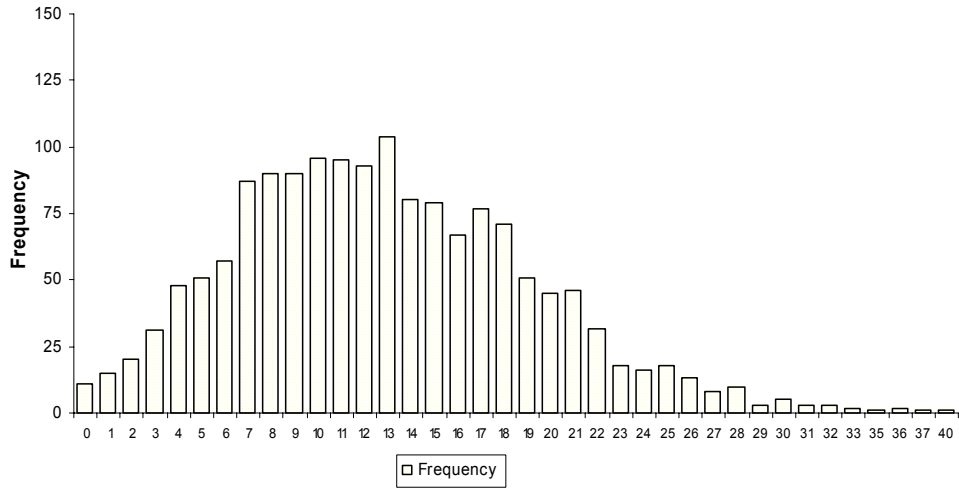


Overall Frequency Distribution by Total Score

Grade 9
Mean=12.86; S.D.=6.32



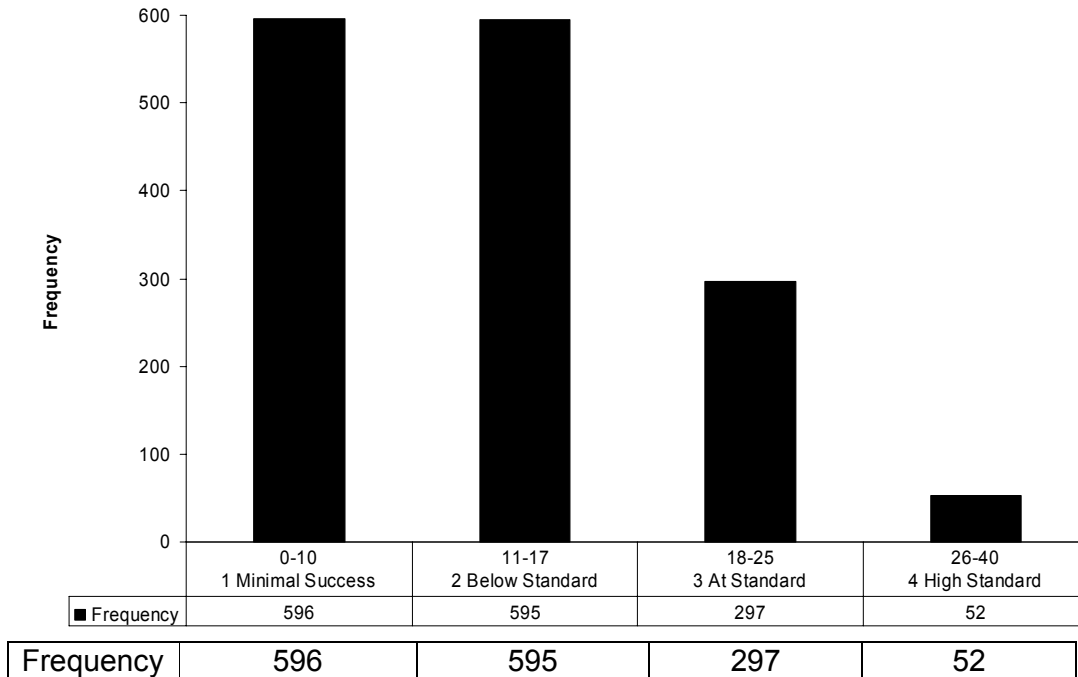
Level Frequency Distribution Chart and Frequency Distribution

2004 - Numbers of students tested in Grade 9: 1540
Grade 9 1999 and
2001

Level	% at ('99)	% at least ('99)	% at ('01)	% at least ('01)
1	37%	100%	22%	100%
2	43%	63%	62%	78%
3	13%	20%	15%	16%
4	7%	7%	1%	1%

Grade 9 2002 - 2004

Level	% at ('02)	% at least ('02)	% at ('03)	% at least ('03)	% at ('04)	% at least ('04)
1	18%	100%	66%	100%	39%	100%
2	61%	82%	29%	34%	39%	61%
3	20%	22%	5%	6%	19%	23%
4	2%	2%	1%	1%	3%	3%



Comparing Student Performance (cont.)

Grade
9

	1999		2001	
	Count	Percentage	Count	Percentage
Level 1	639	36.9%	179	22.3%
Level 2	754	43.5%	496	61.8%
Level 3	219	12.6%	121	15.1%
Level 4	122	7.0%	6	0.7%

	2002		2003		2004	
	Count	Percentage	Count	Percentage	Count	Percentage
Level 1	109	17.5%	1139	65.6%	596	38.7%
Level 2	378	60.8%	501	28.9%	595	38.6%
Level 3	122	19.6%	84	4.8%	297	19.3%
Level 4	13	2.1%	12	0.7%	52	3.4%

	N	Mean	Std. Deviation
2000	1734	1.9	0.88
2001	802	1.94	0.63
2002	622	2.06	0.67
2003	1736	1.41	0.62
2004	1540	1.87	0.84

Fewer students took the test in 2001 and 2002 than in the other administrations, so comparisons of 2001 and 2002 with the other administrations should be treated with caution. No clear patterns emerge in the 1999 to 2004 data. The percentage of students meeting standards increased substantially from 2003 (5.5%) to 2004 (22.7%). The increase in mean performance is statistically significant and corresponds to an effect size of .55 standard deviation units.

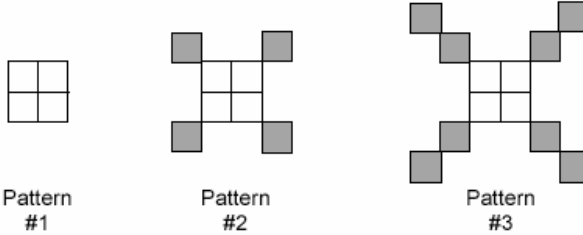
Student Task	Find and extend number patterns in a geometric context. Find and use rules or formulas to solve problems.
Core Idea 1 Functions and Relations	Understand patterns, relations, and functions. <ul style="list-style-type: none">• Generalize patterns using explicitly defined functions.
Core Idea 3 Algebraic Properties and Representations	Represent and analyze mathematical situations and structures using algebraic symbols. <ul style="list-style-type: none">• Use symbolic algebra to represent and explain mathematical relationships.• Use symbolic expressions to represent relationships arising from various contexts.

Square Patterns

This problem gives you the chance to:

- work with a sequence of tile patterns
 - write and use a formula
-

Mary has some white and gray square tiles. She uses them to make a series of patterns like these:



1. How many gray tiles does Mary need to make the next pattern?

2. What is the total number of tiles she needs to make pattern number 6?

Explain how you figured it out.

3. Mary uses 48 tiles in all to make one of the patterns.

What is the number of the pattern she makes?

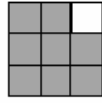
Show your work.

4. Write a formula for finding the total number of tiles Mary needs to make pattern # n .

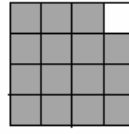
Mary now uses gray and white square tiles to make a different pattern.



Pattern
#1



Pattern
#2



Pattern
#3

5. How many gray tiles will there be in pattern # 10? _____

Explain how you figured it out.

6. Write an algebraic formula linking the pattern number, P, with the number of gray tiles, T.

Square Patterns		Test 9 Rubric	
The core elements of performance required by this task are: <ul style="list-style-type: none"> • work with a sequence of tile patterns • write and use a formula Based on these, credit for specific aspects of performance should be assigned as follows:		points	section points
1.	Gives a correct answer: 12	1	1
2.	Gives a correct answer: 24 Gives a correct explanation such as: Pattern # 6 has 4 arms of length 5 and 4 squares in the middle. or I multiplied the pattern number by 4.	1 1	2
3.	Gives a correct answer: 12 Shows correct work such as: 48 total tiles - 4 white tiles = 44 gray tiles 44 gray tiles ÷ 4 = 11 tiles on each "arm". Add 1 to get pattern number. or I divided the number of tiles by 4 to find the pattern number.	1 1	2
4.	Gives a correct formula: $T = 4(n - 1) + 4$ or equivalent $T = 4n$	1	1
5.	Gives a correct answer: 120 Gives a correct explanation such as: The large square has sides of length 11 and one square is white.	1 1	2
6.	Gives a correct formula such as: $P = \sqrt{(T + 1)} - 1$ or $T = (P + 1)^2 - 1 = P(P + 2)$ Accept equivalent formulas.	1 or 1	1
Total Points			9

Looking at Student Work – Square Patterns

Student A is willing to experiment with patterns in a variety of ways to help make sense of the relationships. In the beginning Student A uses drawings to solve for part 2 and 3. Then the student notices a pattern of adding four every time and uses that relationship to verify the drawing. By doing all this thinking about the relationships, the student is then able to come up with an algebraic expression in part 4 to find for any pattern number. In part 5, the student first notices the differences increase by consecutive odd numbers. Then with further thinking about the pattern number, he sees that you square one more than the pattern number and subtract one. It is interesting to see all the stages of the student's thinking.

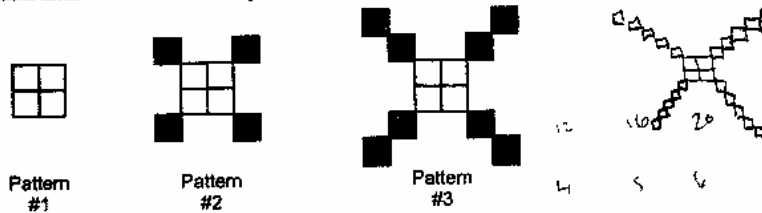
Student A

Square Patterns

This problem gives you the chance to:

- work with a sequence of tile patterns
- write and use a formula

Mary has some white and gray square tiles. She uses them to make a series of patterns like these:



1. How many gray tiles does Mary need to make the next pattern?

12 ✓

2. What is the total number of tiles she needs to make pattern number 6?

Explain how you figured it out.

24 ✓

I drew the pattern for #6. ✓

3. Mary uses 48 tiles in all to make one of the patterns.

What is the number of the pattern she makes?

12 ✓

Show your work.



$$36 + 4 = 40 \text{ (#10)}$$

$$24 + 4 = 28 \text{ (#7)}$$

$$28 + 4 = 32 \text{ (#8)}$$

$$32 + 4 = 36 \text{ (#9)}$$

$$40 + 4 = 44 \text{ (#11)}$$

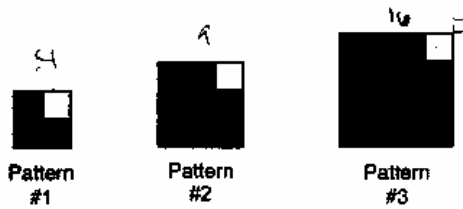
$$44 + 4 = 48 \text{ (#12)}$$

4. Write a formula for finding the total number of tiles Mary needs to make pattern # n.

$4n = \text{total number of tiles.}$ ✓

Student A, part 2

Mary now uses gray and white square tiles to make a different pattern.



$16 + 1 = 25 = 5^2$ $36 = 6^2$

$5 = 6 \times 6$
 $6 = 7 \times 7$

5. How many gray tiles will there be in pattern # 10?

120 ✓ 1

Explain how you figured it out.

I used the formula $(p+1)^2 - 1$. ✓ ~~8~~

6. Write an algebraic formula linking the pattern number, P, with the number of gray tiles, T.

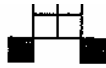
$(p+1)^2 - 1$ ✓ 1

Student B shows work in part three which recognizes that there are gray tiles plus four white tiles. However in trying to define a rule, the student accounts for the white tiles and knows that the gray tiles are 4 times some quantity. Student B fails to describe the relationship of x to the pattern number. Student B also does not seem to understand the variables described in the problem and uses n for tiles instead of for the pattern number. In part 5 Student B notices that each time the pattern increases, the next consecutive odd number is added to the previous total. However the student makes no attempt to quantify this addition in making an algebraic expression. Again the student does not seem to understand variables, using p for the larger quantity and introducing an undefined variable X.

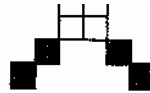
Student B



Pattern #1



Pattern #2



Pattern #3

12	16	20	24
4	5	6	7

1. How many gray tiles does Mary need to make the next pattern?

12 tiles ✓

2. What is the total number of tiles she needs to make pattern number 6?

24 tiles ✓

Explain how you figured it out.

I found that the gray tiles increase by 4 each time, so I knew by pattern 6 there would be 20 gray tiles, and then I added the 4 white tiles to make 24.

3. Mary uses 48 tiles in all to make one of the patterns.

white tiles to make 24.

What is the number of the pattern she makes?

#12 ✓

Show your work.

24 + 4 = 28 #7
28 + 4 = 32 #8
32 + 4 = 36 #9
36 + 4 = 40 #10
40 + 4 = 44 #11
44 + 4 white = 48 #12 ✓

4. Write a formula for finding the total number of tiles Mary needs to make pattern # n.

$4 + 4x = n$ ✗

$\frac{25}{13}$	$\frac{48}{63}$	$\frac{63}{80}$	$\frac{80}{99}$	$\frac{99}{120}$
-----------------	-----------------	-----------------	-----------------	------------------

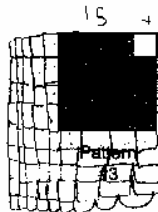
Mary now uses gray and white square tiles to make a different pattern.



Pattern #1



Pattern #2



Pattern #3

24	35	48	63	80	99	120
↑	↑	↑	↑	↑	↑	↑
#2	#5	#6	#7	#8	#9	#10

120 ✓

5. How many gray tiles will there be in pattern # 10?

Explain how you figured it out.

at first I counted, but then I figured out that the tiles go up by odd numbers each time (+3, +5, +7, etc) so I was able to do it faster.

6. Write an algebraic formula linking the pattern number, P, with the number of gray tiles, T.

$P = T \times 2$ ✗

Student C gives a good description of the same pattern used by Student D. The complexity of the pattern makes it too difficult for Student D to describe in terms of variables. One common attempt to describe this pattern symbolically, was to say $p=T+2$. Students are trying to use T as the previous total and are misusing p as a new total rather than pattern number.

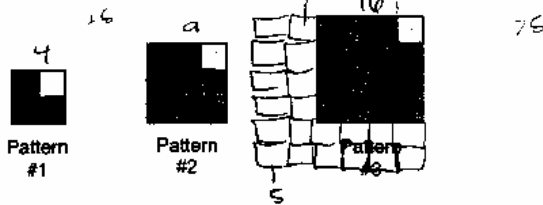
Student C

$$\begin{array}{r} 100 \\ 21 \\ \hline 121 \end{array} \quad \begin{array}{r} 81 \\ 19 \\ \hline 100 \end{array} \quad \begin{array}{r} 64 \\ 17 \\ \hline 81 \end{array} \quad \begin{array}{r} 49 \\ 15 \\ \hline 64 \end{array} \quad \begin{array}{r} 36 \\ 13 \\ \hline 49 \end{array} \quad \begin{array}{r} 25 \\ 11 \\ \hline 36 \end{array}$$

$$\begin{array}{r} 16 \\ 9 \\ \hline 25 \end{array}$$

$$\begin{array}{r} 121 \\ 23 \\ \hline 144 \end{array}$$

Mary now uses gray and white square tiles to make a different pattern.



P-1	4	+5
P-2	9	+7
P-3	16	+9
P-4	25	+11
P-5	36	+13
P-6	49	+15
P-7	64	+17
P-8	81	+19
P-9	100	+21
P-10	121	+23
P-11	144	

5. How many gray tiles will there be in pattern # 10?

$$\begin{array}{r} 121 \\ \hline 0 \end{array}$$

Explain how you figured it out.

each patten goes up two more than the previous.
 ex) patten 1 - has (4) now patten 2 - has (9) x
 then patten 3 - has (16) so 1st you added 5 to get
 9 then you added 7 (+2) to get 16
 4+7=11 11+9=20 20+11=31 etc.

6. Write an algebraic formula linking the pattern number, P, with the number of gray tiles, T.

Student D experiments with the pattern by first drawing the pattern and then playing with number sentences that will yield the total number of tiles. Student D may have noticed a lower solid rectangle of $p(p+1)$ and then a top row of p to give the total number of tiles. Noticing how the pattern is drawn helps students to describe the relationship symbolically.

Student D

Mary now uses gray and white square tiles to make a different pattern.

$10 \times 11 = 110$
 $10 \times 11 = 110$
 $15 \quad 30 \quad 45 \quad 60 \quad 75 \quad 90 \quad 105 \quad 120$
 $\#3: 2 \quad \#6: 3 \quad \#9 \quad \#10$
 $= 110 + 10$
 $\begin{array}{r} 11 \\ \times 11 \\ \hline 11 \\ 110 \\ \hline 121 \end{array}$

5. How many gray tiles will there be in pattern #10?

Explain how you figured it out.

across the # of grey tiles are the same as the pattern number and going down the number of tiles is 1 more than the pattern # So $10 \times 11 = 110$ but then you add the pattern # to that because you have to count the column with the white square in it.

6. Write an algebraic formula linking the pattern number, P , with the number of gray tiles, T .

~~$P(P+1)$~~ $P(P+1) + P = T$

$10: 11 = 110 + 10$
 $10(10+1) + 10 = 120$
 $1: 2 \quad 2+1 = 3$
 $1(1+1) + 1 = 3$
 $3: 4 \quad 12+3$
 $3(3+1) + 3 = 15$
 $2: 3 \quad 6+2$
 $2(2+1) + 2 = 8$

Students had difficulty writing functions and tried to give recursive rules instead. For example to describe the growth of the first pattern, students might write a rule like $t=4+p$. In describing a recursive pattern in part 5 where the finite differences go up by increasing odd numbers, students might write $p=t+2$.

Some students did not seem to understand the variables given in the problem. For example a student in part 4 might use n for the total instead of using n for the pattern number, giving the rule $n=4x$. Some students think that all functions have the same rule and just substituted in the variable giving $y=nx+b$. They weren't bothered that x and b were undefined.

Some students are starting to make sense of the geometric context and how the pattern is growing. For example Student E notices that the gray tiles are one less than a perfect square. However, the student writes $p=t^2-1$. The student is not using the variables as defined in the problem and the student cannot quantify how large the sides of the square should be.

Understanding how to describe the sides of the square by only using the variables in the problem was difficult for students. Student F writes the sides of the square are $P+1$ and $T+1$, but then does not use this to give a formula for the total number of tiles. Student G redefines P from pattern number to an area. $P=(l \times w)$, so $T=P-1$. This is correct, but the Student G needs to describe how to find the length and the width.

Student H notices a good pattern of finding the area of the lower rectangle $(p+1)p$ and then adding the top row. However the student confuses P and T within the equation.

Student H

Mary now uses gray and white square tiles to make a different pattern.

$$2 \times 3 = 6 + 3 = 9 - 1$$



Pattern #1



Pattern #2



Pattern #3

$$3 \times 4 = 12 + 4 = 16 - 1 = 15$$

$$1 + 2 \times 2 = 4 - 1 = 3$$

$$3 + 1 = 4 \times 4 = 16 - 1 = 15$$

5. How many gray tiles will there be in pattern # 10?

120 ✓

Explain how you figured it out.

$$(10+1)10$$

$$\underline{110 + 11 = 121 - 1 = 120} \checkmark$$

6. Write an algebraic formula linking the pattern number, P, with the number of gray tiles, T.

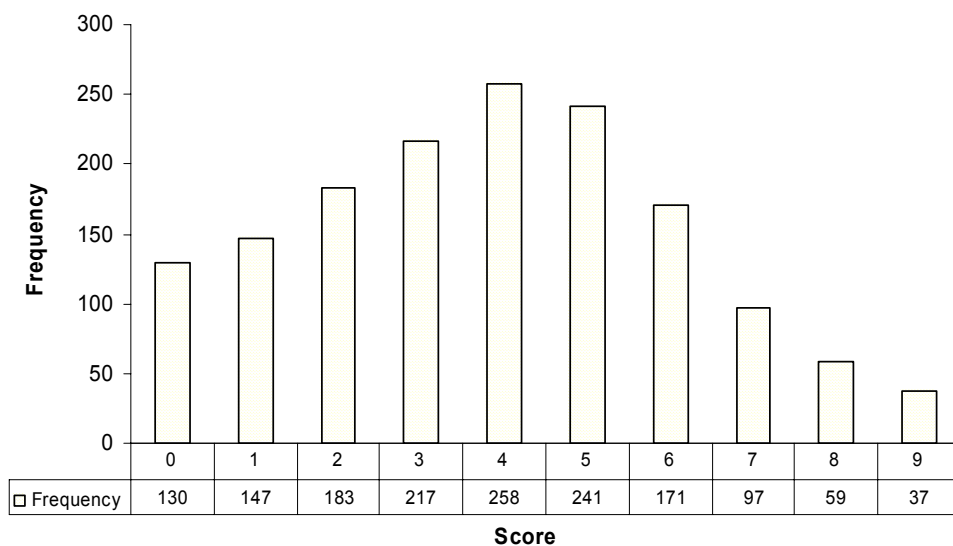
$$\underline{(P+1)P + 1 = T} \checkmark$$

Teacher Notes:

Frequency Distribution for each Task – Grade 9
Grade 9– Square Patterns

Square Patterns

Mean: 3.84, S.D.: 2.28



The maximum score available for this task is 9 points.
The cut score for a level 3 response is 5 points.

Most students (93%) could extend the first pattern from picture 4 to picture 5 and find the total number of gray tiles. Many students (about 71%) could extend the pattern to 6 and find the total tiles. About 38% of the students could extend the first pattern, work back from the total tiles to the pattern number. Only about 10% of the students could use symbolic notation to make pattern generalization. 7% of all students scored no points on this task. All students in the sample who scored zero attempted the problem.

Square Patterns

Points	Understandings	Misunderstandings
0	Almost all students attempted the problem.	Over 30% of all students thought that you only needed 4 gray tiles for the next pattern in part 1. They were thinking of additional tiles instead of total gray tiles.
2	About half the students could extend the pattern to find the total number of tiles to make pattern number 6 or they could work backwards to find the pattern number for 48 tiles.	Most students who missed part two gave the number of gray tiles needed for pattern 6 instead of the total number of tiles. The next most popular answer was 28 (thinking the length of each leg was the same as the pattern number.) Students who missed part three had trouble dealing with the four middle tiles. They either picked one less, forgetting that they had subtracted four tiles, or they picked one more accounting for the middle tiles twice.
5	Students with this score could extend a pattern to find total number of tiles or just gray tiles. They could also work backwards from total tiles to pattern number.	Students had difficulty describing a pattern use symbolic notation. 30% of all students did not even attempt to write an algebraic expression for the first pattern. Many students who attempted the expression did not understand the defined variables. They may have added extra variables, like $4x + 4n$, or just $4x$. Other students used n for the total instead of the pattern number, such as $n=4p$.
7	Students could work successfully with both patterns, distinguishing between gray tiles and total tiles. Students could work backwards to find pattern number given the total number of tiles.	Students could not use symbolic notation to make a formula for either rule. More than 40% of all students did not attempt to write an expression for part 6. Most students did not understand the use of variables in part 6 or even the operation involved. Of the remaining students, 24% wrote expressions using addition or subtraction only. Many students used p as equal to some expression of t . Others put p and t on the same side of the equal sign.
8		Students could not complete an expression for the second pattern.
9	Students could extend patterns, work from total tiles to pattern numbers, and write algebraic formulas to generalize the patterns.	

Questions for Reflection on Square Pattern

- What do you think it is important for student to understand about variables?
- What opportunities or problems have students worked this year, where they have to describe a pattern in symbolic notation? In these situations, was the variable defined for them?
- Make a list of the expressions students used for questions 4 and 6. If you were to sort these expressions into categories by types of misunderstandings how would you label the categories?

For example in part 4, how is the misunderstanding for $p = t+x$ different than the misunderstanding for $4(p-1)$? Which student is showing some understanding of the situation?

In part 6, many students wrote $p = t+1$. What do you think they were looking at in the pattern? In what ways might they have been making sense of the pattern? What types of questions might push their thinking in ways that would be more helpful to developing generalizations?

- Why do you think students were so reluctant to attempt writing generalizations or algebraic expressions? What activities might help them to develop this skill?
- In the beginning of the algebra course, when students are being introduced to variables, how do students learn to describe situations in terms of variables? Do students have opportunities to discuss the number of variables needed to describe a situation? Do students have the opportunity to discuss when one variable is used versus two or three and why this might be important to solving the problem?

Teacher Notes:

Implications for Instruction:

Students at this grade level should have frequent opportunities to use symbolic notation to write or formulas for patterns or to describe a procedure for solving a problem in context. Without the benefit of struggling with context, students often do not grasp the finer points of variables, like thinking about the relationship between independent and dependent variables and understanding that equations are often written in terms of doing calculations to one variable to find the value of the other. By working in context, especially in a rich geometric context, students can see how the different parts of their expressions relate to specific parts of the physical pattern. Questions by the teacher, like what stays the same and what changes, help students to identify and then to quantify the various parts of the patterns. Transference of algebraic skills to useful applications cannot take place without dealing with situations in context.

Course One**Task 2****Population**

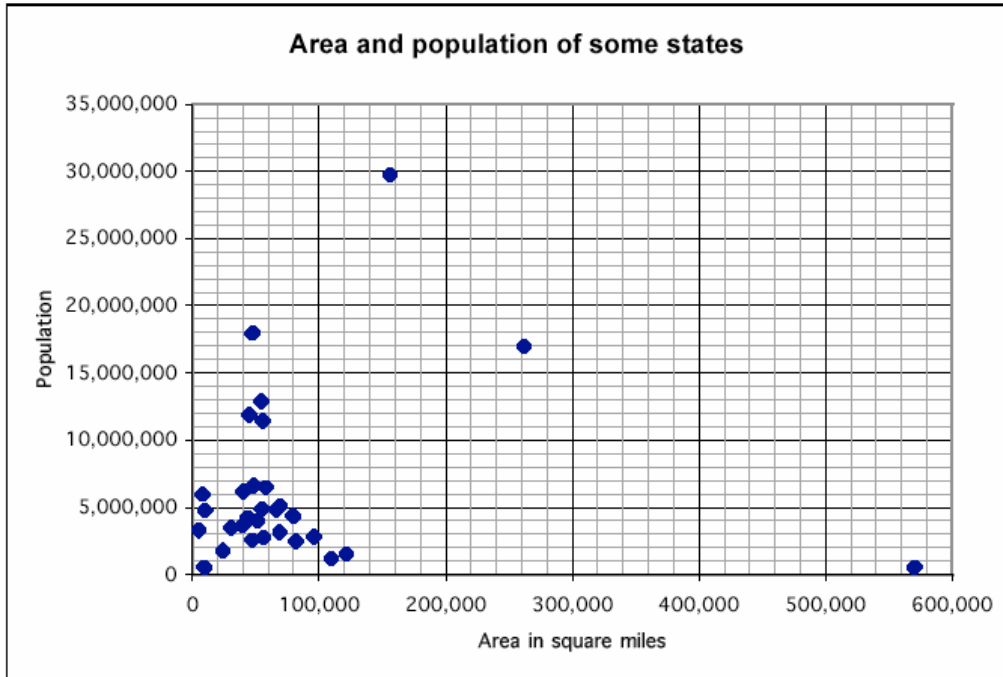
Student Task	Analyze a scatterplot for trend, graph a line represent average density, graph specific point for a given piece of data, locate points on a graph to meet criteria for largest population or lowest density, and calculate density relationships.
Core Idea 5 Data Analysis	<ul style="list-style-type: none">• Understand the relationship between two sets of data, display such data in a scatterplot, and describe trends and shape of the plot including correlations.• Make inferences based on the data and evaluate the validity of conclusions drawn.
Core Idea 3 Algebraic Properties and Representations	<ul style="list-style-type: none">• Approximate and interpret rates of change from graphic and numeric data.

Population

This problem gives you the chance to:

- interpret a scatter plot
- select and perform operations

This graph shows the areas and the populations of some of the states of the USA.



1. Draw a circle around the point that represents the state with the largest population.
2. Draw a square around the point that represents the state with lowest number of people per square mile.
3. Describe the main features of the graph.

4. Draw an X on the graph to show the data for Michigan, with an area of 56,802 square miles and a population of 9,295,277 people.

5. Calculate the number of people per square mile in Michigan, giving your answer to the nearest whole number.

6. The average number of people per square mile for the USA is 70, though no state actually has exactly this population density.

Draw a straight line on the graph to show all the possible positions of points showing 70 people per square mile.

7. The average state area is 68,000 square miles, though no state has exactly this area. If a state with the average area had the average population density, what would its population be?

8. The data in this graph is for the year 1990. Since then, what data will have stayed the same and what data may have changed?

8

Population	Test 9 Rubric	
The core elements of performance required by this task are: <ul style="list-style-type: none"> • interpret a scatter plot • select and perform operations Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Correct point circled. (160,000, 30,000,000)	1	1
2. Correct point boxed. (565,000, 500,000)	1	1
3. Makes a valid point, for example: There is no obvious correlation between area and population. or Most states are closely grouped in the bottom left hand corner of the graph but there are a few outliers. or All states apart from two have an area less than 200,000 square miles.	1 or 1 or 1	1
4. Draws an X in correct position: On horizontal axis $40,000 < X < 60,000$; on vertical axis $8,000,000 < X < 10,000,000$	1	1
5. Gives correct answer: 164 Accept 150 to 170.	1	1
6. Correct line drawn: straight line through origin with gradient 70.	1	1
7. Gives correct answer: 4,760,000	1	1
8. Gives correct answer: Areas will remain the same; and populations may have changed.	1	1
Total Points		8

Looking at Student Work – Population

Student A makes very clear statements about part 3 about the subject of the graph and trends in the data. Although the numbers are not shown, the points on the line show that the line for part 6 was not random but based on the specifics of density.

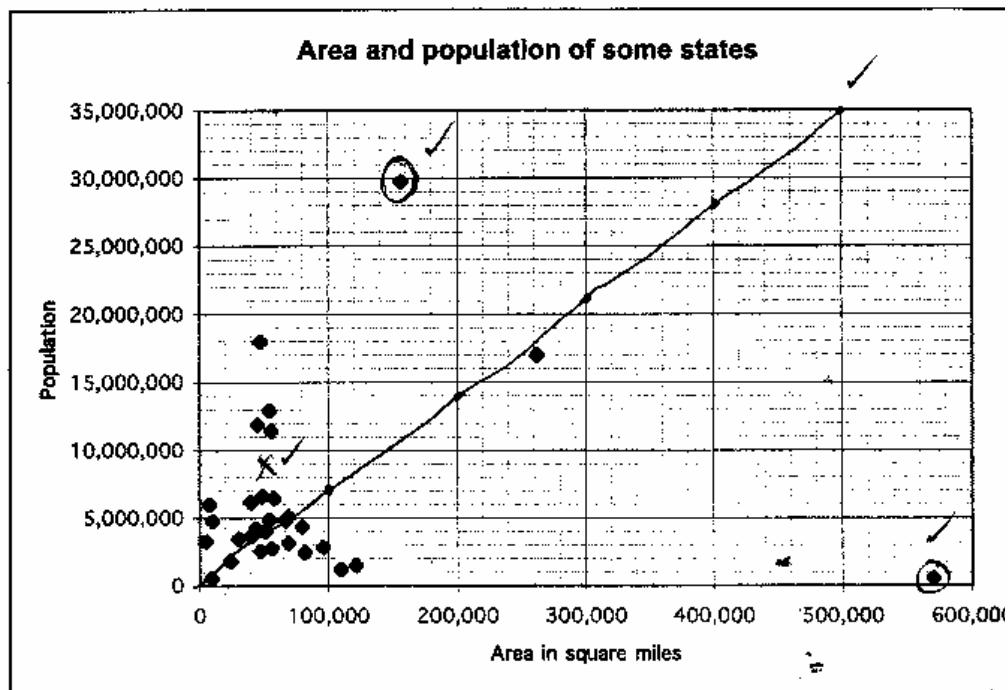
Student A

Population

This problem gives you the chance to:

- interpret a scatter plot
- select and perform operations

This graph shows the areas and the populations of some of the states of the USA.



1. Draw a circle around the point that represents the state with the largest population.
2. Draw a square around the point that represents the state with lowest number of people per square mile.
3. Describe the main features of the graph.

The graph shows the population per square mile of some states, and it looks like most states have about 5,000,000 people for every 60,000 - 100,000 miles.

Student A, part 2

4. Draw an X on the graph to show the data for Michigan, with an area of 56,802 square miles and a population of 9,295,277 people.
5. Calculate the number of people per square mile in Michigan, giving your answer to the nearest whole number.

164 people per square

6. The average number of people per square mile for the USA is 70, though no state actually has exactly this population density.

Draw a straight line on the graph to show all the possible positions of points showing 70 people per square mile.

7. The average state area is 68,000 square miles, though no state has exactly this area. If a state with the average area had the average population density, what would its population be?

$$68,000 \cdot 70$$

$$\begin{array}{r} 68000 \\ \times 70 \\ \hline 476000 \end{array}$$

$$4,760,000$$

8. The data in this graph is for the year 1990. Since then, what data will have stayed the same and what data may have changed?

The square mileage of the states will have stayed the same, but the population may have increased or decreased.

Student B shows confusion about how to draw a line show 70 people per square mile. There are two lines in the lower corner. One shows a vertical trend in the data, but not connected with numerical values. The second line shows a decreasing trend in the data, not connected with numerical values. Student B does show calculations for #5 and #6.

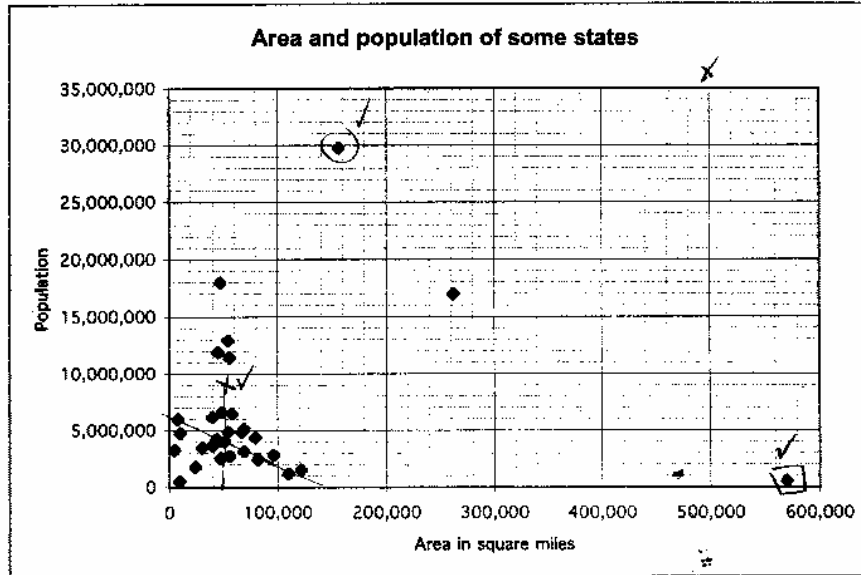
Student B

Population

This problem gives you the chance to:

- interpret a scatter plot
- select and perform operations

This graph shows the areas and the populations of some of the states of the USA.



1. Draw a circle around the point that represents the state with the largest population. /
2. Draw a square around the point that represents the state with lowest number of people per square mile. /
3. Describe the main features of the graph.

This graph shows that most states had less than 5,000,000 people for every 100,000 square miles. /

4. Draw an X on the graph to show the data for Michigan, with an area of 56,802 square miles and a population of 9,295,277 people. /

5. Calculate the number of people per square mile in Michigan, giving your answer to the nearest whole number.

$$\begin{array}{r} 163 \\ 57000 \overline{) 9,295,000} \end{array}$$

163 people X C

6. The average number of people per square mile for the USA is 70, though no state actually has exactly this population density.

Draw a straight line on the graph to show all the possible positions of points showing 70 people per square mile. O

7. The average state area is 68,000 square miles, though no state has exactly this area. If a state with the average area had the average population density, what would its population be?

$$68,000 \times 70 =$$

$$4,760,000 \checkmark /$$

8. The data in this graph is for the year 1990. Since then, what data will have stayed the same and what data may have changed? -

The area would have stayed the same because there was no new land added. The population would have changed to show that there were more people per square mile. ✓ /

Many students had trouble discussing the trends in the data or thinking about the meaning of the data. Student C just describes the content or topic of the graph in part c. In part 8 the student is clearly not thinking about the meaning of the graph. Student C also makes a common error in drawing the density line. The student merely draws a horizontal line connecting the lowest populations, regardless of area. The student misplaces the decimal in part 7.

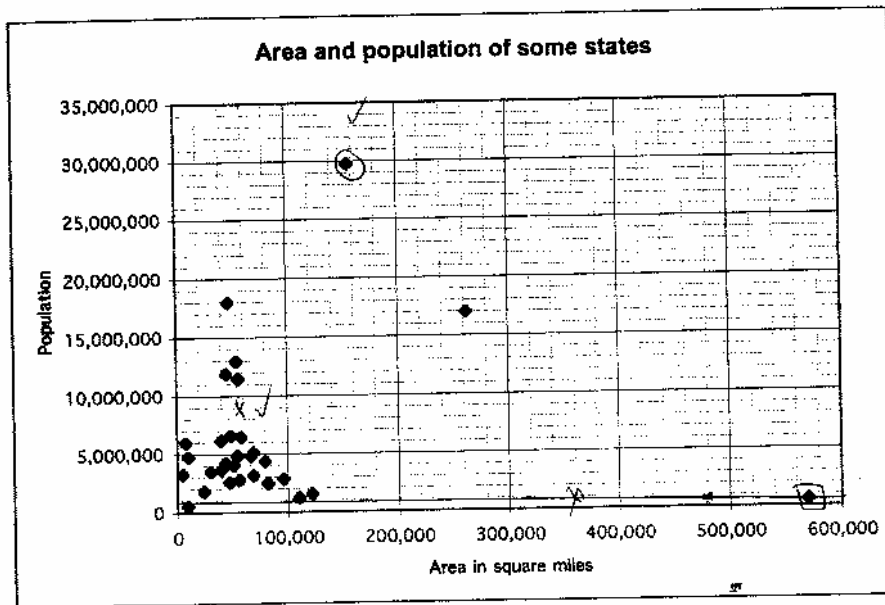
Student C

Population

This problem gives you the chance to:

- interpret a scatter plot
- select and perform operations

This graph shows the areas and the populations of some of the states of the USA.



1. Draw a circle around the point that represents the state with the largest population. (
2. Draw a square around the point that represents the state with lowest number of people per square mile. |
3. Describe the main features of the graph.

the main features are the population & square miles * 0

4. Draw an X on the graph to show the data for Michigan, with an area of 56,802 square miles and a population of 9,295,277 people.

5. Calculate the number of people per square mile in Michigan, giving your answer to the nearest whole number.

61 or 16 0

6. The average number of people per square mile for the USA is 70, though no state actually has exactly this population density.

Draw a straight line on the graph to show all the possible positions of points showing 70 people per square mile.

0

7. The average state area is 68,000 square miles, though no state has exactly this area. If a state with the average area had the average population density, what would its population be?

476,000 0

8. The data in this graph is for the year 1990. Since then, what data will have stayed the same and what data may have changed?

The data would have increasingly changed. 0

Many students did not understand the idea of density. They did not know how to predict points for 70 people per square miles. About 1/3 of the students who missed part 5 drew a horizontal line like Student D. About 1/3 of the students did not attempt to draw a line. About 1/12 of the students drew a vertical line like Student E. 90% of the students missed finding the lowest density in part 2 marked their papers like Student D.

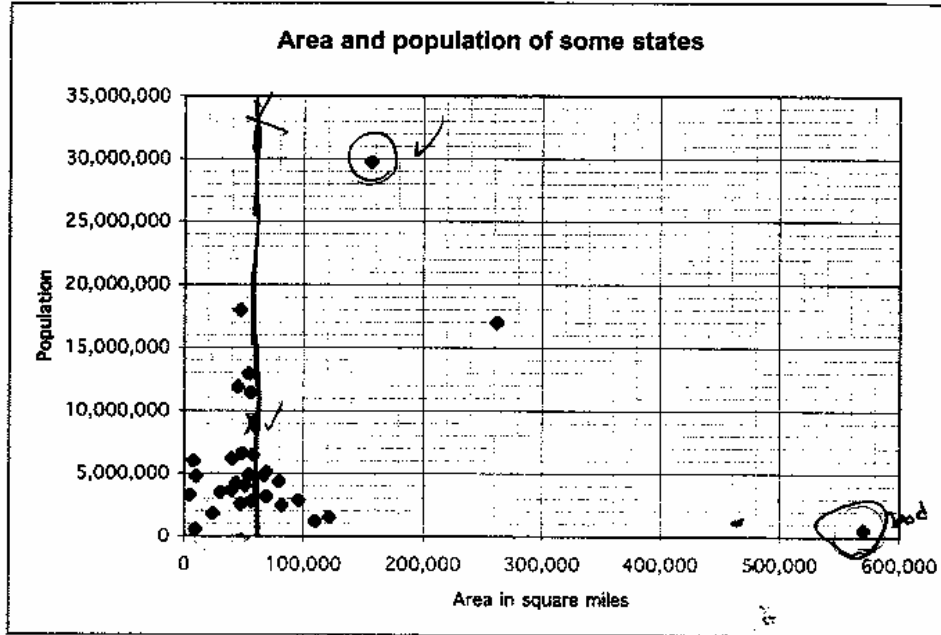
Student E

Population

This problem gives you the chance to:

- interpret a scatter plot
- select and perform operations

This graph shows the areas and the populations of some of the states of the USA.



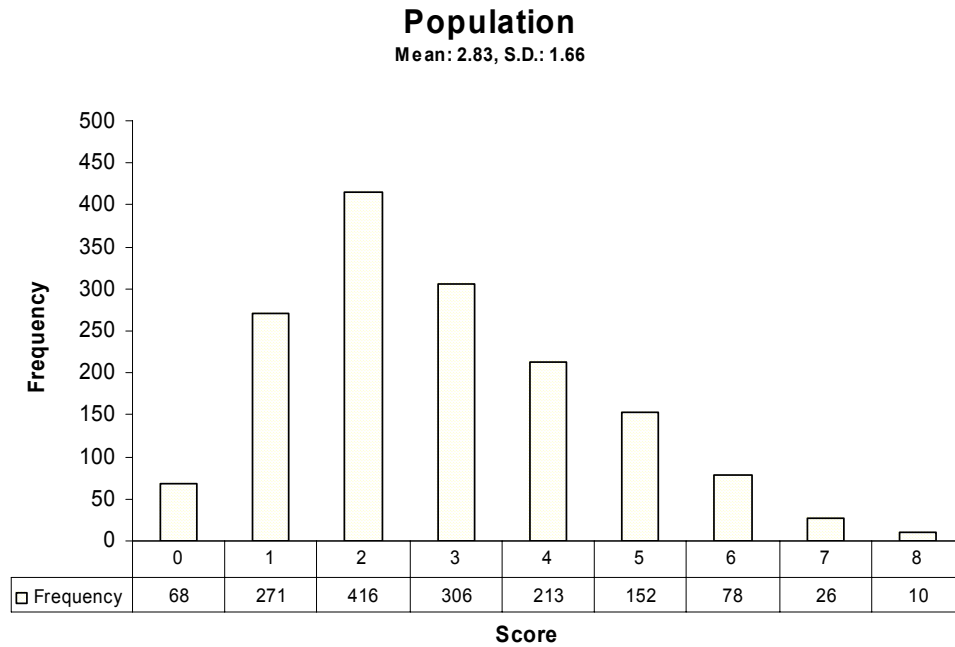
1. Draw a circle around the point that represents the state with the largest population.
2. Draw a square around the point that represents the state with lowest number of people per square mile.
3. Describe the main features of the graph.

Population, Area, Area,

XO

Teacher Notes:

Frequency Distribution for Population



Score:	0	1	2	3	4	5	6	7	8
% <=	4.4%	22.0%	49.0%	68.9%	82.7%	92.6%	97.7%	99.4%	100.0%
% >=	100.0%	95.6%	78.0%	51.0%	31.1%	17.3%	7.4%	2.3%	0.6%

The maximum score available for this task is 8 points.

The cut score for a level 3 response, meeting standards, is 4 points.

Most students (96%) could identify the point on the scatterplot representing the largest population. About half the students could find the largest population, describe the trend in the data, and explain which variable, population or area would change and which would stay the same. About 30% of the students could find largest population, describe data trends, discuss which variables would change, and calculate the population density for Michigan. Less than 3% of the students could meet most of the demands of the tasks. Drawing a line to represent a density of 70 people per square mile and identifying trends in the data caused students at the top end the most difficulty. About 4% of the students scored no points on the task. About half of them attempted the problem.

Population

Points	Understandings	Misunderstandings
0	About half the students with this score attempted the problem.	Some students confused largest area with largest population.
2	Students with this score could identify the largest population and could either plot a point for Michigan or explain which data with change and which would remain the same over time.	Students had trouble with the idea of density (people per square mile). For lowest density they picked the lowest total population.
4	Students with this score could identify the largest population, discuss which data changed, plot a point on the graph and usually calculate the number of people per square mile in Michigan.	Students had difficulty calculating the density for Michigan. Wrong answers ranged from 5 or 6 to 400,000,000. Most students did not attempt this section. Of those that tried, 37% picked numbers under 100. 25% picked numbers over 1,000,000.
6	Students could identify and plot points on the graph those representing density, students could calculate the population for a state with an average area.	Students had difficulty finding the population for an average size state. Most students, who missed this, did not attempt the calculation. Of those who tried, the most common error was 4 or 5 million. The next most common error was 476,000.
8	Students could identify and plot points, describe the trend in the data, draw a line to represent average density, calculate density, and work backwards from density to population. Students could also recognize which data would change over time and which would stay the same.	Students had the most difficulty drawing a line to represent average density of 70. Most students drew a line horizontally across the bottom of the graph connecting the two lowest populations or drew a vertical line at about 70,000 square miles. Students also had difficulty describing the data. Students made statements about the labels and title of the graph rather than trends in the data.

Questions for Reflection on Population

- When looking at graphs, do students understand that graphs are made to provide information? Do they look for ways to describe the shape and meaning of the data?
- Students had difficulty calculating the number of people per square mile. What types of calculation errors did students make? Were they picking inappropriate operations, misplacing decimal points, rounding incorrectly? What made this question difficult for students to interpret?
- When approaching graphing, do students have the opportunity to discuss the slope of a line in context? Do they have an opportunity to make a table of values to graph a line with a given slope? What do you think contributed to their difficulties graphing with a density of 70? What big ideas don't they understand?
- Look at your student papers. How many of your students left more than one section of the task?

Did not attempt the task	Omitted part 4	Omitted part 5	Omitted part 6	Omitted part 7	Omitted part 8

- Why do you think students were unwilling or unable to tackle these parts of the task? How do you develop the habit of mind for persistence in solving math problems?
- Look at your student papers. How many of your students showed their calculations or thinking for solving part 5, 6, and 7? Do you value students showing their work? How do you communicate this value to students?

Teacher Notes:

Implications for Instruction:

Students need to think about using graphs in context as a tool for making sense of or describing the world, for communicating information to an audience, and for making predictions and decisions. By this grade level, students should be frequently moved from the level of locating and identifying information on a graph, to finding relationships in the data, and interpreting those relationships to draw inferences. At this grade level, the expectation should be for students to synthesize information from the graphs and be able to describe trends or highlight important ideas gained from viewing data. Students at this grade level should also be comfortable with more complex data sets, which show greater spread and variation or which deal with scaled intervals as well as scaled frequencies.

Course One**Task 3****From 2 to 3 Dimensions**

Student Task	Reason about a net and how it would fold into a 3-dimensional prism. Find the number of faces, edges, and vertices. Calculate perimeter and area of net and volume of prism. Understand how features in the net relate to features in the 3-dimensional object, deciding which will remain and which will combine when folded.
Core Idea 4 Geometry and Measurement	Describe spatial relationships; develop mathematical arguments about geometric relationships; understand measurable attributes of objects; and understand the units, systems, and process of measurement.

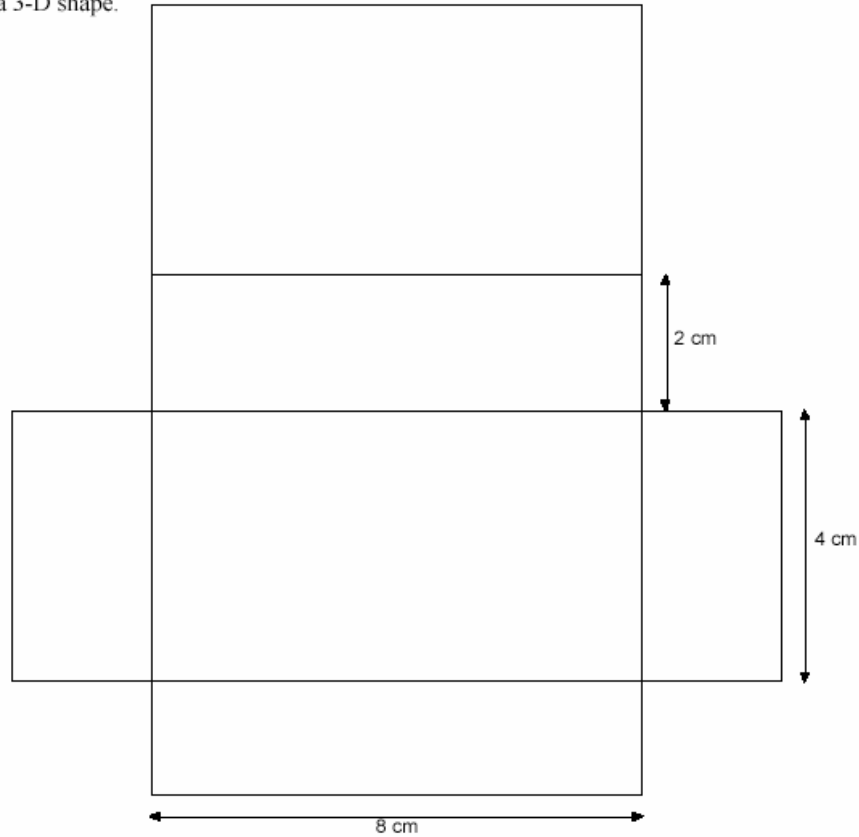
From 2 to 3 Dimensions

This problem gives you the chance to:

- imagine a 3-D shape, given its 2-D net
 - compare areas and lengths in the shape and its net
-

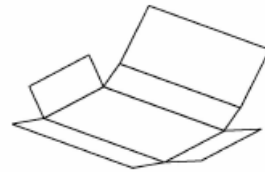
A net is a 2-D pattern that can be cut and folded to make a 3-D shape.

This is a net of a 3-D shape.



Imagine folding this net and making a 3-D shape.

1. How many faces would there be on this 3-D shape?



How many edges would there be on this 3-D shape?

How many corners would there be on this 3-D shape?

2. One student said there would be 14 corners on this 3-D shape.
The student is wrong! What mistake might this student have made?

3. Find the perimeter of the net in centimeters. _____

4. Find the area of the net in square centimeters. _____

5. The total area of this *net* is the same as the total surface area of the *3-D shape*.
Explain why this is true.

6. The total length of all the edges of this *net* is not the same as the total length of all the edges of
this *3-D shape*. Explain why this is true.

7. Find the volume of the 3-D shape in cubic centimeters. _____

8

From 2 to 3 Dimensions		Test 9 Rubric	
The core elements of performance required by this task are: <ul style="list-style-type: none"> • imagine a 3-D shape, given its 2-D net • compare areas and lengths in the shape and its net Based on these, credit for specific aspects of performance should be assigned as follows		points	section points
1. Gives three correct answers: 6 faces 12 edges 8 corners <i>Partial credit</i> Two correct answers.		2 (1)	 2
2. Gives correct explanation such as: The student may have counted every point on the net where lines/edges meet as a corner. Accept any explanation that correctly leads to 14.		1	1
3. Gives correct answer: 48 cm		1	1
4. Gives correct answer: 112 cm²		1	1
5. Gives correct explanation such as: The net becomes the surface area of the 3D shape.		1	1
6. Gives correct explanation such as: Sometimes two of the edges of the 2D shape come together to make a single edge of the 3D shape.		1	1
7. Gives correct answer: 64 cm³		1	1
Total Points			8

Looking at Student Work – From 2 to 3 Dimensions

Students had a very difficult time reasoning about a three-dimensional shape. No students in the sample papers scored a perfect paper. Student A is one of the few students who uses the model to help think about the situation and calculate answers. Student A forgets one side when calculating perimeter and has difficulty counting edges from the 3-dimensional sketch made at the bottom right of the net.

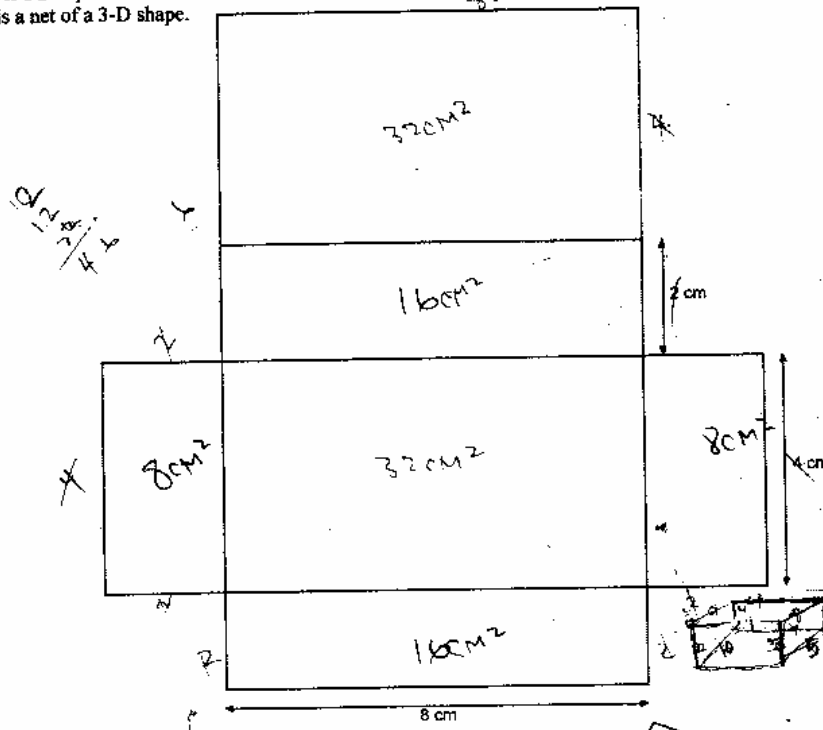
Student A

From 2 to 3 Dimensions

This problem gives you the chance to:

- imagine a 3-D shape, given its 2-D net
- compare areas and lengths in the shape and its net

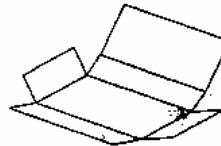
A net is a 2-D pattern that can be cut and folded to make a 3-D shape.
This is a net of a 3-D shape.



Imagine folding this net and making a 3-D shape.

1. How many faces would there be on this 3-D shape?

6 ✓



How many edges would there be on this 3-D shape?

10 ✗
8 ✓

How many corners would there be on this 3-D shape?

Student A, part 2

2. One student said there would be 14 corners on this 3-D shape.
The student is wrong! What mistake might this student have made?

THE STUDENT HAD COUNTED ALL THE CORNERS
IN THE DIAGRAM, THEY FORGOT THAT SOME CORNERS
JOIN TO PROMPTS WHEN IT'S IN A BOX FORM. ✓

3. Find the perimeter of the net in centimeters.

40 CM ✓

4. Find the area of the net in square centimeters.

112 CM² ✓

$$\begin{array}{r} 32 \cdot 2 \\ + 32 \\ \hline 112 \end{array}$$

5. The total area of this net is the same as the total surface area of the 3-D shape.
Explain why this is true.

THIS IS TRUE BECAUSE ~~THE~~ WHEN FOLDED THE NET
DOESN'T USE ANY MORE PAPER. ✓

6. The total length of all the edges of this net is not the same as the total length of all the edges of this 3-D shape. Explain why this is true.

THIS IS TRUE BECAUSE SOME EDGES COME TOGETHER IN
IT IS A RECTANGULAR PRISM AND THOSE EDGES ARE
COUNTED AS ONE. ✓

7. Find the volume of the 3-D shape in cubic centimeters.

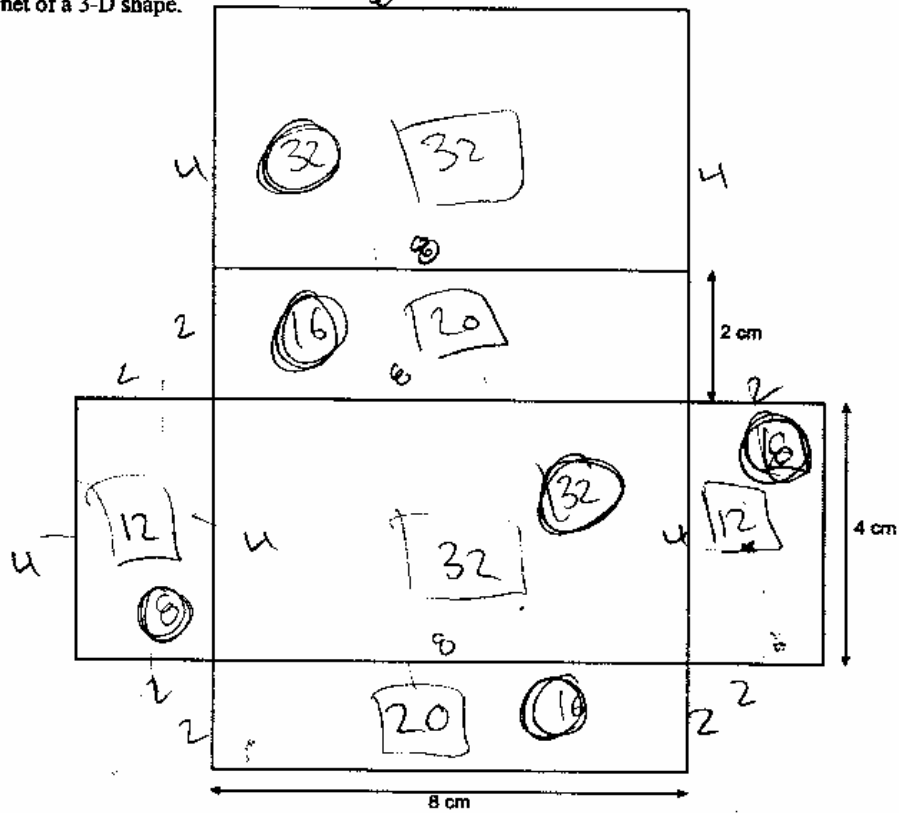
64 CM³ ✓

$$\begin{array}{l} L \cdot W \cdot H \\ 8 \cdot 4 \cdot 2 = 32 \cdot 2 = 64 \end{array}$$

Student B also uses the diagram to help solve the problems. In part 3 the student finds the perimeter of each rectangle in the net instead of thinking about the perimeter of the whole shape. Student B is also one of the few students to show all the calculations, which gives us insight into the types of misunderstandings of students. Student B shows a good understanding of how parts of the shape come together when the net is folded in part 2.

Student B

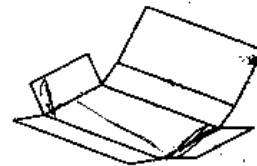
A net is a 2-D pattern that can be cut and folded to make a 3-D shape.
This is a net of a 3-D shape.



Imagine folding this net and making a 3-D shape.

1. How many faces would there be on this 3-D shape?

6 ✓



How many edges would there be on this 3-D shape?

15 X

How many corners would there be on this 3-D shape?

12 X

0

Student B, part 2

2. One student said there would be 14 corners on this 3-D shape.
The student is wrong! What mistake might this student have made?

When you put the two sides that come together to form the width of the box, they start as two angles and become one, maybe they counted it as 2.

3. Find the perimeter of the net in centimeters.

$$\begin{array}{r} 32 \\ 32 \\ \hline 64 \end{array} \quad \begin{array}{r} 40 \\ 24 \\ \hline 64 \end{array} \quad \begin{array}{r} 64 \\ 64 \\ \hline 128 \end{array}$$

128 cm. 0

4. Find the area of the net in square centimeters.

$$\begin{array}{r} 32 \\ 32 \\ \hline 64 \end{array} \quad \begin{array}{r} 16 \\ 16 \\ \hline 32 \end{array} \quad \begin{array}{r} 8 \\ 8 \\ \hline 16 \end{array} \quad \begin{array}{r} 48 \\ 64 \\ \hline 112 \end{array}$$

112 cm² ✓ 1

5. The total area of this net is the same as the total surface area of the 3-D shape.
Explain why this is true.

Because you are taking the same amount of paper and simply manipulating it, and cutting or gluing. ✓

6. The total length of all the edges of this net is not the same as the total length of all the edges of this 3-D shape. Explain why this is true.

Because when you fold it, 2 edges come together to make one, decreasing the actual number of them there is.

7. Find the volume of the 3-D shape in cubic centimeters.

$$\begin{array}{l} lwh \\ l=8 \end{array} \quad \begin{array}{l} w=4 \\ h=2 \end{array} \quad \begin{array}{l} (4)(2)=8 \\ (8)(8)=64 \end{array}$$

64 cm³ ✓ 1



2

Student C also uses a model to help solve the problem and show his thinking. In part 4 the student puts the side flaps at the bottom of the net to simplify finding the surface area. Note the 3-dimensional sketch used in part 7 to calculate volume. Student C doesn't have the words to help explain how the area stays the same or the edges change in part 5 and 6.

Student C

2. One student said there would be 14 corners on this 3-D shape.
The student is wrong! What mistake might this student have made?

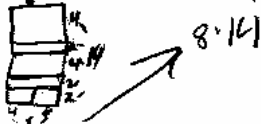
He/she counted when it was 2D ✓ 1

3. Find the perimeter of the net in centimeters.

$$\begin{aligned} 8 \cdot 2 &= 16 \\ 8 \cdot 2 &= 16 \\ 2 \cdot 8 &= 16 \end{aligned}$$

48 cm ✓ 1

4. Find the area of the net in square centimeters.



112 sq cm ✓ 1

5. The total area of this net is the same as the total surface area of the 3-D shape.
Explain why this is true.

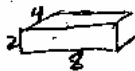
Because the net has the same surface ~~area~~ size as the 3-D shape ✓ 0

6. The total length of all the edges of this net is not the same as the total length of all the edges of this 3-D shape. Explain why this is true.

Because the bend in different places ✓ 0

7. Find the volume of the 3-D shape in cubic centimeters.

64 cb cm ✓ 1



Student D shows more typical work. The student fills in answers with no marks on the diagram or evidence of calculations used to solve part 3, 4, or 7. Student D is not quite complete or accurate enough in the descriptions for part 2 and 6.

Student D

2. One student said there would be 14 corners on this 3-D shape.
The student is wrong! What mistake might this student have made?

They might have accidentally recounted
the top X C

3. Find the perimeter of the net in centimeters.

32 cm X C

4. Find the area of the net in square centimeters.

64 cm² X C

5. The total area of this *net* is the same as the total surface area of the 3-D shape.
Explain why this is true.

Because it is still the same box, it has ✓
changed its shape but it is still the same

6. The total length of all the edges of this *net* is not the same as the total length of all the edges of this 3-D shape. Explain why this is true.

Because when it makes a 3-D shape the
edges touch so you are half of all the edges

7. Find the volume of the 3-D shape in cubic centimeters.

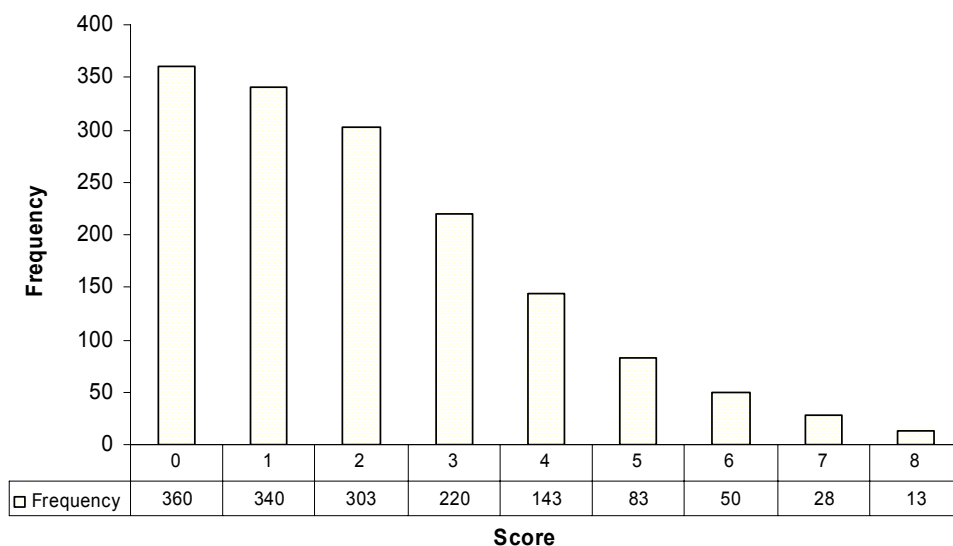
64 cm³ ✓

Teacher Notes:

Frequency Distribution for each Task – Grade 9
Grade 9– From 2 to 3 Dimensions

From 2 to 3 Dimensions

Mean: 2.07, S.D.: 1.85



Score:	0	1	2	3	4	5	6	7	8
% < =	23.4%	45.5%	65.1%	79.4%	88.7%	94.1%	97.3%	99.2%	100.0%
% > =	100.0%	76.6%	54.5%	34.9%	20.6%	11.3%	5.9%	2.7%	0.8%

The maximum score available for this task is 8 points.
The cut score for a level 3 response, meeting standards, is 4 points.

Many students (about 77%) could find the number of faces and the number of corners in the 3-dimensional shape. About half the students could also find the number of edges. About 21% of the students could find the number of faces and vertices, think about how the net folds and explain why a student might miscount to get 14 corners and why the surface area of the net and shape are the same, and find the volume. Less than 1% of the students could meet all the demands of the task. More than 23% of the students scored no points on this task. About 73% of those students attempted the task.

From 2 to 3 Dimensions

Points	Understandings	Misunderstandings
0	73% of the students with this score attempted the task.	Students could not think about how the shape folded together to make a prism. Most students could find the number of faces. Common errors for faces were 4, 12, and 5. 40% of the incorrect responses for corners was 4.
1	Students could imagine the net as a 3-dimensional shape and count the number of faces and corners.	Students had most difficulty counting edges. 32% of the incorrect responses were 8 edges, 25% were 4 edges, and 10% were 6 and 10% were 10.
2	Students could find the number of faces, edges, and corners in the 3-dimensional shape.	Students had difficulty explaining why a student would count 14 corners or why the area of the net was the same as the surface area. They often gave superficial answers like the student miscounted or restated the question in part 5.
4	Students could usually find faces and corners, think about how the net folds and explain why a student might miscount to get 14 corners and why the surface area of the net and shape are the same, and find the volume.	Most students who missed volume did not attempt that part of task. The range for volume was from 4 to $76,176 \text{ cm}^3$. The most common error was a volume of 16 cm^3 .
6	Students could find faces, edges, and corners. Reason about why a student would get 14 corners. They could either find the area of the net or explain why the area of the net was the same as the surface area.	Students had trouble calculating the perimeter of the net. Common errors included 24, 28, 32, 36, 44 and 56. Common errors for area of the net were 64, 16 and 32.
7		Students could not calculate the perimeter of the net.
8	Students could reason about how a 2-dimensional net folded into a rectangular prism and find faces, edges and corners. They could explain how to count corners and why the area of the net was the same as the surface area of the prism. They could use the dimensions of the net to calculate perimeter, area and volume.	

Questions for Reflection on From 2 to 3 Dimensions

- How many of your students could not find the number of edges for the 3-dimensional shapes? What kinds of misconceptions about edges would have led to the different errors that you saw?
- Does looking at the errors in explanations for 2, 5, and 6 give your insights into students understanding or misunderstanding about how the net folds together, show misunderstanding of geometrical terms, or inability to make convincing mathematical arguments. What opportunities have students had to explain why things happen or to reason about a situation?

Look at student work for perimeter. How many students:

Made no response	Put less than 24	24	28	32	35	44 or 46	50 or more

What might have led students to make some of these common errors? How did your students use diagrams to help them figure this out? Did your students show their calculations?

Look at student work for area. How many of your students:

112	16	32	64	No response	Other

What might have led students to make some of these common errors? How did your students use diagrams to help them figure this out? Did your students show their calculations?

- What types of activities or experiences help students develop the habit of mind of persistence in solving difficult problems?
- How do students develop the value of documenting their calculations and seeing a purpose for this?

Teacher Notes:

Implications for Instruction:

Students at this grade level have been working on area, perimeter, edges and faces since fourth and fifth grade. Students start working with 3-dimensional shapes around fifth grade and volume as filling a shape, these ideas deepen in middle school to include finding volume and surface area. At this grade level they should be comfortable and fluent with simple geometric terms and be able to start making convincing mathematical arguments. Frequent classroom discourse is an important part to learning strategies for making mathematical arguments and being able to judge the level of completeness necessary to make it convincing and to know if its will hold for all cases. While geometry is not the main focus of mathematics at this grade level, it provides a nice context for deepening students reasoning skills and developing their logical thinking skills.

Teacher Notes:

Student Task	Convert description of a function from a context to equation and graph. Match function descriptions and equations to their graphical representation.
Core Idea 1 Functions and Relations	Understand patterns, relations, and functions. <ul style="list-style-type: none">• Understand relations and functions and select, convert flexibly among, and use various representations for them.

Graphs

This problem gives you the chance to:

- relate equations, descriptions and graphs of a relationship
-

On the opposite page are four graphs.

Below are four descriptions and four equations.

Choose the description and the equation that fits each graph.

Write the correct description and the correct equation under each graph.

Descriptions:

x is the width and y is the length of a rectangle with a constant perimeter.

x is the radius and y is the area of a circle.

x is the length and y is the width of a rectangle with a constant area.

x is the radius and y is the circumference of a circle.

Equations:

$$y = \frac{k}{x}$$

$$y = kx$$

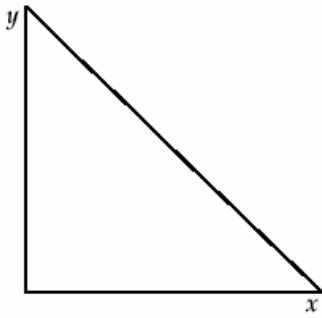
$$y = kx^2$$

$$y = k - x$$

In each equation, k is a fixed number.

1. Explain how you made your choices.

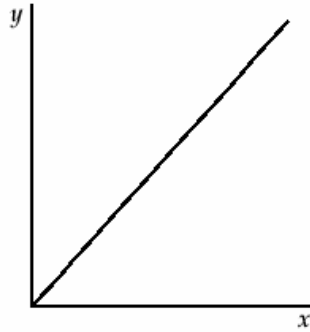
Graph A



Description _____

Equation _____

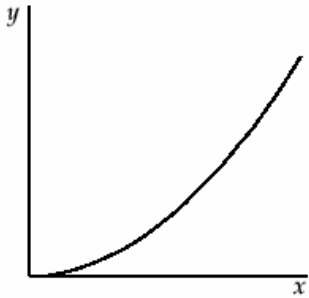
Graph B



Description _____

Equation _____

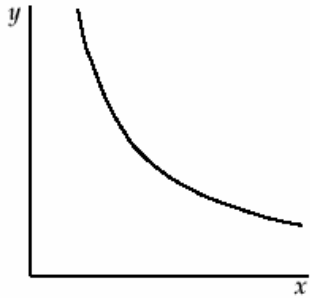
Graph C



Description _____

Equation _____

Graph D



Description _____

Equation _____

Graphs	Test 9 Rubric	
<p>The core elements of performance required by this task are:</p> <ul style="list-style-type: none"> relate equations, descriptions and graphs of a relationship <p>Based on these, credit for specific aspects of performance should be assigned as follows</p>	points	section points
<p>1. Gives correct explanations such as:</p> <p>Graphs A and B are straight lines.</p> <p>Graphs C and D are curves.</p> <p>Graphs B and C pass through the origin.</p> <p>Graph D: the axes are asymptotes.</p> <p>Any correct statement about the mathematics 1 point.</p>	1	max 1
<p>2. Correct allocation of descriptions and equations to graphs.</p> <p>Graph A x is the width and y is the length of a rectangle with a constant perimeter. $y = k - x$</p> <p>Graph B x is the radius and y is the circumference of a circle. $y = kx$</p> <p>Graph C x is the radius and y is the area of a circle. $y = kx^2$</p> <p>Graph D x is the length and y is the width of a rectangle with a constant area. $y = \frac{k}{x}$</p> <p>Max 4 points for correct descriptions. Max 3 points for correct equations.</p>	<p>1 1</p> <p>1 1</p> <p>1 1</p> <p>1 1</p>	<p>max 7</p>
Total Points		8

Looking at Student Work – Graphs

Out of 165 students in the sample, no student scored above a 5. No student showed evidence of trying to write out an equation to match the descriptions given in the problem. No student in the sample showed evidence of making a t-chart to put in values for x and y to match and help graph the equations given in the problem.

Student A is a typical 5 paper. The student could not articulate using mathematics to think about how to match descriptions or equations to their graphs. The student is able to correctly identify all 4 equations, but struggles with understanding the descriptions.

Student A

Descriptions:

x is the width and y is the length of a rectangle with a constant perimeter.

x is the radius and y is the area of a circle.

x is the length and y is the width of a rectangle with a constant area.

x is the radius and y is the circumference of a circle.

Equations:

$$y = \frac{k}{x}$$

$$y = kx$$

$$y = kx^2$$

$$y = k - x$$

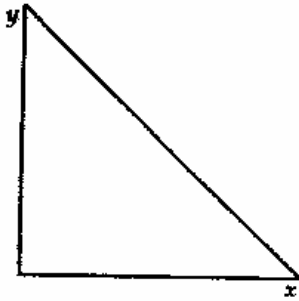
In each equation, k is a fixed number.

1. Explain how you made your choices.

I honestly didn't understand this
so I made educated guesses

Student A, part 2

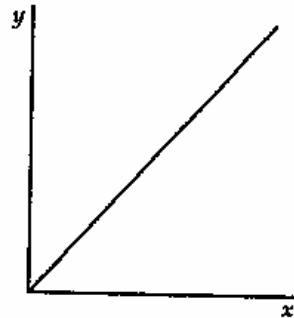
Graph A



Description x is the width and
y is the length of a rectangle
with a constant perimeter.

Equation $y = k - x$ ✓

Graph B



Description x is the radius
y is area of a circle

Equation $y = kx$ ✓

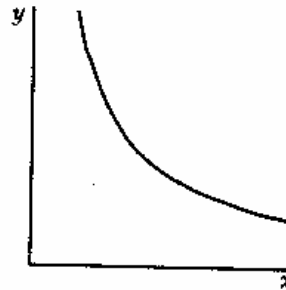
Graph C



Description x is the radius
and y is the circumference
of a circle

Equation $y = kx^2$ ✓

Graph D



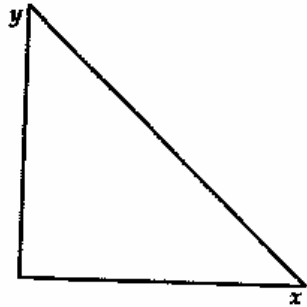
Description x is the length
and y is the width of a
rectangle with a constant area

Equation $y = \frac{k}{x}$ ✓

Student B also states, "I guessed. I've never seen this before." The student is able to correctly guess all the equations, but misses all the descriptors.

Student B

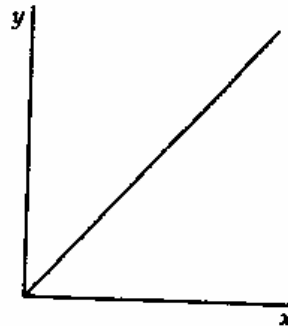
Graph A



Description x is the width & y is the length of a rec.

Equation ~~$y = kx^2$~~ ✓
 $y = k - x$

Graph B



Description x is the length y is the width

Equation $y = kx$ ✓

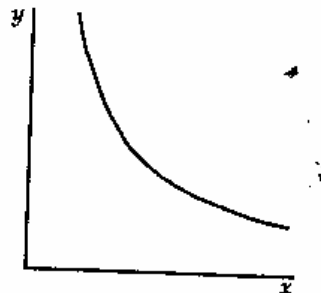
Graph C



Description x is radius y is circumference

Equation ~~$y = kx$~~ ✓
 $y = kx^2$

Graph D



Description y is radius x is circumference

Equation ~~$y = kx$~~ ✓
 $y = \frac{k}{x}$

Student C expresses the idea of thinking about whether the equations are going to be increasing or decreasing. In examining the error patterns, it seems many students were able to make this level of distinction between the equations and the graphs. However this only gave them a 50% chance of being right. It also does help students to think about the matching descriptors to the graph.

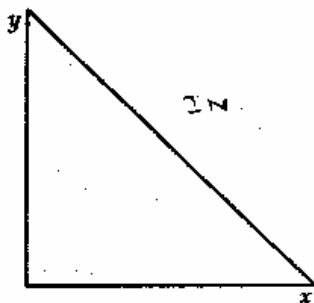
Student C

In each equation, k is a fixed number.

1. Explain how you made your choices.

FIRST I WROTE WHETHER IT WAS INVERSE OR DIRECT. THEN I THOUGHT OF TWO NUMBERS AND PUT IT IN THE EQUATION FOR THE DESCRIPTION I THOUGHT UP. I DESCRIBED THROUGH AND WHICH GRAPH IT WOULD APPLY TO BEST.

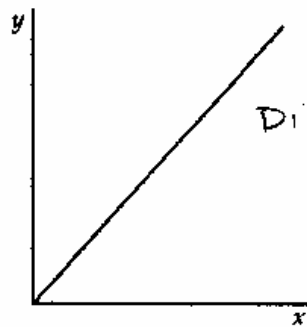
Graph A



Description x is the length and y is the width of a rectangle with a

Equation $y = \frac{k}{x}$ x

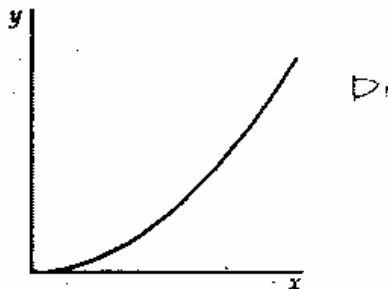
Graph B



Description x is the width y is the length of a rectangle w/ a constant perimeter.

Equation $x - y = k$ x

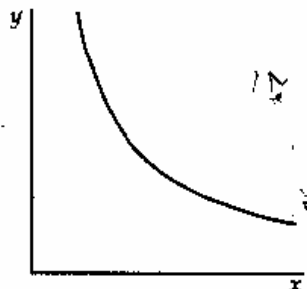
Graph C



Description x is the radius and y is the area of a circle ✓

Equation $kx^2 = y$ ✓

Graph D



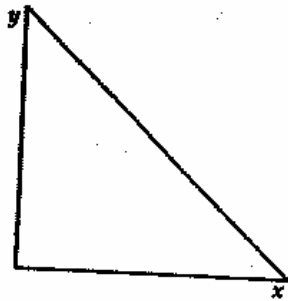
Description x is the radius and y is the circumference of a circle x

Equation $y = k - x$ x

Some students, like D and E, did not use the equations and/ or descriptions provided in the prompt. Student D tries to describe the shape of the graphs and make up equations. Student E also describes the shape of the graph.

Student D

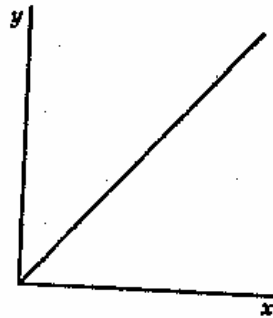
Graph A



Description curved horizontal
up and down

Equation $y = -x$

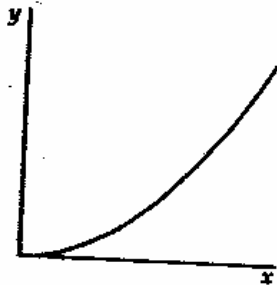
Graph B



Description Strait up to
the right.

Equation $y = x$

Graph C



Description curved down
left & right

Equation $x = y$

Graph D

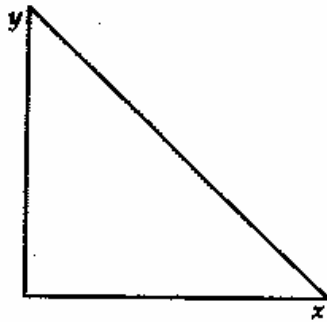


Description curved left
and right

Equation $y = x$

Student E

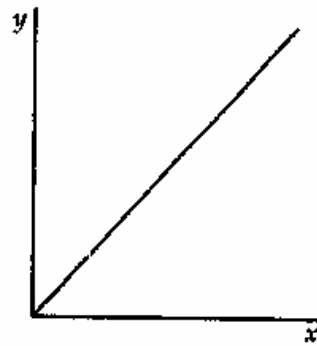
Graph A



Description It is a
triangle ✓

Equation $y = kx$ ✓

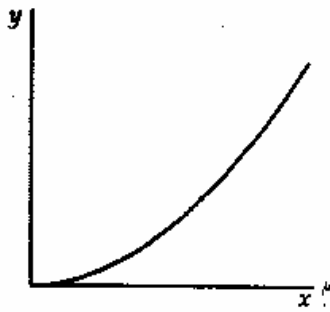
Graph B



Description the line
increase quickly ✓

Equation $y = \frac{k}{x}$ ✓

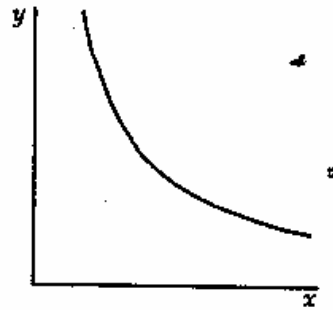
Graph C



Description the line increase
gradually ✓

Equation $y = kx^2$ ✓

Graph D



Description the line
decrease slowly ✓

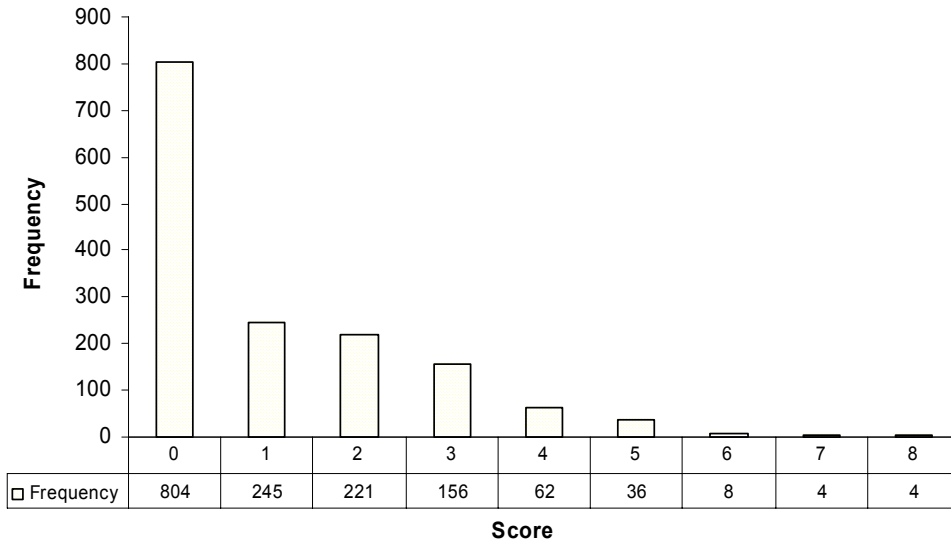
Equation $y = k - x$ ✓

Teacher Notes:

Frequency Distribution for each Task – Grade 9
Grade 9– Graphs

Graphs

Mean: 1.10, S.D.: 1.47



Score:	0	1	2	3	4	5	6	7	8
% <=	52.2%	68.1%	82.5%	92.6%	96.6%	99.0%	99.5%	99.7%	100.0%
% >=	100.0%	47.8%	31.9%	17.5%	7.4%	31.9%	1.0%	0.5%	0.3%

The maximum score available for this task is 8 points.
The cut score for a level 3 response, meeting standards, is 3 points.

About half the students could give a description for either graph A or C or give an equation for B or C. A little less than 20% of the students could correctly identify a couple of the descriptions and at least one equation. Less than 1% could match all the descriptions and equations. Over 52% of the students scored no points on this task. 32% of them attempted the task. 46% did not attempt this task, but were willing to attempt the task following graphs. 22% did not attempt this task or the final task of the exam.

Graphs

Points	Understandings	Misunderstandings
0	Only 32% of the students with this score attempted the task.	Students in the entire sample did not seem to have a strategy for connecting the equations or descriptions to the graphs. Students seemed to imply in part 1 that they guessed. There is an underlying belief that they should just know it without doing any work. Math answers are quick.
1	The easiest answers for students were the descriptions for A and C or the equation for C.	Students did not show evidence of trying to write equations for the descriptions or of making t-charts of values for x and y to help match equations to graphs.
3	Students with this score could generally match two descriptions and one equation. Their correct descriptions were not necessarily the same as their correct equation.	Some students seemed to know which equations would create graphs with positive or negative slopes, but then did not necessarily make the appropriate guess between the two choices.
5	Students could generally identify all of the correct equations and only two correct descriptions.	
6/7		No students in the sample had this score.
8	Less than 1% of the students could describe correctly how they matched descriptions and equations to graphs and then successfully match all the descriptions and equations to the 4 given graphs.	The equations and descriptions about circumference and constant area were most difficult for students to think about.

Questions for Reflection on Graphs

- How is graphing taught in your classroom?
- Do students have the opportunity to make t-charts for x and y values to help plot graphs?
- Do students have the opportunity to talk about how the equations for curves are different from the equations for straight lines?
- What type of student work would you have expected to see from students to help solve this problem or as a rationale for part 1? Why do you think students tried to guess instead of using mathematics to work out the correct solutions?
- Why do you think so few students were willing to even attempt the problem when all the answers were given?
- Why do you think students didn't see a connection between the graphs in the problem and the types of graphs in their textbooks?
- What types of problems or investigations have students done this year that might have developed the type of logic or strategies needed to solve this task? What other resources do you have that might have these types of problems or investigations?

Look at the papers for your best student. Was there anything significant between the types of responses they gave and those responses for other students in the classroom?

Teacher Notes:

Implications for Instruction:

Students need to be able to make connections between multiple representations of the same idea. They should have frequent opportunities to work with situations, turn those situations into equations, and make graphs from the equations. In reading the sample, students did not show evidence of trying to turn the words in the descriptions into equations. No student in the sample attempted to make a t-chart of values for any of the equations to help them match the equation to a graphic representation. Learning skills without being able to make connections is not helpful in transferring knowledge to any application outside the classroom. Students working in elementary algebra should be quite fluid with this level of making connections.

Teacher Notes:

Student Task	Extend a pattern, work a pattern backwards, and generate a sequence using a given pattern. Add and divide algebraic terms with two variables, solve simultaneous equations and use substitution to find missing expressions.
Core Idea 1 Functions and Relations	Understand patterns, relations, and functions.
Core Idea 3 Algebraic Properties and Representations	Represent and analyze mathematical situations and structures using algebraic symbols.

Fibonacci Sequences

This problem gives you the chance to:

- work with algebra to solve number problems
-

Here is a Fibonacci sequence.

2	5	7	12	19	31
---	---	---	----	----	----

To make a Fibonacci sequence you write two numbers. **2 and 5**

To get the next number you add your first two numbers. **$2 + 5 = 7$**

Then you continue in this way. **$5 + 7 = 12$** **$7 + 12 = 19$** **etc.**

1. Here are two different Fibonacci sequences.

In the empty boxes, write the two missing numbers in each of these sequences.

3	4	7	11		
---	---	---	----	--	--

5		11		28	45
---	--	----	--	----	----

2. Write the first six numbers of your own Fibonacci sequence.

--	--	--	--	--	--

3. Here are the first six terms of a Fibonacci sequence

x	y	$x + y$	$x + 2y$	$2x + 3y$	$3x + 5y$
-----	-----	---------	----------	-----------	-----------

Add these six terms and simplify your answer.

Complete the following sentence.

This shows that the sum of the first six terms is always times the fifth term.

4. Below are two numbers of a Fibonacci sequence.

		6		17	
--	--	---	--	----	--

We can write two equations linking the expressions on the previous page and the numbers in the boxes above.

These two equations are:

$$x + y = 6 \quad \text{and} \quad 2x + 3y = 17$$

By solving these two equations simultaneously, find the values of x and y .

Then fill in the empty boxes in the sequence above, and check that they fit the Fibonacci pattern.

7

Fibonacci Sequences		Test 9 Rubric	
The core elements of performance required by this task are: • work with algebra to solve number problems Based on these, credit for specific aspects of performance could be assigned as follows		points	section points
1.	Gives correct answers: 18 and 29 6 and 17	1 1	2
2.	Gives a correct sequence.	1	1
3.	Gives correct answer: $8x + 12y$ Gives correct answer: four	1 1	2
4.	Writes a correct sequence: 1, 5, ..., 11, ..., 28 Shows that $x = 1$ and $y = 5$ are solutions to the equation.	1 1	2
Total Points			7

Looking at Student Work on Fibonacci Sequences

Student A solved all the parts of the task. The student was able to combine terms successfully in part 3 of the task. However the student did not solve part 4 using simultaneous equations. Student A appears to have used a combination of guess and check and substitution.

Student A

Fibonacci Sequences

This problem gives you the chance to:

- work with algebra to solve number problems

Here is a Fibonacci sequence.

2	5	7	12	19	31
---	---	---	----	----	----

To make a Fibonacci sequence you write two numbers. **2 and 5**

To get the next number you add your first two numbers. **$2 + 5 = 7$**

Then you continue in this way. **$5 + 7 = 12$ $7 + 12 = 19$ etc.**

1. Here are two different Fibonacci sequences.

In the empty boxes, write the two missing numbers in each of these sequences.

3	4	7	11	18	29	✓
---	---	---	----	----	----	---

5	6	11	17	28	45	✓
---	---	----	----	----	----	---

2. Write the first six numbers of your own Fibonacci sequence.

1	1	2	3	5	8	✓
---	---	---	---	---	---	---

3. Here are the first six terms of a Fibonacci sequence

x	y	$x + y$	$x + 2y$	$2x + 3y$	$3x + 5y$
-----	-----	---------	----------	-----------	-----------

Add these six terms and simplify your answer.

$$x + y + x + y + x + 2y + 2x + 3y + 3x + 5y$$

$$8x + 12y \quad \checkmark$$

Complete the following sentence.

This shows that the sum of the first six terms is always times the fifth term.

4 ✓

4. Below are two numbers of a Fibonacci sequence.

1	5	6	11	17	28
---	---	---	----	----	----

We can write two equations linking the expressions on the previous page and the numbers in the boxes above.

These two equations are:

$$x + y = 6 \quad \text{and} \quad 2x + 3y = 17$$

By solving these two equations simultaneously, find the values of x and y .

Then fill in the empty boxes in the sequence above, and check that they fit the Fibonacci pattern.

$$3 + 3 = 6 \quad 2(3) + 3(3) = 17$$

$$6 = 6$$

$$6 + 9 \neq 17$$

$$4 + 2 = 6 \quad 2(4) + 3(2) = 17$$

$$8 + 6 \neq 17$$

$$7 + 5 = 6 \quad 2(1) + 3(5) = 17 \quad \checkmark$$

$$2 + 15 = 17$$

$$6 = 6$$

$$17 = 17$$

Student B can combine like terms in part 3. Student B does not use guess and test to solve for part 4, but again uses guess and check and substitution.

Student B

3. Here are the first six terms of a Fibonacci sequence

x	y	$x + y$	$x + 2y$	$2x + 3y$	$3x + 5y$
-----	-----	---------	----------	-----------	-----------

Add these six terms and simplify your answer.

$$x + y + (x + y) + (x + 2y) + (2x + 3y) + (3x + 5y) \\ = 8x + 12y \quad \checkmark$$

Complete the following sentence.

This shows that the sum of the first six terms is always times the fifth term.

Student B, part 2

4. Below are two numbers of a Fibonacci sequence.

1	5		6		17	29
---	---	--	---	--	----	----

We can write two equations linking the expressions on the previous page and the numbers in the boxes above.

These two equations are:

$$x + y = 6 \quad \text{and} \quad 2x + 3y = 17$$

$$x = 1 \quad y = 5$$

By solving these two equations simultaneously, find the values of x and y .

Then fill in the empty boxes in the sequence above, and check that they fit the Fibonacci pattern.

$$x + y = 6$$

$$x + 0 = 6$$

$$x =$$

$$2 + 4 = 6$$

$$\begin{array}{r} 17 \\ - 6 \\ \hline 11 \end{array}$$

$$3 + 3 = 6$$

$$4 + 6 =$$

$$3 + 6 =$$

$$2 + 4 = 6 \quad 10$$

$$1 + 5 = 6 = 11 \quad \checkmark$$

$$x + y = 6$$

$$1 + 5 = 6$$

$$2x + 3y = 17 \quad \checkmark$$

$$2(1) + 3(5) = 17$$

$$2 + 15 = 17$$

Student C gives shows another example of using substitution to find the missing numbers in the sequence. Student C

4. Below are two numbers of a Fibonacci sequence.

	5	6	11	17	28
--	---	---	----	----	----

We can write two equations linking the expressions on the previous page and the numbers in the boxes above.

These two equations are:

$$x + y = 6 \quad \text{and} \quad 2x + 3y = 17$$

By solving these two equations simultaneously, find the values of x and y .

Then fill in the empty boxes in the sequence above, and check that they fit the Fibonacci pattern.

$$\begin{array}{r}
 x + y = 6 \\
 \downarrow \quad \downarrow \\
 1 + 5 = 6
 \end{array}
 \quad
 \begin{array}{r}
 2x + 3y = 17 \\
 2(1) + 3(5) \\
 2 + 15 = 17 \\
 \checkmark
 \end{array}$$

Some students do not know how to combine like terms. Student C gets exponents when adding like terms in part 3. Student D gets exponents when adding the equations in part 4.

Student C

... of a Fibonacci sequence

x	y	$x + y$	$x + 2y$	$2x + 3y$	$3x + 5y$
-----	-----	---------	----------	-----------	-----------

Add these six terms and simplify your answer.

$$\begin{array}{l}
 x^2 + 2y^2 \\
 2x^2 + 5y^2 \\
 (3x^2 + 10y^2) - 5x^2(2) - 2x
 \end{array}$$

Complete the following sentence.

This shows that the sum of the first ...

Student D

Then fill in the empty boxes in the sequence above, and check that they fit the Fibonacci pattern.

$$x + y = 6 \quad 2x + 3y = 17$$

$$2x^2 + 3y^2 = 23$$

Student E does not seem to understand the pattern on the first page. The student tries increasing the difference by 1 in the first sequence (starting with +3, +4, +5, +6). In the sequence in part 2 the student is just adding on by 2's. The student tries to use the numbers from the incorrect first sequence to estimate the times 5 in the 5th term. In part 4, Student E is one of the few students who uses simultaneous equations to solve x and y. However, because the student does not see the pattern he can't use it to fill in the sequence.

Student E

1. Here are two different Fibonacci sequences.

In the empty boxes, write the two missing numbers in each of these sequences.

3	4	7	11	16	22
---	---	---	----	----	----

5		11		28	45
---	--	----	--	----	----

2. Write the first six numbers of your own Fibonacci sequence.

2	4	6	8	10	12
---	---	---	---	----	----

3. Here are the first six terms of a Fibonacci sequence

x	y	x+y	x+2y	2x+3y	3x+5y
---	---	-----	------	-------	-------

Add these six terms and simplify your answer.

Complete the following sentence.

This shows that the sum of the first six terms is always 5 times the fifth term.

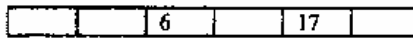
Handwritten calculations on the right side of the page:

$$\begin{array}{r} 2 \\ 22 \\ 16 \\ 11 \\ 7 \\ 4 \\ 2 \\ + 3 \\ \hline 63 \end{array}$$

$$\begin{array}{r} 3 \\ 16 \\ 11 \\ 7 \\ 4 \\ 2 \\ + 3 \\ \hline 63 \end{array}$$

$$\begin{array}{r} 30 \\ 16 \\ 7 \\ + 4 \\ \hline 64 \end{array}$$

4. Below are two numbers of a Fibonacci sequence.



We can write two equations linking the expressions on the previous page and the numbers in the boxes above.

These two equations are:

$$x + y = 6 \quad \text{and} \quad 2x + 3y = 17$$

By solving these two equations simultaneously, find the values of x and y .

Then fill in the empty boxes in the sequence above, and check that they fit the Fibonacci pattern.

$$\begin{array}{l}
 y = -x + 6 \quad 2x + 3(-x + 6) = 17 \\
 x = -y + 6 \quad 2x + 3x + 18 = 17 \\
 \qquad \qquad \qquad -18 \quad \checkmark \\
 \hline
 -x = -1 \\
 x = 1 \\
 2(1) + 3y = 17 \\
 -2 \quad -2 \\
 \hline
 3y = 15 \\
 y = 5
 \end{array}$$

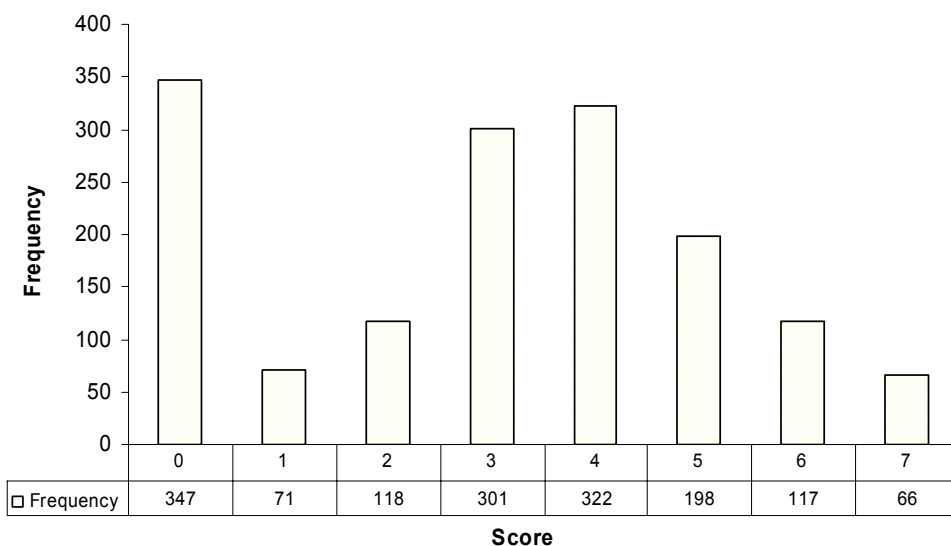
$S + 1 = 6$
 $(x + y) = 6$

Teacher Notes:

Frequency Distribution for each Task – Grade 9
Grade 9– Fibonacci Sequences

Fibonacci Sequences

Mean: 3.02, S.D.: 2.09



Score:	0	1	2	3	4	5	6	7
% ≤	22.5%	27.1%	34.8%	54.4%	75.3%	88.1%	95.7%	100.0%
% ≥	100.0%	77.5%	72.9%	65.2%	45.6%	24.7%	11.9%	65.2%

The maximum score available for this task is 7 points.

The cut score for a level 3 response, meeting standards, is 3 points.

Many students (about 78%) could solve the 2nd sequence in part one or combine like terms in part 3. Many students (about 73%) could fill in both sequences in part 1. More than half the students (65%) could fill in the sequence in part 1 and part 2 or in part 1 and part 4. A little less than half the students (45%) could fill in all the sequences in part 1, 2 and 4. About 6.5% of the students could meet all the demands of the task, including using guess and check, substitution, or solving simultaneous equations to find a value for x and y in part 4. Almost 23% of the students scored no points on this task. 41% of the students with this score attempted the task.

Fibonacci Sequences

Points	Understandings	Misunderstandings
0	About 40% of the students with this score attempted the problem.	Students did not use the rule to extend the pattern. They may have seen a pattern of increasing differences or a constant difference of 5 or 6. When making their own sequence they may have just counted by 2's.
1	Students with this score could generally work the pattern backward to fill in the 2 nd sequence in part 1 or they could combine like terms in part 3.	Some students made exponents when adding x's or y's together in part 3. Others did not add all the terms in the sequence, maybe stopping after the 5 th term.
2	Students use the pattern to fill in the sequences in part 1.	Students did not use this pattern when making their own sequences.
3	Students could use the pattern to fill in sequences and either make their own sequence or fill in sequence 4.	Many students did not attempt sequence 4 or did not try part 3 of the task.
4	Students could use the pattern to fill in sequences in part 1 and 4, and write their own sequence.	Students could not use algebraic symbols to combine like terms or to solve simultaneous equations.
6	Students could fill in sequences, make their own sequence, combine like terms, and divide one algebraic expression into another.	Students used some form of guess and check and mental math to fill in the sequence in 4. They showed no evidence or incorrect evidence of solving the equations provided.
7	Students with this score could fill in sequences, combine like terms, divide with algebraic expressions, and use guess and check, substitution, or solving simultaneous equations to find a value for x and y in part 4.	Most students with this score used guess and check or substitution to solve for x and y in part 4. There was little evidence of understanding of how to solve simultaneous equations.

Questions for Reflection on Fibonacci Sequences

- How many of your students could use the rules provided to fill in the Fibonacci sequences in part 1?
- Do your students get frequent opportunities to look at sequences or patterns and try to find rules about how they work?
- Could students combine like terms in part 3? How many of them had solutions with exponents? How many were unwilling to attempt this part of the task?
- What evidence did you see of practical application of algebra skills in part 4? How many of your students in part 4:

Solved the sequence with no work	Used guess and check	Used substitution	Solved for x and y using simultaneous equations	Made severe algebraic errors	Did not attempt the task or part 4 of the task

- Were you surprised that students didn't attempt to use algebra to solve for x and y? What experiences have students had solving problems with simultaneous equations in context? What activities help students to see the efficiency and convenience of using algebra over guess and check?

Teacher Notes:

Implications for Instruction:

Students at this grade level should be comfortable solving simple equations with two unknowns. They should have some basic sense-making around the equations that allows them to use strategies like guess and check. But, at this point they should start to be able to use other strategies for solving two equations with two unknowns. They should begin to demonstrate transference from algebraic skills and symbol manipulation into problem-solving tools. Students should also develop a norm around solving equations of substituting values back into the original equations to see if the values are correct or meet the constraints of the problem.

Teacher Notes:
