

2nd grade

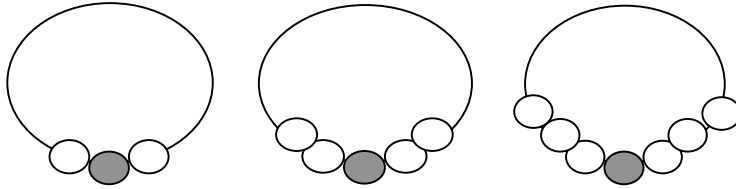
Task 4

Growing Necklaces

Student Task	Find and describe the pattern generated when “growing” a necklace of beads. Determine why a given number of beads would not fit with this necklace pattern.
Core Idea 3 Patterns, Functions, and Algebra	Understand patterns and use mathematical models to represent and to understand qualitative and quantitative relationships. <ul style="list-style-type: none">• Describe and extend growing patterns and translate from one representation to another• Communicate reasoning using numbers, pictures and/or words• Communicate mathematical thinking clearly and coherently

Growing Necklaces

Every year Matthew makes a necklace for his sister, Janci. Here are drawings of the necklaces.



Year 1

Year 2

Year 3

Year 4

Beads	3 beads	5 beads	7 beads	
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1. Draw a picture of the necklace for Janci's 4th year next to the 3rd necklace.
2. How many beads would Matthew need to make a necklace for Janci's 6th year?

_____beads in all

Show how you figured it out.

3. Would Matthew ever make a necklace like this for Janci with 12 beads?

Yes or No _____

Why or why not?

Growing Necklaces		Grade 2		Rubric	
The core elements of the performance required by this task are: <ul style="list-style-type: none"> • Describe and extend growing patterns and translate from one representation to another • Communicate reasoning using numbers, pictures and/or words 					
Based on these credit for specific aspects of performance should be assigned as follows				points	section points
1	Draws the 4 th necklace with 9 beads			2	2
2	Gives correct answer as: 13 Explains thinking such as: $6 + 1 + 6 = 13$, presents a series of numbers 3, 5, 7, 9, 11, 13 or explains their thinking in words			2 1	3
3	Gives correct answer as: No Explains how they got the answer of No such as: There are always an odd number of beads, Matt made one with 11 and 13 but not 12. Allow answer such as: it is not the same pattern			1 2 (1)	3
Total Points					8

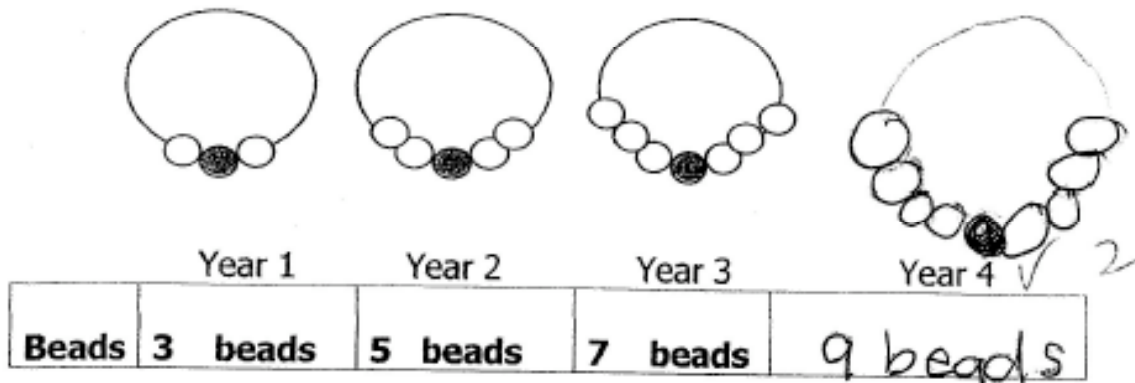
Looking At Student Work – Growing Necklaces:

Many students did well on this task. Student A continued the bead pattern to the next year's necklace and was able to predict the number of beads for the 6th year. She used a justification of $6 + 6 + 1$ for the number of beads in the 6th year. She argued that a necklace of 12 would not work for this pattern because there would be twelve beads plus the one in the middle.

Student A

Growing Necklaces

Every year Matthew makes a necklace for his sister, Janci. Here are drawings of the necklaces.



1. Draw a picture of the necklace for Janci's 4th year next to the 3rd necklace.

2. How many beads would Matthew need to make a necklace for Janci's 6th year?

13 beads in all

Show how you figured it out.

✓ EQ

$$\begin{array}{cccc}
 1 & 2 & 3 & 4 \\
 \circ & \circ & \circ & \circ \\
 \circ & \circ & \circ & \circ
 \end{array}$$

$$6 + 6 = 12 + 1 = 13$$

3. Would Matthew ever make a necklace like this for Janci with 12 beads?

Yes or No no ✓

Why or why not? ✓

Beacuse the neckoker has
twelv beads and one in the
midel.

Students B and C spoke to the number pattern of odd numbers. Student B showed that pattern by drawing and by identifying the total number of beads each year.

Student B

3. Would Matthew ever make a necklace like this for Janci with 12 beads?

Yes or No no ✓

Why or why not?

I said no because there's
always an odd num ber of
beads and 12 is an even num ber

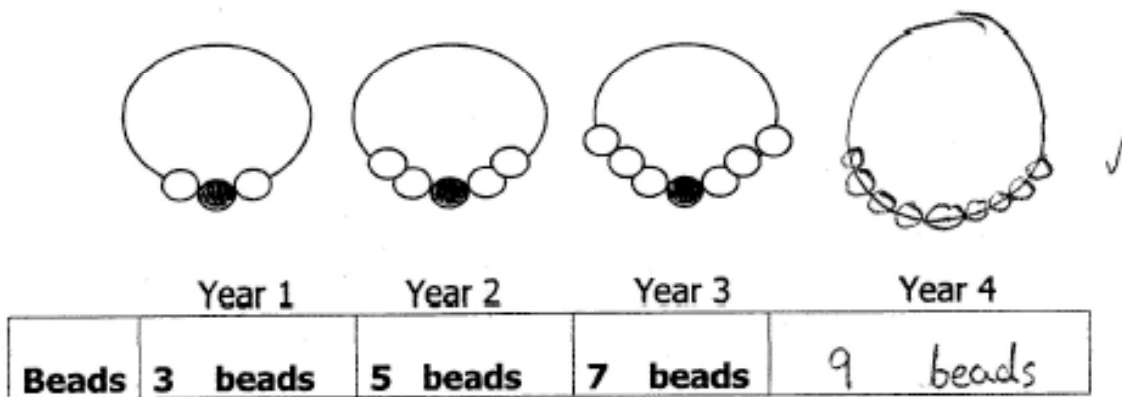


Student C successfully continued the bead necklace pattern by drawing all the years through the 6th year. He understands that 2 beads are added each year as well as realizing that the number of beads is always an odd number. Student C's proof also includes a drawing of why 12 won't work.

Student C

Growing Necklaces

Every year Matthew makes a necklace for his sister, Janci. Here are drawings of the necklaces.



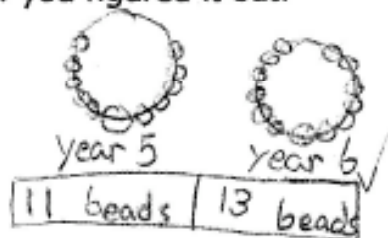
1. Draw a picture of the necklace for Janci's 4th year next to the 3rd necklace.

each year add two more beads.

2. How many beads would Matthew need to make a necklace for Janci's 6th year?

13 ✓ beads in all

Show how you figured it out.



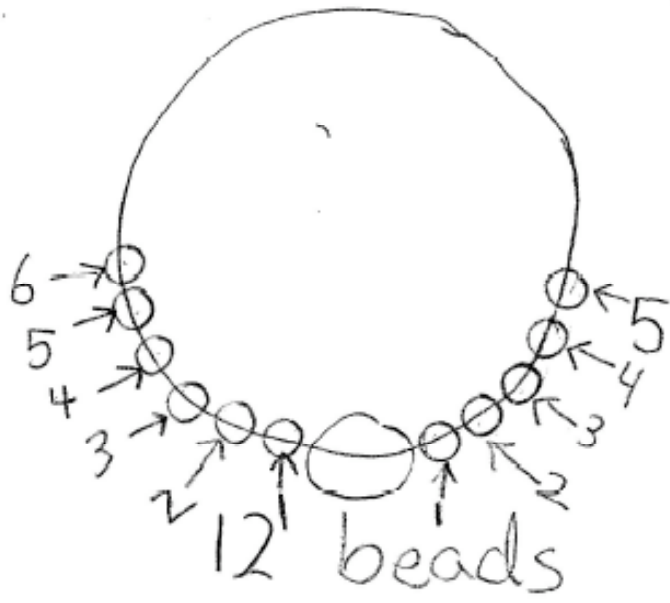
3. Would Matthew ever make a necklace like this for Janci with 12 beads?

Yes or No No ✓



Why or why not?

Because the sides aren't
even they will be odd.
One side has 6 other side has 5.



Students D, E, and F justify their ideas about the growth pattern by addressing the addition of two beads each year but starting with an odd number of beads. Student D knows that if you are starting at 3 beads, as year one does, and adding two beads each year, then 12 beads will be skipped.

Student D



	Year 1	Year 2	Year 3	Year 4
Beads	3 beads	5 beads	7 beads	9 beads ✓

2. How many beads would Matthew need to make a necklace for Janci's 6th year?

13 beads in all ✓

Show how you figured it out.

Year 4 + 2 beads = 11 beads
 Year 5 + 2 beads = 13 beads
 Year 6 = 13 beads

3. Would Matthew ever make a necklace like this for Janci with 12 beads?

Yes or No NO ✓

Why or why not?

I think no because it's counting 2 by 2's starting from 3 and it skipps 12.

Student E adds two beads each year from three beads and knows that the number used will always be odd.

Student E

2. How many beads would Matthew need to make a necklace for Janci's 6th year?

13 beads in all ✓

Show how you figured it out.

1st year
3 beads

2nd year
5 beads

3rd year ✓
7 beads

4th year
9 beads

5th year
11 beads

6th year
13 beads

3. Would Matthew ever make a necklace like this for Janci with 12 beads?

Yes or No No! ✓

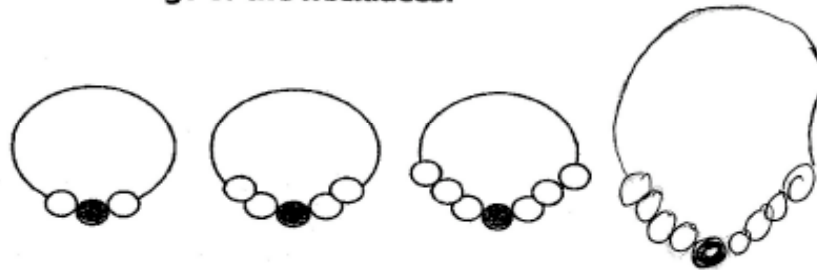
Why or why not?

~~Because if you start with an~~
~~odd number like 3 if you~~
~~add 2 its another odd number~~
like $3 + 2 = 5$. ✓ 2

Student F adds two beads each year starting from one black bead. This explanation of starting with the one black bead is directly identifying the constant in the growth pattern. The work of Students A-F reflects the work of approximately 20% of all students – those who met all of the demands of the task.

Student F

Every year Matthew makes a necklace for his sister, Janci. Here are drawings of the necklaces.



	Year 1	Year 2	Year 3	Year 4
Beads	3 beads	5 beads	7 beads	9 beads

1. Draw a picture of the necklace for Janci's 4th year next to the 3rd necklace.
2. How many beads would Matthew need to make a necklace for Janci's 6th year?

13 beads in all ✓ 2

Show how you figured it out.

They are counting ✓
by twos except starting
with 1 then 3.

3. Would Matthew ever make a necklace like this for Janci with 12 beads?

Yes or No ✓ 1

Why or why not?

Because we are counting
by twos but starting ✓
with 1 not 2.

The task required a deep level of explanation in order to receive full credit. Student G's explanation received partial credit as it speaks to adding 2 beads each year but does not explain the oddness of the number of beads each time nor why 12 beads would not work in this pattern.

Student G

3. Would Matthew ever make a necklace like this for Janci with 12 beads?

Yes or No NO ✓ |

Why or why not?

Because he keeps on
adding by 2 each
and every year.

The work of Students H and I clearly shows understanding of the 2 beads added each year as well as the pattern of odd numbers. Unfortunately in continuing the necklace pattern, Student H repeated year 4 at year 5. Although we gave 2nd graders a difficult assignment - to give only the 4th and 6th year as answers (skipping the 5th year) - only 7% of all students made an error similar to this repeating or just gave the number of beads for the 5th year rather than the 6th.

Student H

Growing Necklaces

Every year Matthew makes a necklace for his sister, Janci. Here are drawings of the necklaces.

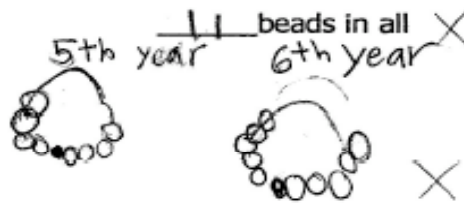
	Year 1	Year 2	Year 3	Year 4
Beads	3 beads	5 beads	7 beads	9 beads

1. Draw a picture of the necklace for Janci's 4th year next to the 3rd necklace.



2. How many beads would Matthew need to make a necklace for Janci's 6th year?

Show how you figured it out.



3. Would Matthew ever make a necklace like this for Janci with 12 beads?

Yes or No no ✓

Why or why not?

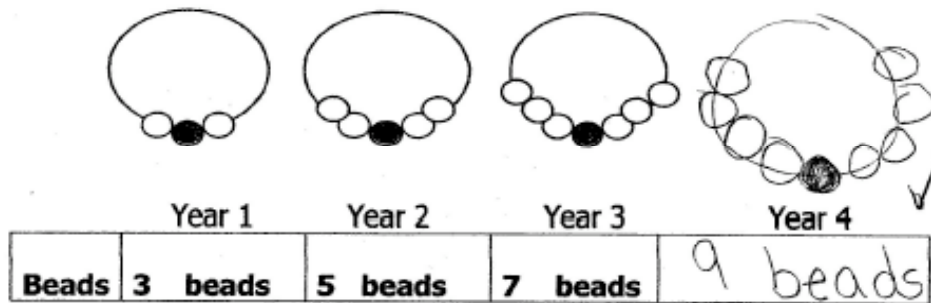
because he adds 2 from
an odd number each
year ✓

In part 3, Student I gave what would be the total number of beads needed for the 12th year. It is not clear if that is what was meant by 25 beads but it would be the correct answer in that case. Unfortunately, that doesn't answer what was asked in part 3.

Student I

Growing Necklaces

Every year Matthew makes a necklace for his sister, Janci. Here are drawings of the necklaces.



1. Draw a picture of the necklace for Janci's 4th year next to the 3rd necklace.

2. How many beads would Matthew need to make a necklace for Janci's 6th year?

13 beads in all

Show how you figured it out.



3. Would Matthew ever make a necklace like this for Janci with 12 beads?

Why or why not?

It will use the whole space for 25 beads.

For approximately 60% of the 2nd graders, providing an explanation in part 3 proved challenging. Many, like Students J and K could continue the pattern and show how the pattern grew by 2 beads each year but their explanations for why or why not 12 beads did not connect back to the mathematics.

Student J

Every year Matthew makes a necklace for his sister, Janci. Here are drawings of the necklaces.

	Year 1	Year 2	Year 3	Year 4
Beads	3 beads	5 beads	7 beads	9 beads

1. Draw a picture of the necklace for Janci's 4th year next to the 3rd necklace.

2. How many beads would Matthew need to make a necklace for Janci's 6th year?

13 beads in all

Show how you figured it out.

I added 4 beads to the necklace for the 6th year.

3. Would Matthew ever make a necklace like this for Janci with 12 beads?

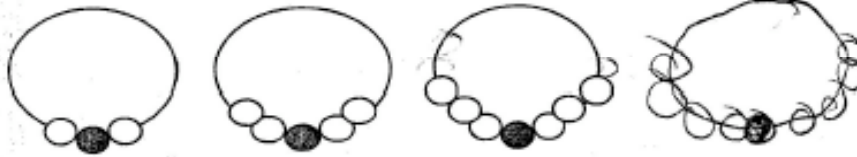
Yes or No yes x

Why or why not?

Yes he could make a necklace with 12 beads for Janci because there is enuf room. x

Student K

Every year Matthew makes a necklace for his sister, Janci.
Here are drawings of the necklaces.



	Year 1	Year 2	Year 3	Year 4
Beads	3 beads	5 beads	7 beads	9 beads

1. Draw a picture of the necklace for Janci's 4th year next to the 3rd necklace.



2. How many beads would Matthew need to make a necklace for Janci's 6th year?

~~13~~ beads in all ✓

Show how you figured it out.

$$3 + 2 = 5 + 2 = 7 + 2 = 9 + 2 = 11 + 2 = 13$$

✓ eq

3. Would Matthew ever make a necklace like this for Janci with 12 beads?

Yes or No No ✓

Why or why not?

No because you do not
have all the beads
to finish the necklace. *

The explanations of Students L-N lack mathematical connections as well.

Student L

3. Would Matthew ever make a necklace like this for Janci with 12 beads?

Yes or No NO ✓

Why or why not?

I + will be hard of do
and it is to much to
do. x.

Student M

3. Would Matthew ever make a necklace like this for Janci with 12 beads?

Yes or No Yes x ○

Why or why not?

Matthew will make a necklace
with 12 beads because years^x
will pass.

Student N

3. Would Matthew ever make a necklace like this for Janci with 12 beads?

Yes or No yes ✓ ○

Why or why not?

Because he love her with
all his heart x ○

Approximately 45% of students were unable to meet the essential demands for this task. The work of Students O, P, Q, and R is reflective of some of the misconceptions around this task. Student O was able to continue the necklace pattern to the 4th year but gave the total number of beads for the 1st through the 4th year necklaces rather than continuing the pattern. The explanation in part 3 is not related to the pattern or to the number of beads.

Student O

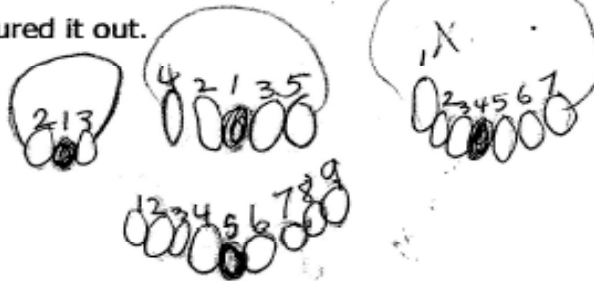
	Year 1	Year 2	Year 3	Year 4
Beads	3 beads	5 beads	7 beads	9 beads

1. Draw a picture of the necklace for Janci's 4th year next to the 3rd necklace.

2. How many beads would Matthew need to make a necklace for Janci's 6th year?

24 beads in all

Show how you figured it out.



3. Would Matthew ever make a necklace like this for Janci with 12 beads?

Yes or No no

Why or why not?

Because he mite for got that he
makes necklaces for his sister Janci
each year.

Student P continues the pattern through the 5th year by drawing the necklaces and by showing the odd number of beads. The explanation, however, avoids mentioning why 12 beads will not make a necklace of this pattern.

Student P

	Year 1	Year 2	Year 3	Year 4
Beads	3 beads	5 beads	7 beads	9 beads

1. Draw a picture of the necklace for Janci's 4th year next to the 3rd necklace.

2. How many beads would Matthew need to make a necklace for Janci's 6th year?

11 beads in all^{*}

Show how you figured it out.



$$6 + 5 = 11^*$$

$$5 + 6 = 11$$

3. Would Matthew ever make a necklace like this for Janci with 12 beads?

Yes or No No ✓

Why or why not?

No because she would be too
old for necklaces like that and
she might not even want a necklaces.

Student Q correctly sites the odd number in this pattern but fails to follow the growth pattern at the beginning.

Student Q

	Year 1	Year 2	Year 3	Year 4
Beads	3 beads	5 beads	7 beads	13 Beads

1. Draw a picture of the necklace for Janci's 4th year next to the 3rd necklace.

2. How many beads would Matthew need to make a necklace for Janci's 6th year?

15 beads in all X

Show how you figured it out.

because it was a odd number; Xi

3. Would Matthew ever make a necklace like this for Janci with 12 beads?

Yes or No No ✓

Why or why not?

Because Matthew would ✓
make only odd beads not
even.

Student R continues the necklace to the 4th year. In predicting and proving subsequent necklaces, however, Student R fails to include the one constant black bead.

Student R



	Year 1	Year 2	Year 3	Year 4
Beads	3 beads	5 beads	7 beads	

1. Draw a picture of the necklace for Janci's 4th year next to the 3rd necklace.

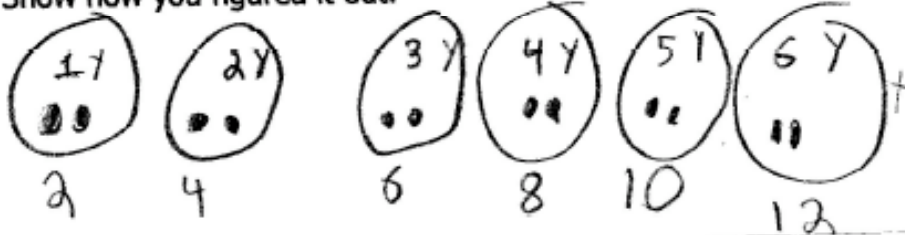


9 Beads ✓ ~

2. How many beads would Matthew need to make a necklace for Janci's 6th year?

12 beads in all

Show how you figured it out.



3. Would Matthew ever make a necklace like this for Janci with 12 beads?

Yes or No Yes x

0

Why or why not?

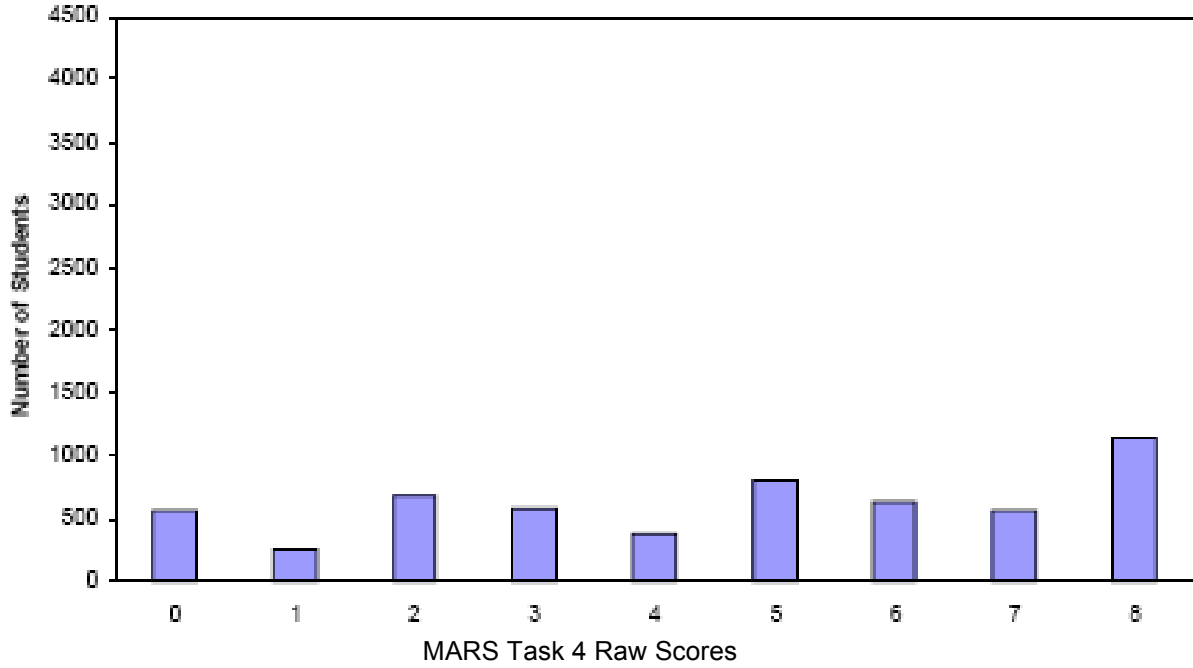
The pattern is by 2's. x
Problem two shows the
answer. Yes 12 is possible!

Teacher Notes:

Frequency Distribution for Task 4 – Grade 2 – Growing Necklaces

Growing Necklaces

Mean: 4.59 StdDev: 2.66



Score:	0	1	2	3	4	5	6	7	8
Student Count	568	255	684	578	368	792	621	555	1139
% ≤	10.2%	14.8%	27.1%	37.5%	44.1%	58.4%	69.5%	79.5%	100.0%
% ≥	100.0%	89.8%	85.2%	72.9%	62.5%	55.9%	41.6%	30.5%	20.5%

There is a maximum of 8 points available for this task.
The cut score for a level 3 response, meeting standards, is 4 points.

Many students, about 80% could correctly draw the next beaded necklace in the sequence. A little more than half of the students could predict the correct number of beads needed for the necklace for the 6th year. 20.5% of the students could meet all the demands of the task including making a mathematical justification for why 12 beads would not be used to make a necklace of this pattern. Approximately 10% of the students scored no points on this task. All the students in the survey attempted this task.

Growing Necklaces

Points	Understandings	Misunderstandings
0	All the students with this score attempted the task.	Some students repeated the same pattern and did not represent the growth pattern. Several of the students with a score of “0” showed a growth pattern but used an even numbers of beads.
2	Students could draw the 4 th year necklace correctly.	When predicting the number of beads for the 6 th year necklace, many students added all the beads for the first four necklaces and gave that as an answer.
4	Students could draw the 4 th year necklace, predict the 13 beads needed for the 6 th year, and justify the 13 beads by drawing or presenting the number pattern.	Students showed a growing pattern but not one that matched this necklace pattern. Many students gave the bead count for the 5 th year necklace instead of the 6 th year.
6	Students could continue the pattern to the 4 th and 6 th year, show their work for the extension of the pattern and state that 12 beads would not work in this pattern.	Many students struggled to give a mathematical explanation as to why 12 beads would not fit this pattern. Students who missed the explanation cited reasons having to do with the logistics of constructing any kind of necklace or the relationship between Matthew and Janci rather than to the mathematics in the pattern.
7		Students could do the entire task but had an explanation that did not fully communicate the mathematics of this pattern. Most students said “It is not the same pattern” and did not clarify what about the number 12 did not fit this pattern.
8	Students could extend the pattern using pictures, the table, and number patterns. They could make a mathematical justification for why a number of beads did not fit in the pattern of beaded necklaces.	

Based on teacher observations, this is what second grade students seemed to know and be able to do:

Areas of difficulty for second graders, students struggled with:

- Explaining why 12 did not fit the pattern
- Writing equality statements
- Using mathematical connections to explain why 12 beads did not follow the pattern

Strategies used by successful students:

- Extending the pattern by drawing
- Adding on with the pattern to find the number of beads for the 6th necklace
- Showing or explaining their thinking for extending the pattern

Questions for reflection on Growing Necklaces:

- What types of growing pattern problems have your students worked on this year?
- What activities have students had to help build their spatial visualization? Are students encouraged to record their spatial visualizations?
- Are students given the opportunity to predict the next item in a sequence? Are students encouraged to form predictions for **any** item that would be in the subsequent pattern?
- When working with patterns, are students asked to describe what is changing and what stays the same?
- Are students able to give examples and non-examples of patterns in a sequence?
- When predicting the extension of patterns, are students encouraged to justify their thinking? Are students encouraged to site mathematical reasoning in their justifications?
- What types of activities do students have to share their mathematical thinking around patterns? Are students encouraged to describe the patterns mathematically?

Teacher Notes:

Implications for Instruction:

Young children begin their work with patterns and functions by looking at repeating patterns. They are very excited to recognize, analyze, and extend patterns of color, motion, shape, design and size (to name a few). Once children begin to recognize the underlying order in patterns, they will be encouraged to predict how the pattern will extend. Children learn to make connections and to see relationships and to recognize the same pattern idea portrayed in many different ways. Even though growing patterns are more difficult for some children to see than are repeating patterns, keep children open to many different kinds of patterns of varying complexity. When they see a pattern represented in different ways they will be better able to find the underlying structure of similar patterns. Ask probing questions to help form generalizations around patterns: How could you describe this pattern? How can it be extended? How is each necklace the same? Where does it grow/change? Use language and notation to model the relationships in patterns. The quantitative change can be described mathematically and this representation will lead the young student to correct predictions of the extension of the pattern. In the beginning, students will look at the pattern to predict the numbers that come next on the list. After they have had many experiences like that, provide the questions that will encourage discussion around prediction further out in the pattern: How many beads will be on the necklace made for the 10th year? Will 12 beads ever be on a necklace of this pattern? Why? Why not? The search for patterns is very important in mathematics because it helps to reveal the underlying order that exists in situations and events. It is a way to build and strengthen young children’s understanding of number relationships as well as an exciting and satisfying experience of the beauty of mathematics!

Teacher Notes:
