, how many steps will he take in all? 12 footsteps ✓

Show how you figured it out.
I made a smaller foot and it ended out to be 12 footsteps.

4. If Aaron and his dad each took 6 steps, who would walk farther? Aaron's dad would walk farther. ✓

Students C-E show their proof of answer without drawing on the rug. Student C quantifies Dad's foot size as equal to three lines and Aaron's equal to two lines. We have to assume that the student means the rectangular areas covered by each foot as it takes a step. Student D quantifies each step differently than Student C. In this case it's a relationship of 4:3. The explanation is lacking in clarity but Dad's foot does cover four lines and Aaron's covers three lines.

Student C



Dad Aaron

1. Who has to take more steps to walk across the whole rug? Aaron ✓

2. If Dad walks across the whole rug, how many steps will he take in all? eight ✓

Show how you figured it out.
Dad's foot fits in three lines so I just counted up. ✓

3. If Aaron walks across the whole rug, how many steps will he take in all? twelve ✓

Show how you figured it out.
Aaron's foot fits in two lines so I just counted up. ✓

4. If Aaron and his dad each took 6 steps, who would walk farther? Dad ✓

Student D



Dad Aaron

1. Who has to take more steps to walk across the whole rug? Aaron ✓

2. If Dad walks across the whole rug, how many steps will he take in all? 8 ✓

Show how you figured it out.
because Aaron's take four lines every step. The lines are the things in the middle.

3. If Aaron walks across the whole rug, how many steps will he take in all? 12 ✓

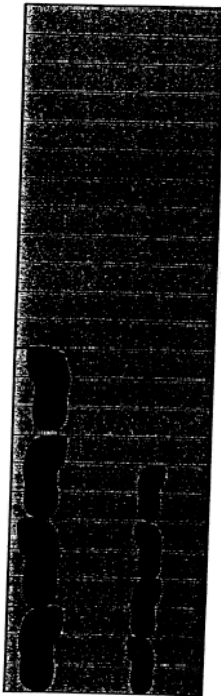
Show how you figured it out.
Aaron's step are three lines.

4. If Aaron and his dad each took 6 steps, who would walk farther? Dad ✓

Student E shows beginning multiplicative relationships in their complete explanation that it is eight steps of three lines each to cover the total 24 lines in the diagram! In this case, the student explains the rectangular bar as a line.

Student E

Look at the footsteps on the rug. Dad measures the rug by counting his steps. Aaron measures the rug by counting his steps.



1. Who has to take more steps to walk across the whole rug? Aaron ✓

2. If Dad walks across the whole rug, how many steps will he take in all?
8 steps ✓

Show how you figured it out.

Each of dad's steps = 3 lines
and there are 24 lines ✓

3. If Aaron walks across the whole rug, how many steps will he take in all?
Twelve ✓

Show how you figured it out.

There are 24 lines and each of his footsteps = 2 lines. ✓

Dad Aaron

4. If Aaron and his dad each took 6 steps, who would walk farther? Dad ✓

Approximately 10% of the students misread questions number two and three. Student F is an example of those students who, when asked to find the total steps to get across the rug, answered the number of steps remaining to get across the rug. The student shows complete understanding for the issues of measurement and so were only marked wrong the first time they made this mistake.

Student F



1. Who has to take more steps to walk across the whole rug? Aaron ✓ 1
 I made mor footsteps.

2. If Dad walks across the whole rug, how many steps will he take in all?
4 x mr. 1-1 = 0

Show how you figured it out.
 I countid the footsteps 1

3. If Aaron walks across the whole rug, how many steps will he take in all?
8 x ✓ 10

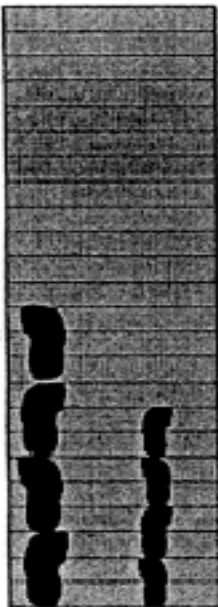
Show how you figured it out.
 I countid the footsteps ✓ 1

4. If Aaron and his dad each took 6 steps, who would walk farther? Dad ✓

He has 6 steps

The following three students have an understanding of the proportionality in the two footstep sizes. Each accounts for the size but errs in counting. Student G defines Dad's foot as equal to 3 boxes and Aaron's foot as equal to 2 boxes but miscounts Aaron's footsteps.

Student G



1. Who has to take more steps to walk across the whole rug? Aaron ✓

2. If Dad walks across the whole rug, how many steps will he take in all?
8 steps ✓

Show how you figured it out.
 Each step is worth 3 boxes. ✓

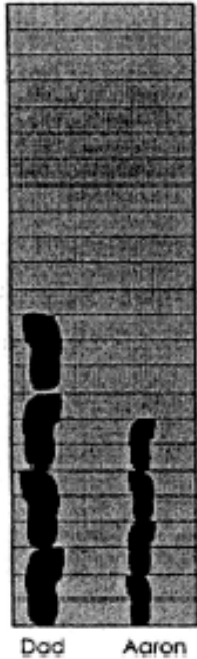
3. If Aaron walks across the whole rug, how many steps will he take in all?
14 steps X

Show how you figured it out.
 Aarons feet is worth 2 boxes. ✓

Dad Aaron

Student H has used an unstructured "T" chart. For Dad's footsteps the student was successful but in figuring Aaron's footsteps, she went one footstep past the end of the rug showing the relationship for 26 lines rather than for 24.

Student H



1. Who has to take more steps to walk across the whole rug? Aaron ✓

2. If Dad walks across the whole rug, how many steps will he take in all?
8 steps in all ✓

Show how you figured it out.

3, 6, 9, 12, 15, 18, 21, 24 ✓
1, 2, 3, 4, 5, 6, 7, 8 ✓

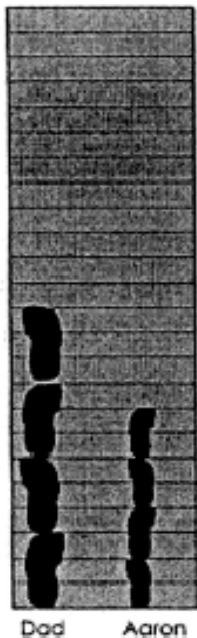
3. If Aaron walks across the whole rug, how many steps will he take in all?
13 steps in all X

Show how you figured it out.

2, 4, 6, 8, 9, 12, 14, 16,
1, 2, 3, 4, 5, 6, 7, 8,
18, 20, 22, 24, 26
9, 10, 11, 12, 13

Student I errs in counting the number of steps for Aaron. This student correctly quantifies Dad's steps and is able to make a beginning attempt to describe a relationship between the footsteps and the space each footstep covers.

Student I



1. Who has to take more steps to walk across the whole rug? Aaron ✓

2. If Dad walks across the whole rug, how many steps will he take in all?
8 ✓

Show how you figured it out.

when he takes a step he covers up two lines in the middle

3. If Aaron walks across the whole rug, how many steps will he take in all?
X 0

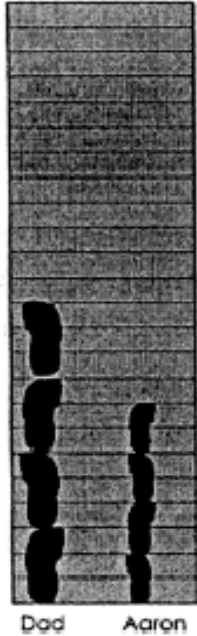
Show how you figured it out.

because he covers line in the middle

In answering questions number two and three, 5% of the respondents said that Dad would take 12 steps and Aaron would take 16 steps. Students J and K both said that they counted Dad as three bars and Aaron as two bars.

Did they count in this way and just recount the initial four steps for each so that Dad is $4 + 8$ to get across and Aaron is $4 + 12$ to get across? Did they see the relationship of 3:2 but then just count the remaining blank bars as Dad has 12 remaining bars and Aaron has 16 remaining bars?

Student J



1. Who has to take more steps to walk across the whole rug? Aaron ✓

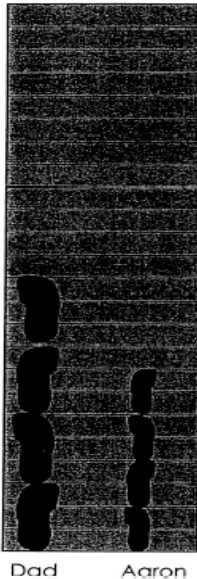
2. If Dad walks across the whole rug, how many steps will he take (in all)?
twelve steps X

Show how you figured it out.
I just counted three of the bars ✓
and go again + by using

3. If Aaron walks across the whole rug, how many steps will he take (in all)?
sixteen steps X

Show how you figured it out.
By counting twos ✓

Student K



1. Who has to take more steps to walk across the whole rug? Dad X

2. If Dad walks across the whole rug, how many steps will he take in all?
12 X

Show how you figured it out.
counting by threes. ✓

3. If Aaron walks across the whole rug, how many steps will he take in all?
16 X

Show how you figured it out.
By counting by twos ✓

4. If Aaron and his dad each took 6 steps, who would walk farther? Dad ✓

58% of the 2nd grade students had some level of difficulty in providing evidence of how they figured out the correct number of steps for each to walk across the rug. Student L is an example of an incomplete explanation and representation to correct calculations. Student M makes a comparison of foot sizes but fails to quantify the relationship.

Student L

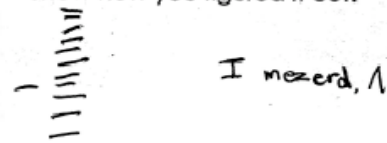


Dad Aaron

1. Who has to take more steps to walk across the whole rug? Aaron ✓

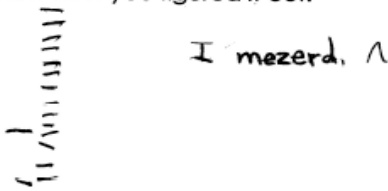
2. If Dad walks across the whole rug, how many steps will he take in all?
8 ✓

Show how you figured it out.



3. If Aaron walks across the whole rug, how many steps will he take in all?
12 ✓

Show how you figured it out.



Student M



Dad Aaron

1. Who has to take more steps to walk across the whole rug? dad X

2. If Dad walks across the whole rug, how many steps will he take in all?
8 steps ✓

Show how you figured it out.

because dad feet are big. x

3. If Aaron walks across the whole rug, how many steps will he take in all?
12 steps ✓

Show how you figured it out.

Aaron feet small than dad

Student N's explanation is similar to many students. In each case, they knew to add the beginning four steps and to add more steps for Aaron than for Dad. It appears, though, that the number of extra steps for Dad and Aaron to cross the rug was an arbitrary number.

Student N



Dad Aaron

1. Who has to take more steps to walk across the whole rug? Aaron ✓

2. If Dad walks across the whole rug, how many steps will he take in all?
9 X

Show how you figured it out.

$$\begin{array}{r} 4 \\ + 5 \\ \hline 9 \end{array} X$$

3. If Aaron walks across the whole rug, how many steps will he take in all?
11 X

Show how you figured it out.

$$\begin{array}{r} 4 \\ + 7 \\ \hline 11 \end{array} X$$

4. If Aaron and his dad each took 6 steps, who would walk farther? Aaron X

Student O is representative of less than 5% of the second grade responses. In most of these cases, there appears to be little or no understanding of what the questions are asking. In this example, the student doesn't give up but rather shows something he/she does know about addition.

Student O



Dad Aaron

1. Who has to take more steps to walk across the whole rug? Dad 0

2. If Dad walks across the whole rug, how many steps will he take in all? 23 0

Show how you figured it out.
I got 23 because
 $20 + 3 = 23$

3. If Aaron walks across the whole rug, how many steps will he take in all? 20 0

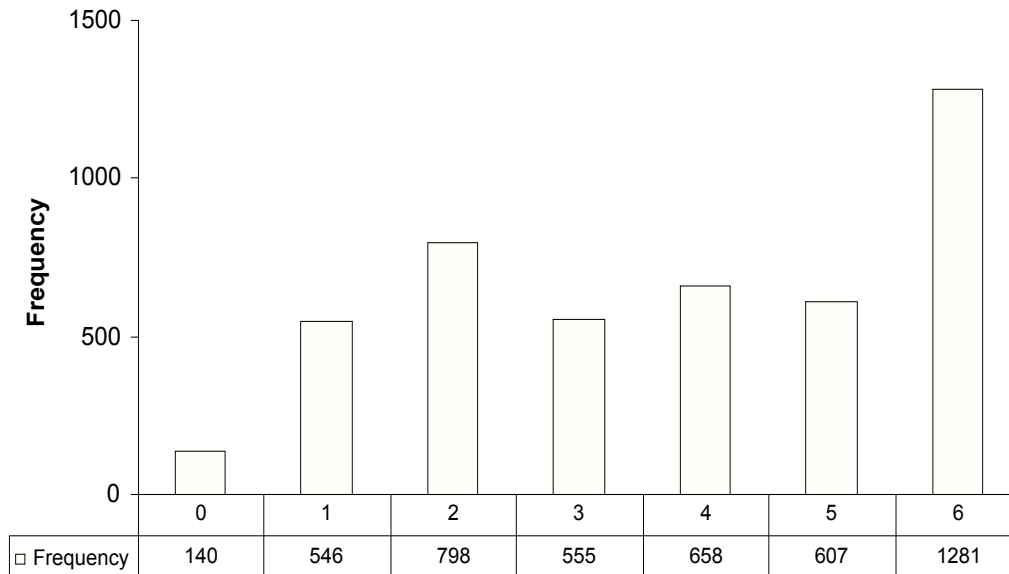
Show how you figured it out. X
||||| + ||||| = 20

Teacher Notes:

Frequency Distribution for each Task – Footsteps on the Rug

Footsteps on the Rug

Mean: 3.74, S.D.: 1.89



Score:	0	1	2	3	4	5	6
% ≤	3.1%	15.0%	32.4%	44.5%	58.8%	72.1%	100.0%
% ≥	100.0%	96.9%	85.0%	67.6%	55.5%	41.2%	27.9%

The maximum score available for this task is 6 points.
The cut score for a level 3 response is 3 points.

Almost 68% of the students were able to see a relationship between the footsteps and the number of iterations necessary to cross the rug. Just over 41% of the students could find the number of steps Dan and Aaron would take to walk across the rug as well as show or explain their answers. 3% of the students scored a zero on this task. All students attempted to solve this problem.

Footsteps on the Rug

Points	Understandings	Misunderstandings
0		Most of these students did not find a relationship between the sizes of the two footsteps.
1	Students scoring one on this task knew that Dad would walk farther than Aaron after six steps. About 1/3 of these students knew that the smaller the unit, the more steps would be taken.	Although many students gave Aaron's steps as being more than Dad's they simply added a few more steps without a relationship behind the amount added.
2	Most students knew Dad would walk farther after 6 steps and that Aaron would take more steps to cross the rug. A few students could explain a relationship between Dad's footsteps and Aaron's. 25% of all students had success finding the total number of footsteps for Aaron OR Dad by drawing units on the rug.	Students were equally as likely to misunderstand or misinterpret question 1 as they were question 4– Who takes more steps to cross the same distance? and Given the same number of steps, who would walk further?
3-4	At this score, students are seeing a relationship between footstep and space – either 4:3 lines or 3:2 spaces. Slightly more than 50% figured that Dad would take 8 steps and that Aaron would take 12 steps.	Explaining how they figured out the amount of footsteps was difficult for students. Many students compared the footstep size rather than quantifying the size of each. The explanations were either incomplete or did not justify their answer.
5	Most students correctly found the number of total steps for both Dad and Aaron and were able to mathematically explain the relationship between feet and steps. 10% of the students understood all four questions but misread the problem and wrote how many <i>more</i> steps were needed for each.	Most of the students made a calculation error for one of the two measurements.
6	Students successfully drew the total steps taken to cross the rug and/or explained the relationship between footstep size and number of steps to go across this rug.	

Based on teacher observation, this is what second grade students seemed to know and be able to do:

- Use pictures and drawings to help calculations
- Compare proportions
- Tell that Aaron would take more steps to get across the rug
- Tell that if both took 6 steps, Dad would walk farther
- Know that larger units result in smaller numbers of units
- Show a relationship of 3:2 or 4:3 for Dad:Aaron's footsteps

Areas of difficulty for second graders, these students struggled with:

- Using consistent sized units
- Explaining their answers
- Counting steps already on the rug as well as those left to be counted
- Drawing measurements on a grid

Questions for Reflection-Footsteps on the Rug

Carefully look over your student work in the context of this measurement learning framework:

		Number of successful students	Number of unsuccessful students
Stage 1: Identification of length	Directly compares Dad's footsteps to Aaron's		
Stage 2: Informal Measurement	Finds the number of units to cover the rug without gaps or overlays		
	Quantifies the number of units to cover the rug		
	Uses these measurements to compare Dad's footstep to Aaron's		
Stage 3: Unit structure	Replicates the footsteps to cover the rug by drawing or showing the size or the relationship		
	Knows the larger the unit, the fewer units needed		

- What opportunities do your students have to measure common items and look for relationships between the measurement and the measurement tool?
- If students are comparing two items directly do they have a strategy for comparing two items when they are not side by side?

- If students are iterating units with multiples of that unit, how are they placing the units? Do they line them up end to end? Are they leaving gaps or placing them with overlays?
- If students are iterating a single unit, how are they placing that unit each time they move it?
- What effective ways do they have for keeping track of how many times they have iterated the unit?

Instructional Implications:

Traditionally, the goal of measurement instruction has been to help students learn how to use conventional measurement tools such as the ruler. Although this is an eventual goal we all have for our students, fundamental measurement understandings should be built first. In a research project (Bragg and Outhred 2000a, 2000b) designed to assess the understanding of measurement concepts, students of ages six to ten years were found to be able to count units, unit marks, or unit spaces on a ruler but often erred by naming the number of marks instead of spaces or by aligning the ruler incorrectly. Those students who correctly used procedural instructions often could not use them to solve practical problems. When we emphasis techniques rather than concepts we may obscure the big ideas of linear measurement such as: partitioning of length, unit iteration, transitivity, conservation of length, accumulation of length, and relation to number.*

If we can focus classroom conversations and thoughts on students’ own meaning of measurement concepts, we can make those implicit ideas to measurement an explicit part of our discussions. Students can then apply them to different problem solving situations.

Resources: Navigating Through Measurement (NCTM Publication Pre-K – 2nd Grade), Developing Number Sense (K. Richardson), Investigations in Number, Data and Space- How Long? How Far? (TERC 2nd Grade)

Teacher Notes:

* for more on this subject, see “Linear and Area Measurement in Pre-Kindergarten to Grade 2” Stephan and Clements Learning and Teaching Measurement NCTM Yearbook 2003 pgs 3-14