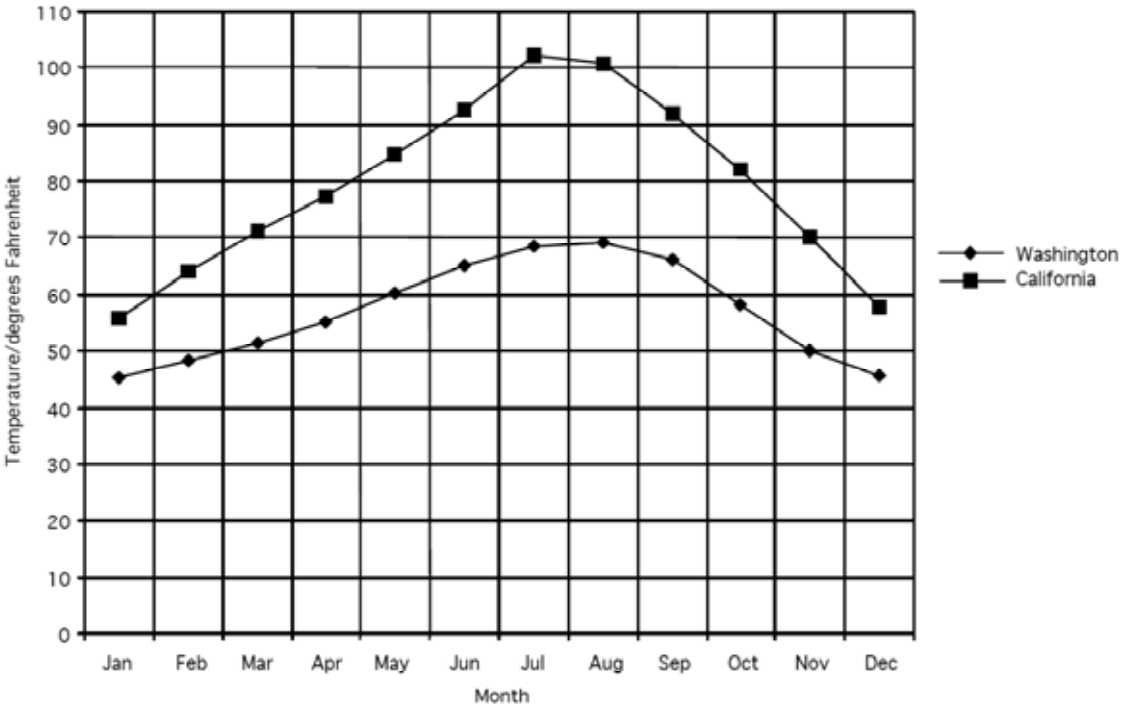
Temperatures

This problem gives you the chance to:
• understand and interpret statistical graphs and diagrams showing real data

This graph shows the highest average temperatures for each month of the year for one place in Washington and one place in California.



1. Write two statements about what is the same and what is different in the two sets of temperatures.

(i) _____

_____

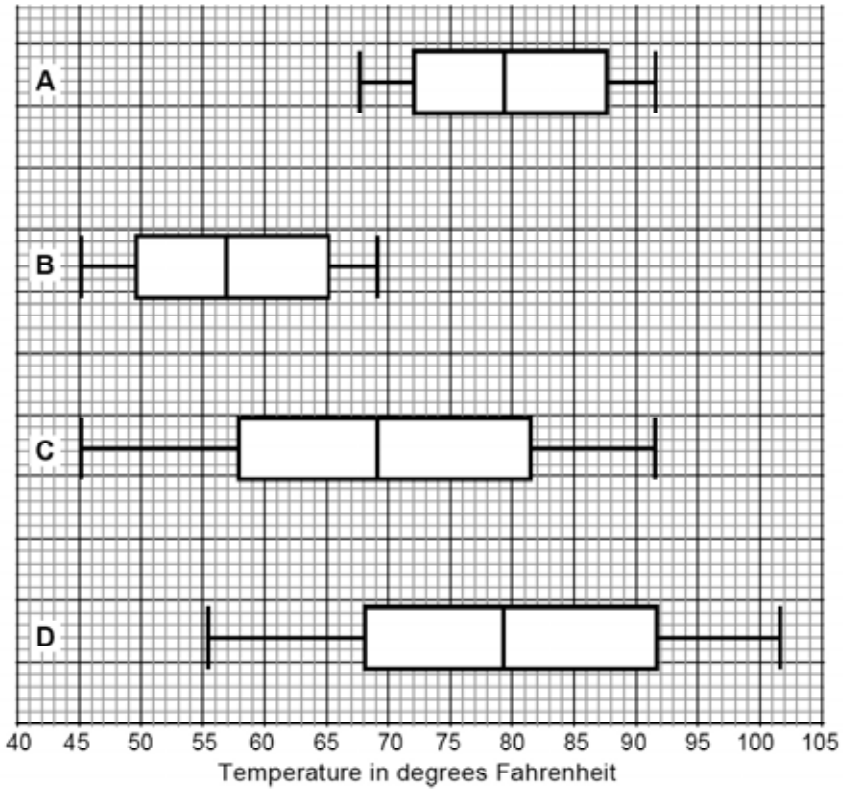
_____

(ii) _____

_____

_____

Box and whisker temperature diagrams



2. Which of the four box diagrams shows the Washington temperatures? _____

   Explain how you decided.

   _____

   _____

3. Which of the four box diagrams shows the California temperatures? _____

   For which months of the year is the maximum monthly temperature for California between the upper and the lower quartiles?

   _____

   _____

   Explain how you figured it out.

   _____

   _____

7

Temperatures	Rubric	
The core elements of performance required by this task are: • understand and interpret statistical graphs and diagrams showing real data Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives two correct statements such as: Similarities: January has the lowest temperature in both states. June and July have the greatest temperatures. Temperatures increase from the beginning of the year to the middle of the year, then decrease again. Differences: California's temperatures are higher than Washington's for every month. The range of temperatures is greater for California than for Washington.	2x1	2
2. Gives correct answer: **B** Gives a correct explanation such as: Lowest temperature is 45º and highest is 69º. Numbers are not essential dependent on correct answer B.	1 1	2
3. Gives correct answer: **D** Gives correct answer: **March, April, May,** June, September**, October, November** Gives correct reason such as: The temperatures are between 68º and 92º. Numbers are not essential dependent on correct answer D.	1 1 1	3
Total Points		**7**

Temperatures

Work the task and look at the rubric. What are the key features of each graph that you would want students to notice?

Look at student work for part one. How many of your students discussed:

Shape of data	Steepness: rise or fall	Highs/lows	Range	Nonmathematical ideas: both states, both about temp.	Confused Wash. for Calif.	Other

What are the instructional implications for the data you have just collected?

Now look at student work for two and three.

- Were you students able to put a quantity with their explanations for 2 and 3b?
- Were the quantities given enough to eliminate other choices?
- How many of your students could list all the correct months?
- Why do you think naming the months was difficult for students?
- What conjectures can you make for their errors? (pick three students who scored fairly well on other parts of the task and see if you can figure out what caused their error on this part)

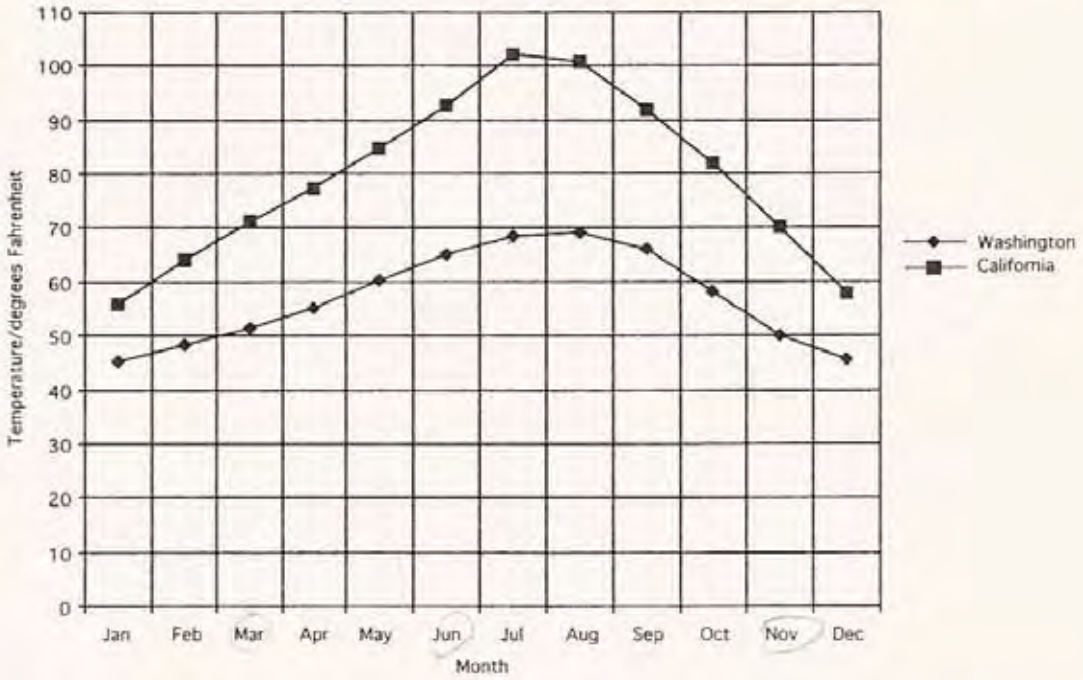
What types of data do students work with in your class?
When working with graphs, do students frequently look at key features of the graph? Make comparisons between data sets?
How do classroom discussions help to build the logic needed to eliminate all the other choices? Do students understand that B and C had the same low temperature, that A and C had the same high temperature, or that B and A were both compact?

Looking at Student Work on Temperatures

Student A marks both graphs to find the months between upper and lower quartiles. The student is able to quantify why B is the best choice for Washington's temperatures. The student is able to make two good comparisons: shape of data, range of data.

Student A

This graph shows the average highest temperatures for each month of the year for one place in Washington and one place in California.

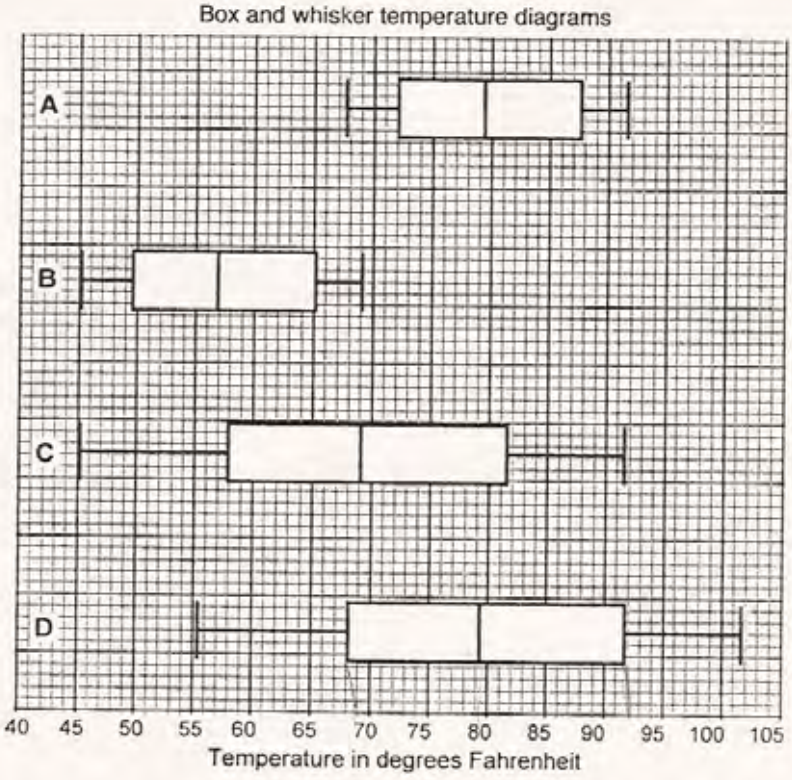


1. Write two statements about what is the same and what is different in the two sets of temperatures.

(i) Both the tempreture r'se till July/Aug. then it cools gets down.

(ii) California has lower averege tempretrve than Wasington, Washington has less defference in change of tempreture than the California's change in Tempreture

Student A, continued:



2. Which of the four box diagrams shows the Washington temperatures? B ✓✓

Explain how you decided.

B diagram has lowest temperature at 45F° which is same as the chart and peak temperature is at 75F° which also is same as the chart ✓✓

3. Which of the four box diagrams shows the California temperatures? D. ✓✓

For which months of the year is the monthly temperature for California between the upper and the lower quartiles?

~~May~~ Mar. – Jun., Sep. – Nov. ✓✓

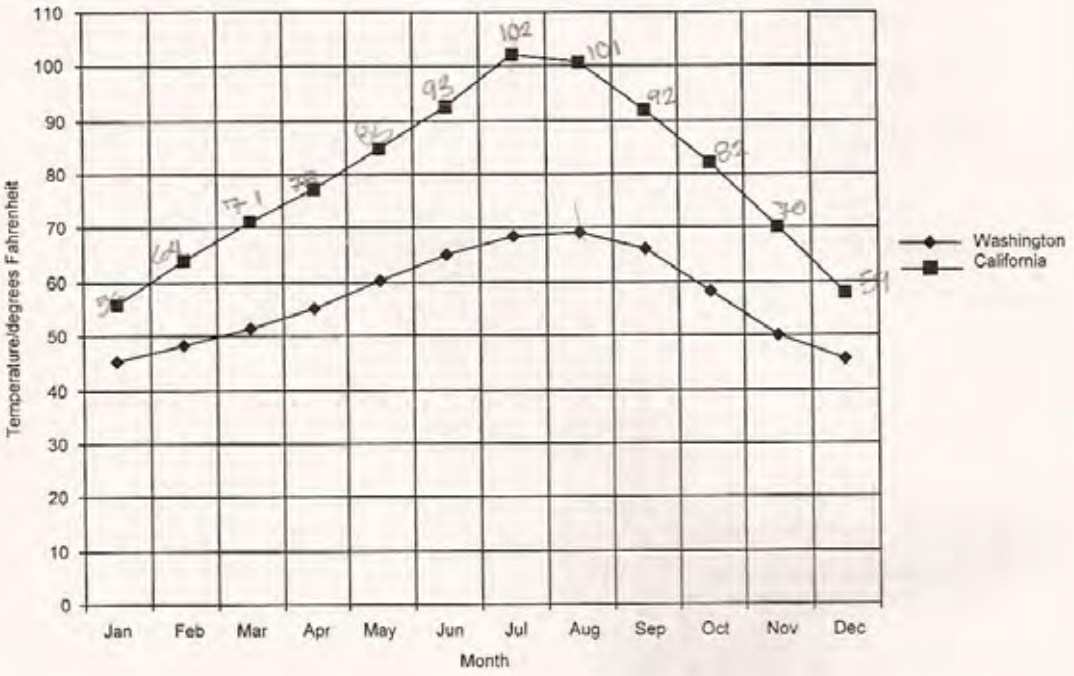
Explain how you figured it out.

First figure out the temperature by looking at the box diagram then checked it on the chart. ✓✓

Student B marks the temperature readings for California on the graph to help aid with the comparisons. The student seems to think that between the upper and lower quartiles refers to center of the box plot or median rather than the range of scores within the box. The student confuses median and average in the final part of three.

Student B

This graph shows the average highest temperatures for each month of the year for one place in Washington and one place in California.

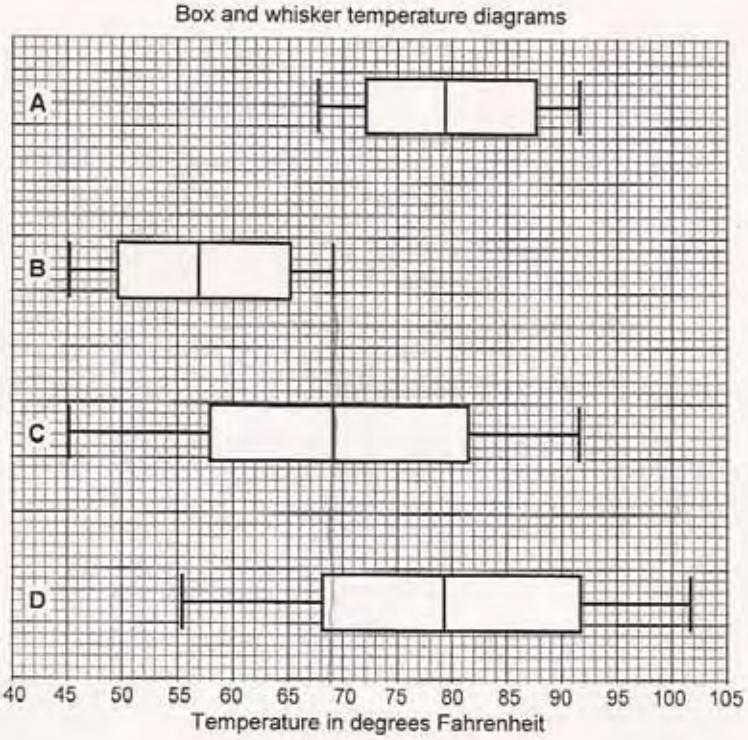


1. Write two statements about what is the same and what is different in the two sets of temperatures.

(i) SAME: THEY BOTH ARCH AROUND THE SAME AREA. ✓ ✓ 1

(ii) WASHINGTONS TEMPERATURES ARE LOWER THAN CALIFORNIA'S. ✓ ✓ 1

Student B, continued:



102
101
93
92
85
82
78
71
70
64
+59
956
953

79.4
12 1953

2. Which of the four box diagrams shows the Washington temperatures? B ✓ 1

Explain how you decided.

THE LOW WAS 45°F & THE HIGH ✓ 1
WAS 79°F. bold ✓ 1

3. Which of the four box diagrams shows the California temperatures? D ✓ 1 1

For which months of the year is the monthly temperature for California between the upper and the lower quartiles?

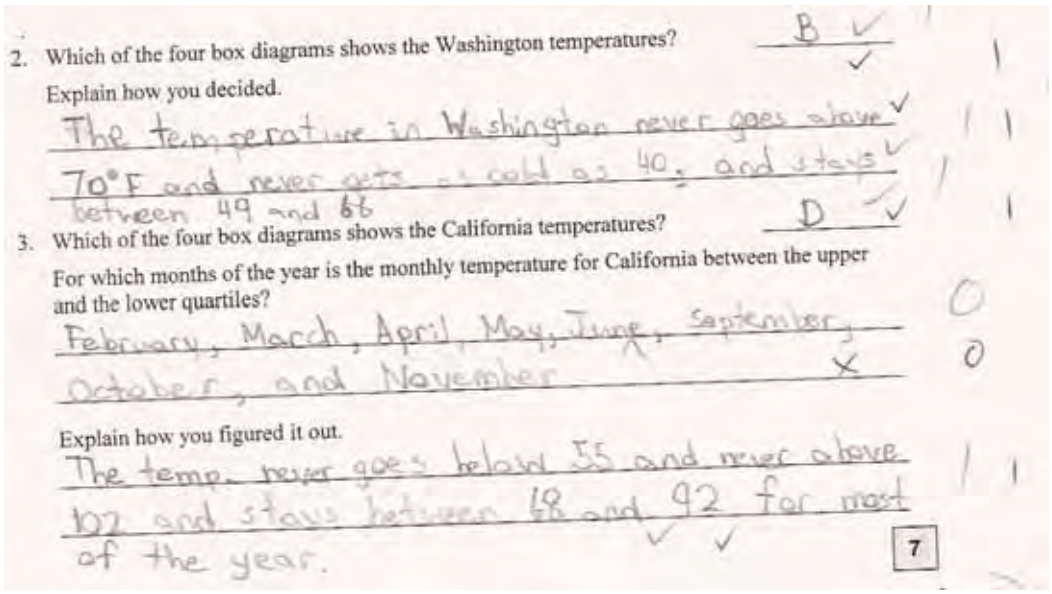
AROUND APRIL. X 0 0

Explain how you figured it out.

I FOUND THE AVERAGE OF THE 0
TEMPERATURES. X 0

Student C seems to think that between the highest and lowest quartiles refers to the end points or extremes of the temperature range. The student leaves out only the two highest temperatures and two lowest temperatures. In further digging into the explanation, while the student mentions the values demarking the middle quartiles, the emphasis for this student is still the extremes.

Student C



2. Which of the four box diagrams shows the Washington temperatures? B ✓ ✓

Explain how you decided.

The temperature in Washington never goes above 70°F and never gets as cold as 40, and stays between 49 and 66 ✓ ✓

3. Which of the four box diagrams shows the California temperatures? D ✓

For which months of the year is the monthly temperature for California between the upper and the lower quartiles?

February, March, April, May, June, September, October, and November ✓

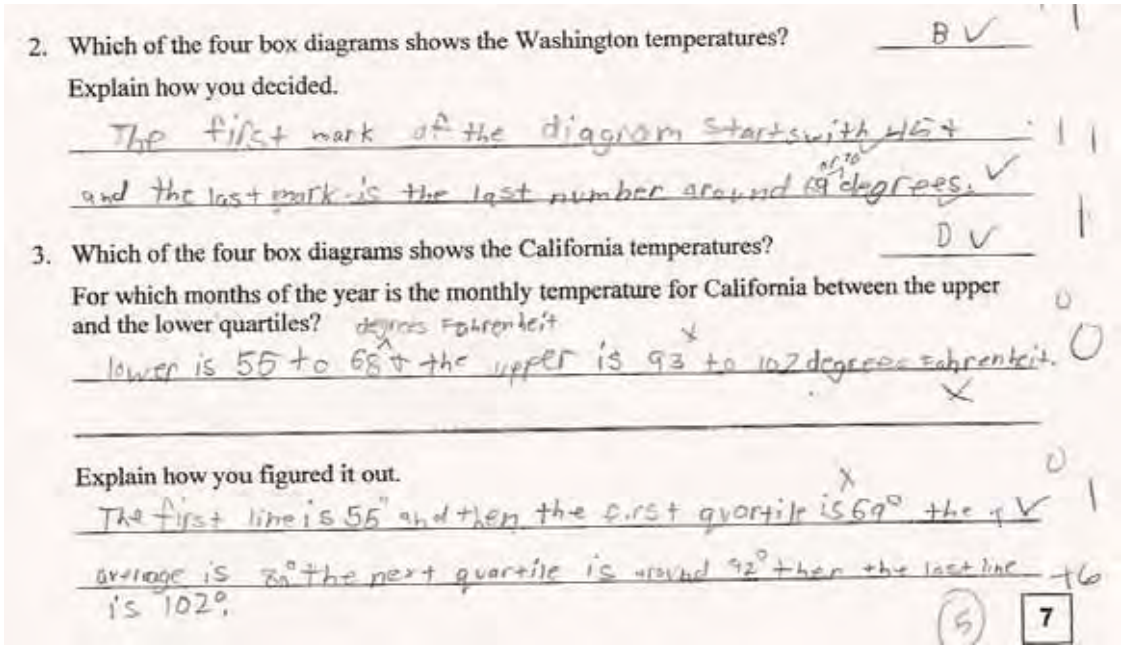
Explain how you figured it out.

The temp. never goes below 55 and never above 102 and stays between 68 and 92 for most of the year. ✓ ✓

1 1 1 1 1 0 0 7

Student D seems to be clear on reading the values on the box and whisker plot and defining the range for each quartile. The student doesn't give the months that fall into the middle quartiles. *So you think the student is only thinking about the limits of each quartile or do you the student understands idea of all the values within the range? What question would you like to ask this student?*

Student D



2. Which of the four box diagrams shows the Washington temperatures? B ✓

Explain how you decided.

The first mark of the diagram start with 46° and the last mark is the last number around 69 degrees. ✓

1 1 1

3. Which of the four box diagrams shows the California temperatures? D ✓

For which months of the year is the monthly temperature for California between the upper and the lower quartiles? degrees Fahrenheit

lower is 55 to 68 & the upper is 93 to 102 degrees Fahrenheit. ✓

0 0

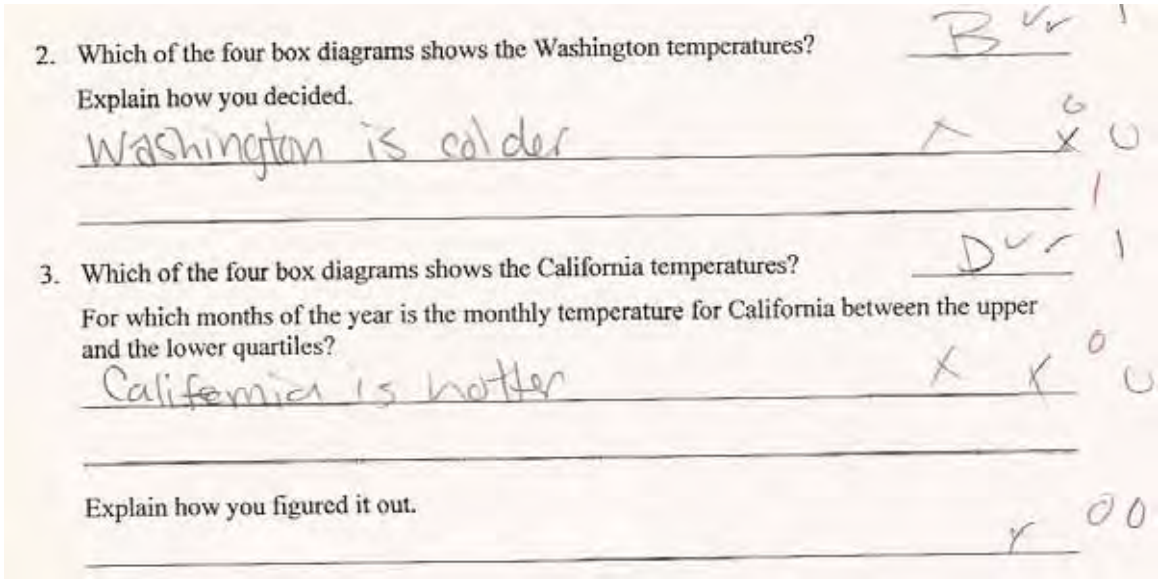
Explain how you figured it out.

The first line is 55° and then the first quartile is 69° the ✓ average is 80° the next quartile is around 92° then the last line is 102°.

1 +6 (5) 7

Student E is able to identify the correct box plots for each state. However the student gives no quantities. Notice that plot C has the same lowest temperature as B. In part three the students does not think about the middle quartiles, but just the upper limit of the temperatures.

Student E



2. Which of the four box diagrams shows the Washington temperatures? B

Explain how you decided.

Washington is colder

3. Which of the four box diagrams shows the California temperatures? D

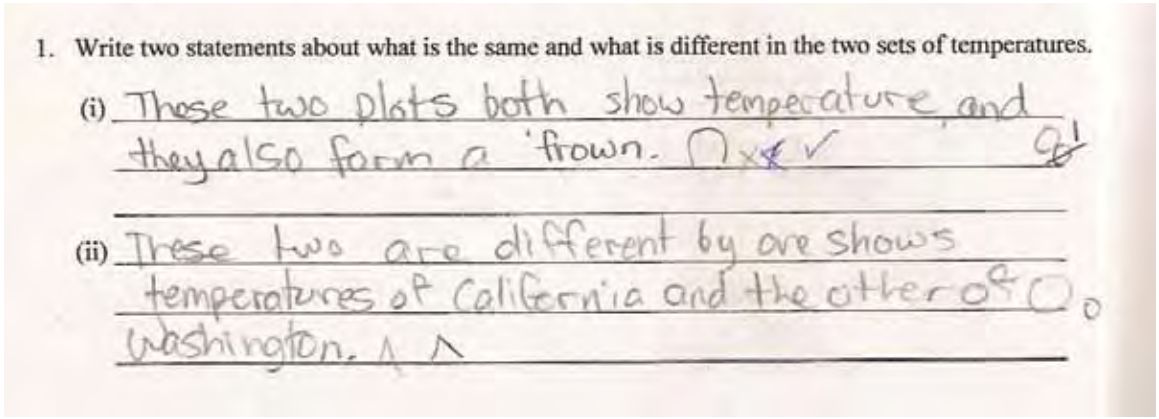
For which months of the year is the monthly temperature for California between the upper and the lower quartiles?

Californica is hotter

Explain how you figured it out.

As students move down in score points the quality of comments about features of the graph drop off dramatically. Student F does not talk about features of the graph so much as describe the topic of the graphs. While the student only gives one quantity for justifying B, that quantity is enough to distinguish it from all the other choices. In part three the student does not seem to understand the idea of quartiles.

Student F

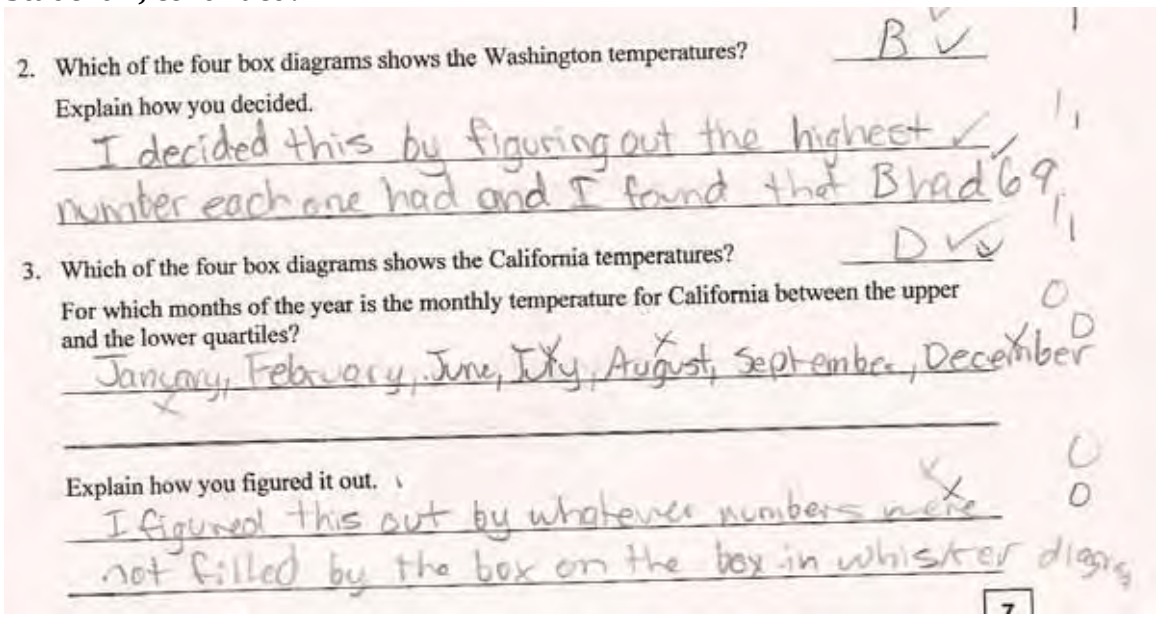


1. Write two statements about what is the same and what is different in the two sets of temperatures.

(i) These two plots both show temperature and they also form a 'frown.'

(ii) These two are different by one shows temperatures of California and the other of Washington.

Student F, continued:


2. Which of the four box diagrams shows the Washington temperatures? B

Explain how you decided.

I decided this by figuring out the highest number each one had and I found that B had 69.

3. Which of the four box diagrams shows the California temperatures? D

For which months of the year is the monthly temperature for California between the upper and the lower quartiles?

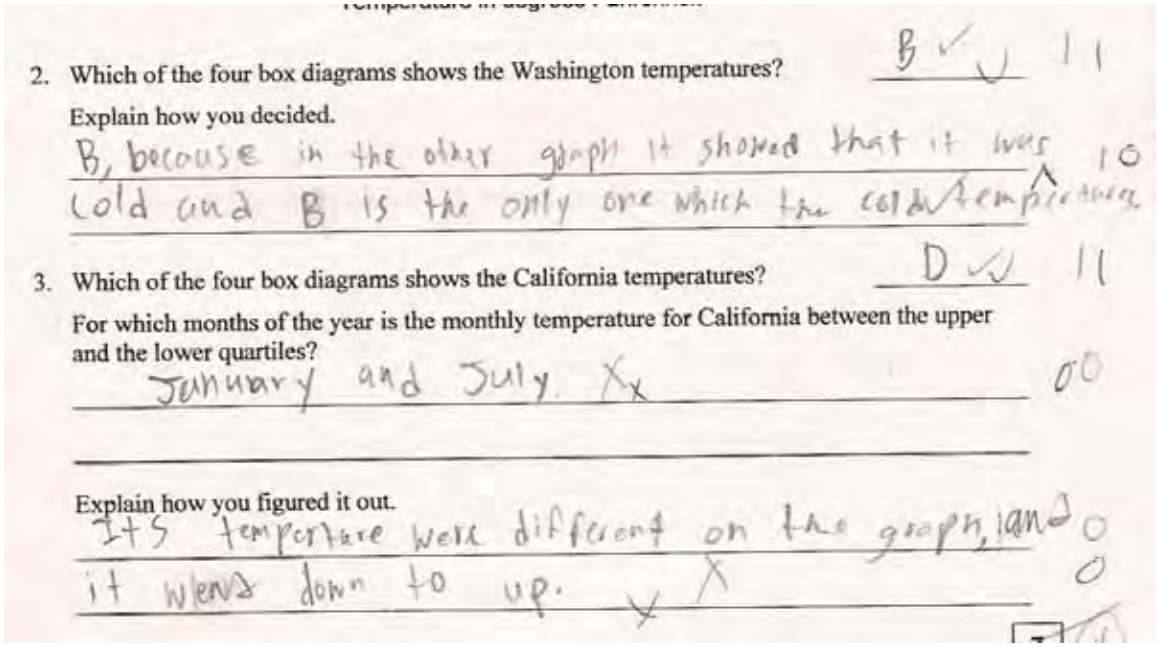
January, February, June, July, August, September, December

Explain how you figured it out.

I figured this out by whatever numbers were not filled by the box on the box in whisker diagrams

Student G again thinks only about cold as a reason for picking graph B. This answer does not distinguish between choice B and C. For part 3 the student only makes sense of the extremes and not of the middle quartiles.

Student G


2. Which of the four box diagrams shows the Washington temperatures? B

Explain how you decided.

B, because in the other graph it showed that it was cold and B is the only one which has cold temperatures.

3. Which of the four box diagrams shows the California temperatures? D

For which months of the year is the monthly temperature for California between the upper and the lower quartiles?

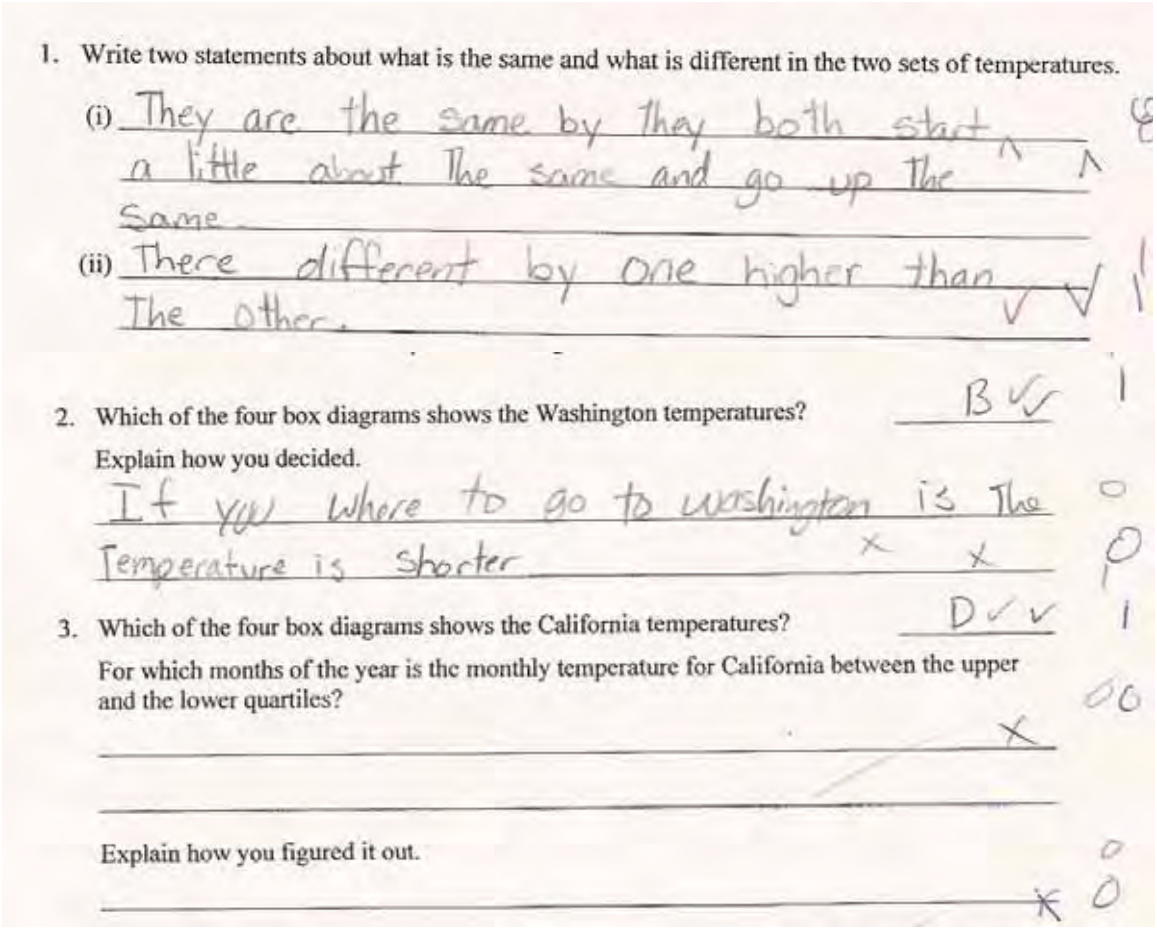
January and July.

Explain how you figured it out.

Its temperature were different on the graph, and it went down to up.

Student H has difficulty with quantifying relationships. In part one, the student makes an attempt to discuss the rise in temperature during the same time period, but can't quite complete the statement. In part two, shorter is not a justification for B because A is also short. In part three the student can identify D, but not justify the choice.

Student H



1. Write two statements about what is the same and what is different in the two sets of temperatures.

(i) They are the same by they both start a little about the same and go up the same

(ii) There different by one higher than The other.

2. Which of the four box diagrams shows the Washington temperatures? B

Explain how you decided.

If you where to go to washington is the Temperature is shorter

3. Which of the four box diagrams shows the California temperatures? D

For which months of the year is the monthly temperature for California between the upper and the lower quartiles?

Explain how you figured it out.

Eighth Grade

8th Grade Task 3 Temperatures

Student Task	Understand and interpret statistical graphs and diagrams showing real data. Compare and contrast data sets.
Core Idea 5 Data Analysis	**Collect, organize, analyze, and display relevant data.** • Select, create, and use appropriate graphical representations of data, including box plots and scatterplots. • Find, use, and interpret measures of center and spread, including interquartile range. • Discuss and understand the correspondence between data sets and their graphical representations, especially box plots and scatterplots.

Based on teacher observation, this is what eighth graders knew and were able to do:

- Identify similarities and differences
- Match the box and whisker plots to the appropriate states

Areas of difficulty for eighth graders:

- Understanding the distance between upper and lower quartiles as a range of values represented by the in the box plot
- Understanding that the line in the box represents median instead of average
- Reading the key incorrectly and therefore confusing Washington and California
- Quantifying their reasons for choosing the box and whisker plots
- Giving enough detail to eliminate other options

MARS Test Task 3 Frequency Distribution and Bar Graph, Grade 8

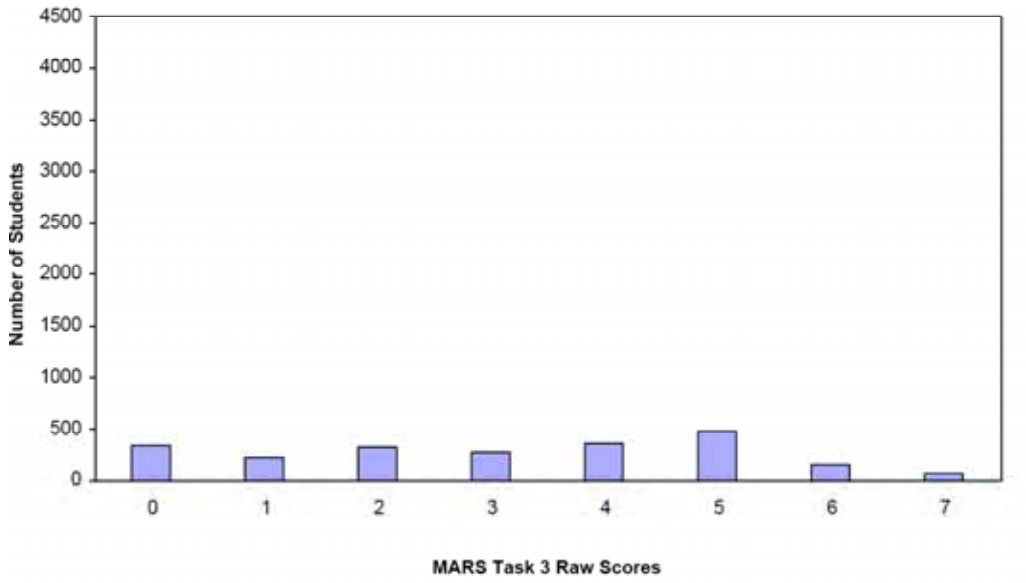
Task 3 - Temperatures

Mean: 3.14 StdDev: 2.02

Table 42: Frequency Distribution of MARS Test Task 3, Grade 8

Task 3 Scores	Student Count	% at or below	% at or above
0	342	15.3%	100.0%
1	218	25.1%	84.7%
2	323	39.5%	74.9%
3	273	51.7%	60.5%
4	363	68.0%	48.3%
5	482	89.5%	32.0%
6	161	96.7%	10.5%
7	73	100.0%	3.3%

Figure 51: Bar Graph of MARS Test Task 3 Raw Scores, Grade 8



The maximum score available for this task is 7 points.
The minimum score for a level 3 response, meeting standards, is 5 points.

Most students, 85%, could identify one similarity or difference between the temperature graphs for California and Washington. Many students could describe both a similarity and a difference between the two graphs. Less than half the students, 32%, could write a similarity and difference, choose the appropriate box plot to represent Washington and quantify the reason for their choice, and choose the box plot for California. Less than 5% of the students could pick the months where the temperatures fell between the upper and lower quartiles and explain how they knew. 15% of the students scored no points on this task. 63% of the students with this score attempted the task.

Temperatures

Points	Understandings	Misunderstandings
0	63% of the students with this score attempted the task.	Many students confused the graph for Washington and California. Students could not pick out important features of the graph for making comparisons or could not attach enough detail to their statements. Students talked about nonmathematical features of the graph: they're both about temperature, they're both states.
1	Students could find one similarity or difference between the two graphs.	
2	Students could find one similarity and one difference for the graphs.	Students reversed the box plots for California and Washington. They could not attach quantity to their explanations.
5	Students could write a similarity and difference, choose the appropriate box plot to represent Washington and quantify the reason for their choice, and choose the box plot for California.	Students did not understand that between the upper and lower quartile referred to the range of values within the box on the box plot. Many students looked only at the extremes of the plot or the outside edge of the box. Some students confused median and average.
7	Students could write a similarity and difference, choose the appropriate box plot to represent Washington and quantify the reason for their choice, and choose the box plot for California. They also could pick the months where the temperatures fell between the upper and lower quartiles and explain how they figured it out.	

Implications for Instruction

Students need exposure to a variety of graphs, including line graphs and box and whisker diagrams. Students, to be literate in reading and interpreting graphs, should be able to compare and contrast key statistical features of a graph, such as range, highest and lowest, slope, direction of slope. Students should understand that the box and whisker diagram divides the data into four equal-size groups to help the reader understand the distribution of the data points. They should also understand the line dividing the box represents the median of the data.

Students need to work with data in context to think about why it might be important to see or think about the distribution of data. For example, students might look at test scores, ages of people attending a movie, temperatures for a vacation destination and then discuss what different box and whisker plots for those choices might represent. The purpose and nuances of box and whisker plots only becomes clear within a situation. It is also important for students to think about box and whisker plots with the idea of large amounts of data. There is not so much purpose in understanding distribution for a small number of data points, which can easily be sorted by a cursory glance. During discussions, it might be important to ask why might this be useful? How might it help in making decisions? Students need to connect making, reading, and interpreting graphs with conveying information and answering questions or making decisions.

25% Sale

This problem gives you the chance to:
- work with percentage increase and decrease

In a sale, all the prices are reduced by 25%.

1. Julie sees a jacket that cost $32 before the sale.
   How much does it cost in the sale?

   $ _____

   Show your calculations.

In the second week of the sale, the prices are reduced by 25% of the previous week's price.
In the third week of the sale, the prices are again reduced by 25% of the previous week's price.
In the fourth week of the sale, the prices are again reduced by 25% of the previous week's price.

2. Julie thinks this will mean that the prices will be reduced to $0 after the four reductions because 4 x 25% = 100%.

   Explain why Julie is wrong.

3. If Julie is able to buy her jacket after the four reductions, how much will she have to pay?

$\$$_____

Show your calculations.

4. Julie buys her jacket after the four reductions.

What percentage of the original price does she save?

_____%

Show your calculations.

8

25% Sale	Rubric	
The core elements of performance required by this task are: • work with percentage increase and decrease Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: $**24** Shows correct work such as: 32 ÷ 4 = 8 and 32 - 8	1 1	2
2. Gives a correct explanation such as: Each reduction is 25% of the previous week's price, and as the price goes down each week, the 25% will be a smaller amount each week.	1	1
3. Gives correct answer: $**10.12** or $**10.13** (accept $10.10) Shows correct work such as: 32 x 0.75^4 or 24 x 0.75^3 **or** 24 – (24 x 0.25) = 18 18 – (18 x 0.25) = 13.5 13.5 – (13.5 x 0.25) *Partial credit* Correct as far as/such as, 24 – (24 x 0.25) = 18; 18 – (18 x 0.25) = 13.5	1 2 (1)	3
4. Gives correct answer: **68.3**% or **68.4**% (accept 68%) Shows correct work such as: 32 – 10.12(or 3) = 21.88(7) **and** 21.88(7) / 32 x 100 **or** 10.12(or 3) / 32 x 100 = 31.6(7) **and** 100 – 31.6(7) *Partial credit* Gives answer 31.6% or 31.7% with some correct work (accept 32%)	1 1ft (1)	2
Total Points		**8**

25% Sale

Work the task and look at the rubric. What strategies might your students have for finding percents?

Look at student work for part one of the task. How many of your students put:

$24	$8	$26	$22	$31.75	Other

Can you figure out why a student might get each of the wrong answers above? What are they misunderstanding about the context and percents?

Now look at student responses to part two.

- How many of your students thought that it can't be right because things aren't free?
- How many responses show students didn't understand the situation of taking a reduction each week?
- What other misconceptions did you see?

Did your students have access to calculators?

Look at student work in part three:

- How many of your students did not attempt this part? Why do you think they were reluctant to do this part of the task?
- How many of your students still thought the answer was $24?
- How many lost track of the number of reductions (stopped at $13.50 or went too far at $7.59?

In part four, many students found the percent of the sale price to original price instead of the amount saved. What might have helped them to find the amount saved? What do you think they didn't understand?

How do you present percents to students?

Do they frequently work problems in context?

How does context change or add to the complexity of the problem?

Are students familiar with models to help them track the action of the story?
Do they use double number lines or bar models to help them make sense of percents?
Out of all the tools available for finding percents (benchmark numbers, multiplying by decimals, using proportions, etc.), what seemed most comfortable for students?
What are the implications for instruction based on this data?

Looking at Student Work on 25% Sale

Student A is able to reason about part/whole relationships in section one of the task, subtracting the 25% from 100% and then using a proportion to solve for the sale price. In section three the student makes a list of the cost starting at the original price and then after each successive reduction. This helps the student know when the 4 reductions are finished. Notice also that each reduction is clearly sectioned off to make reading easier. That same understanding of part/whole is shown by initial subtraction in part 4.

Student A

In a sale, all the prices are reduced by 25%.

1. Julie sees a jacket that cost $32 before the sale.
   How much does it cost in the sale?

   $ 24 ✓ √1

   Show your calculations.

100-25=75 $\frac{x}{32}=\frac{75}{100}$ 75 32 150 2250 2400 ✓ 100√2400 200 400 24 √ 1

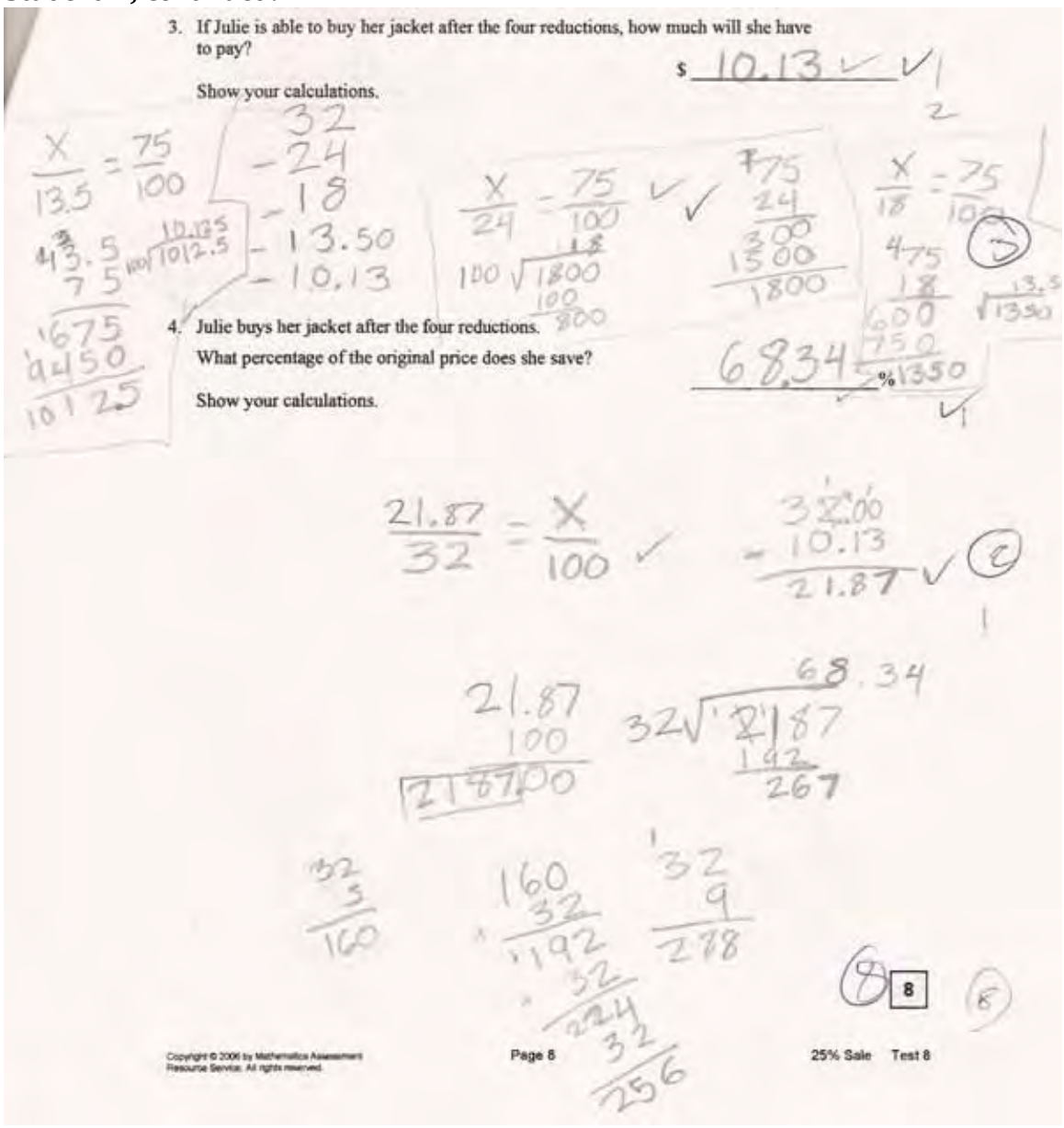
In the second week of the sale, the prices are reduced by 25% of the previous week's price.
In the third week of the sale, the prices are again reduced by 25% of the previous week's price.
In the fourth week of the sale, the prices are again reduced by 25% of the previous week's price.

2. Julie thinks this will mean that the prices will be reduced to $0 after the four reductions because 4 x 25% = 100%.

   Explain why Julie is wrong.

Julie is wrong because she's ✓ reducing 25% of the original price. She should reduce 25% of the already reduced price. 1

Student A, continued:



3. If Julie is able to buy her jacket after the four reductions, how much will she have to pay?

$ 10.13

Show your calculations.

4. Julie buys her jacket after the four reductions.

What percentage of the original price does she save?

68.34 %

Show your calculations.

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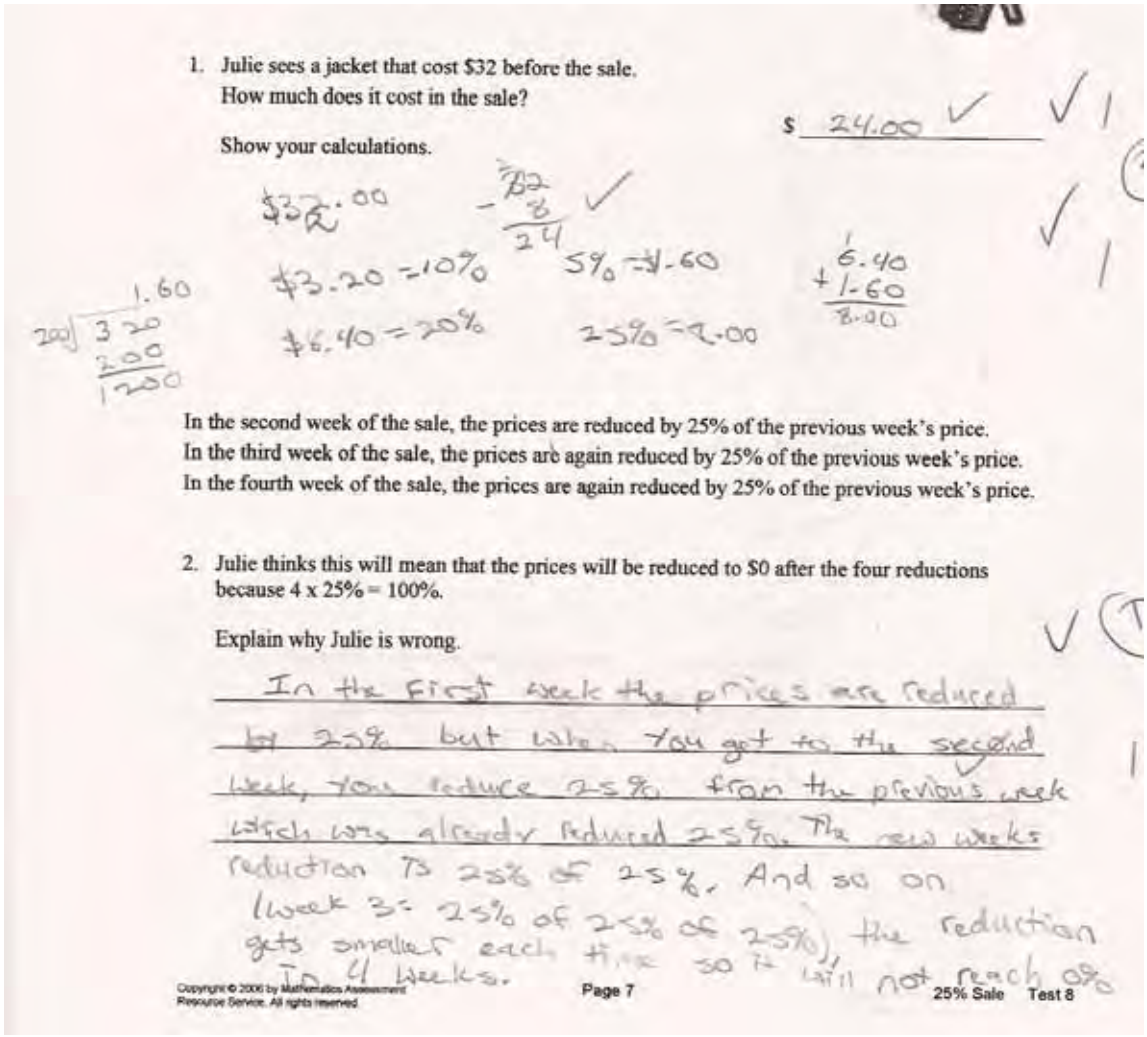
25% Sale Test 8

Student B used benchmark numbers to find 25% in part one and three. The student gives a good explanation for why the jacket is not free in part two. In the final part of the task, the student finds the percent of the new price rather than the percent saved.

Student B



1. Julie sees a jacket that cost $32 before the sale.
How much does it cost in the sale?

Show your calculations.

$ 24.00 ✓ ✓1

$32.00 − 8 = 24 ✓

10% = $3.20 → 1.60, 200 | 320, 200, 1200

5% = $1.60

20% = $6.40

25% = $8.00

$6.40 + 1.60 = 8.00 ✓ 1

In the second week of the sale, the prices are reduced by 25% of the previous week's price.
In the third week of the sale, the prices are again reduced by 25% of the previous week's price.
In the fourth week of the sale, the prices are again reduced by 25% of the previous week's price.

2. Julie thinks this will mean that the prices will be reduced to $0 after the four reductions because 4 x 25% = 100%.

Explain why Julie is wrong.

✓ (1)

In the first week the prices are reduced by 25% but when you get to the second week, you reduce 25% from the previous week which was already reduced 25%. The new weeks reduction is 25% of 25%. And so on (week 3: 25% of 25% of 25%) the reduction gets smaller each time so it will not reach 0% in 4 weeks. 1

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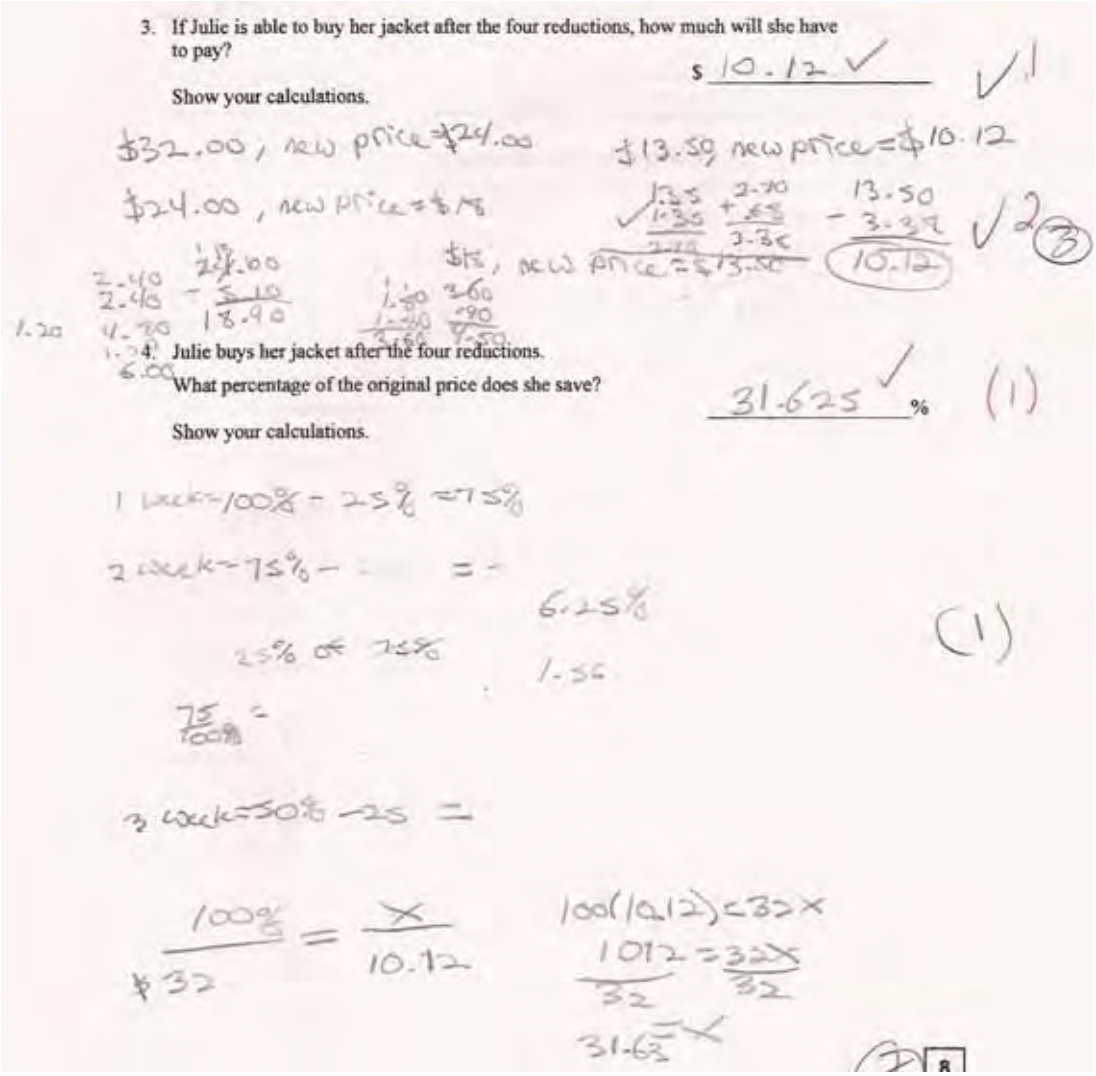
25% Sale Test 8

Student B, continued



3. If Julie is able to buy her jacket after the four reductions, how much will she have to pay?

$ 10.12 ✓ ✓1

Show your calculations.

$32.00, new price $24.00

$24.00, new price = $18

$13.50, new price = $10.12

13.50 − 3.38 = 10.12 ✓2 (3)

4. Julie buys her jacket after the four reductions.

What percentage of the original price does she save?

31.625 ✓ % (1)

Show your calculations.

1 week = 100% − 25% = 75%

2 week = 75% − = =

6.25%

25% of 75%

1.56

(1)

3 week = 50% − 25 =

100% / $32 = x / 10.12

100(10.12) = 32x

1012 = 32x

31.63 = x

(2) 8

Student C uses the fractional equivalent of 25% to solve for the sale price in part 1 and 3. The student does not take enough discounts in part 3 and stops after the 3rd discount. In part 4 the student appears to be using benchmark percents to find the percent of the new price compared to the original price. The student's answer is not correct.

Student C

1. Julie sees a jacket that cost $32 before the sale.
   How much does it cost in the sale?

   $ 24.00 ✓

   Show your calculations.

   32÷4=8 32-8=24 ✓

In the second week of the sale, the prices are reduced by 25% of the previous week's price.
In the third week of the sale, the prices are again reduced by 25% of the previous week's price.
In the fourth week of the sale, the prices are again reduced by 25% of the previous week's price.

2. Julie thinks this will mean that the prices will be reduced to $0 after the four reductions because 4 x 25% = 100%.

   Explain why Julie is wrong.

   Julie is wrong because the second week they don't take 25% off of 32. they take 25% off of 24. Then it is 18, then 25% of 18 is 13.5, ect... ✓

3. If Julie is able to buy her jacket after the four reductions, how much will she have to pay?

   $ 13.50 X

   Show your calculations.

   24, 18, 13.5
   24÷4=6 24-6=18
   18÷4=4.5 18-4.5=13.5 ✓

4. Julie buys her jacket after the four reductions.
   What percentage of the original price does she save?

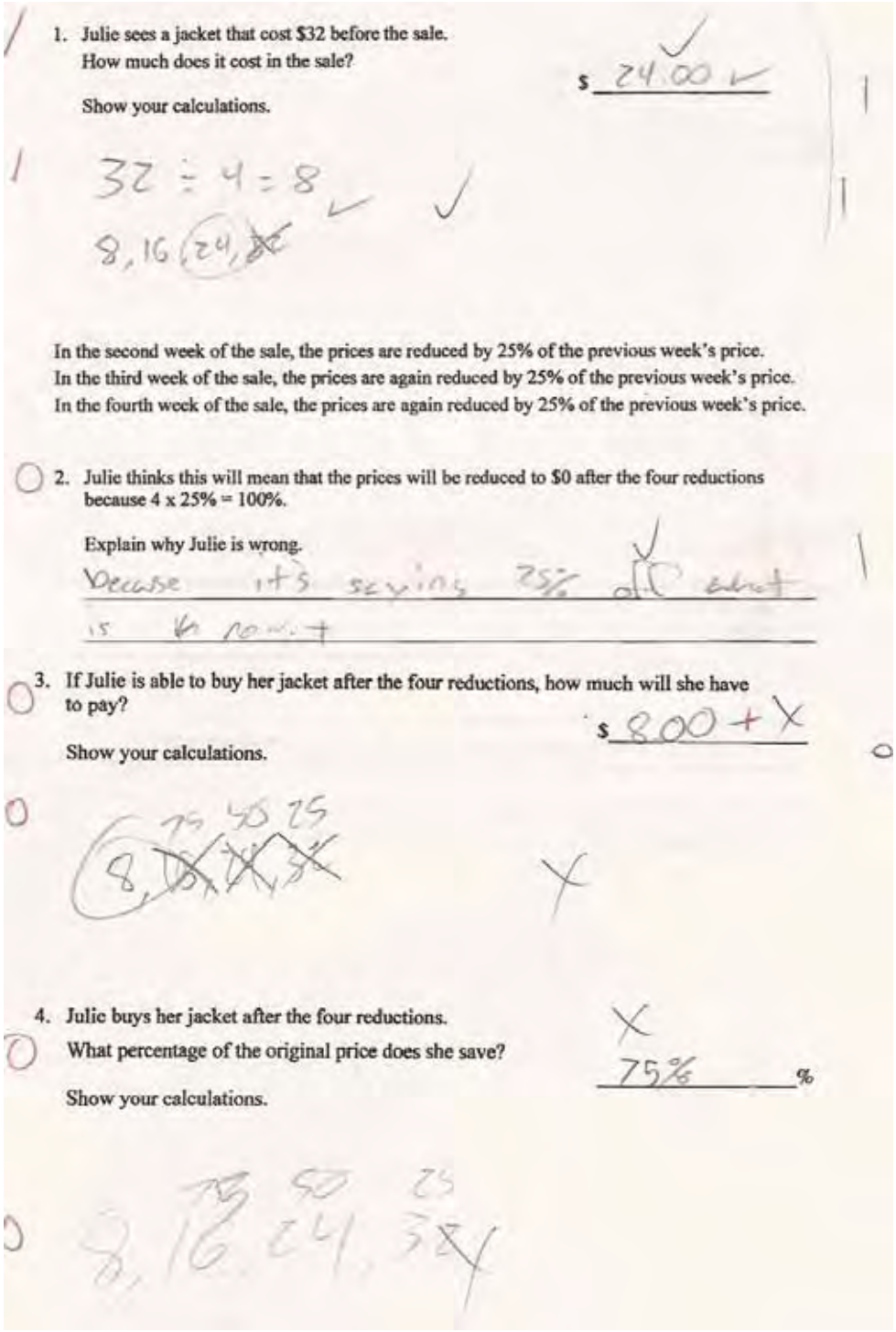
   32.05 X%

   Show your calculations.

   32
   13.5
   .25
   .125
   .0125
   .00125
   .34375 X

Student D understands that 25% is 1/4 of the value. In part two the student seems to understand that 25% is off the "new" price. But in reading further work, the student seems to just keep subtracting 1/4 for each reduction. *If you were writing a hint to this student to stimulate their thinking, what might it be?*

Student D



1. Julie sees a jacket that cost $32 before the sale.
How much does it cost in the sale?

$ 24.00

Show your calculations.

32 ÷ 4 = 8

8, 16, 24, 32

In the second week of the sale, the prices are reduced by 25% of the previous week's price.
In the third week of the sale, the prices are again reduced by 25% of the previous week's price.
In the fourth week of the sale, the prices are again reduced by 25% of the previous week's price.

2. Julie thinks this will mean that the prices will be reduced to $0 after the four reductions because 4 x 25% = 100%.

Explain why Julie is wrong.

Because it's saying 25% of the whole is not now

3. If Julie is able to buy her jacket after the four reductions, how much will she have to pay?

$ 8.00 + X

Show your calculations.

75 50 25

8, 16, 24, 32

4. Julie buys her jacket after the four reductions.
What percentage of the original price does she save?

75 %

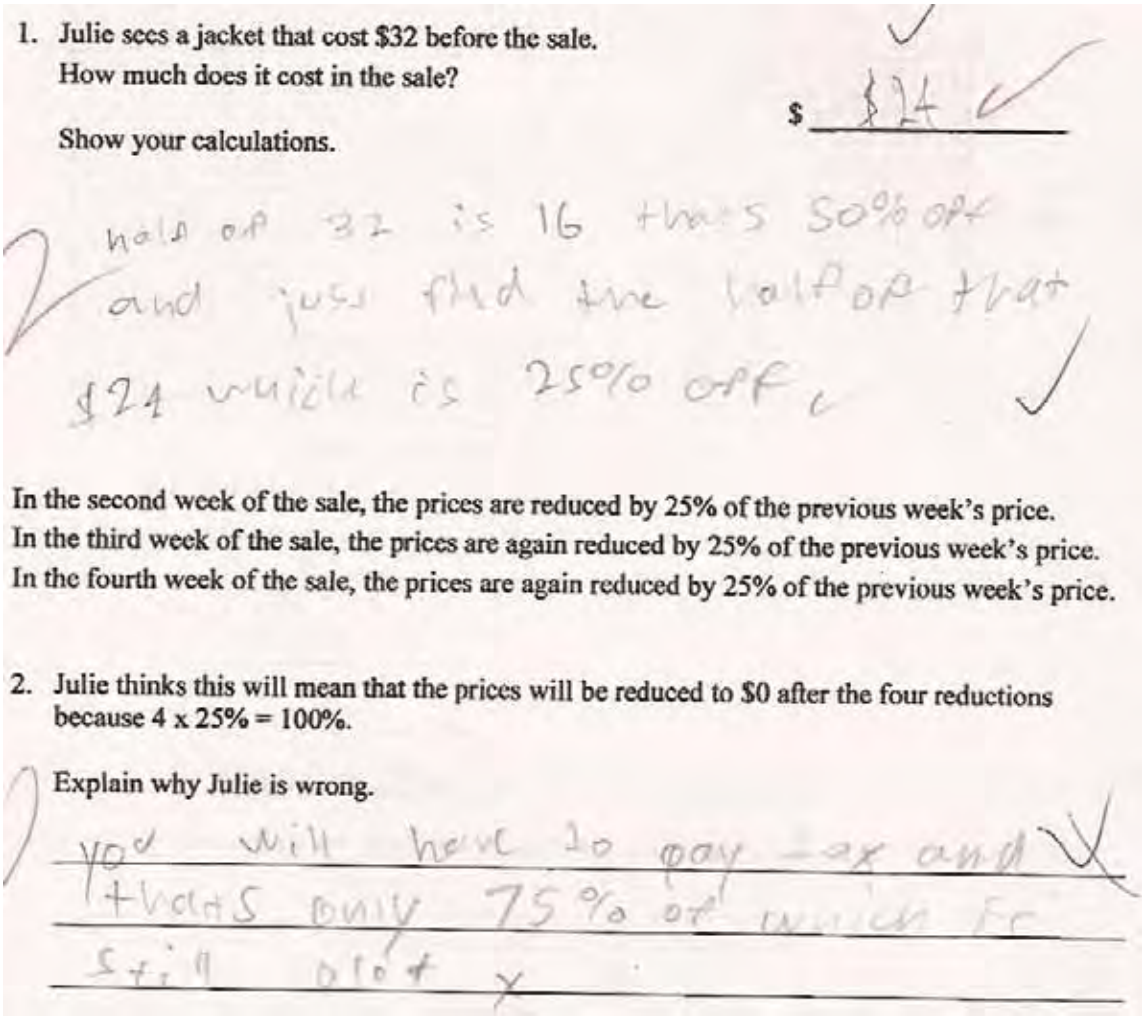
Show your calculations.

75 50 25

8, 16, 24, 32

Student E shows some of the thinking of students with lower scores. The student is able to use an understanding of fractions to find the new price in part 1. The student does not understand the notion of successive reductions in part two and does not attempt part three or four of the task.

Student E



1. Julie sees a jacket that cost $32 before the sale.
   How much does it cost in the sale?

   $ $24 ✓

   Show your calculations.

half of 32 is 16 thats 50% off and just find the half of that $24 which is 25% off ✓

In the second week of the sale, the prices are reduced by 25% of the previous week's price.
In the third week of the sale, the prices are again reduced by 25% of the previous week's price.
In the fourth week of the sale, the prices are again reduced by 25% of the previous week's price.

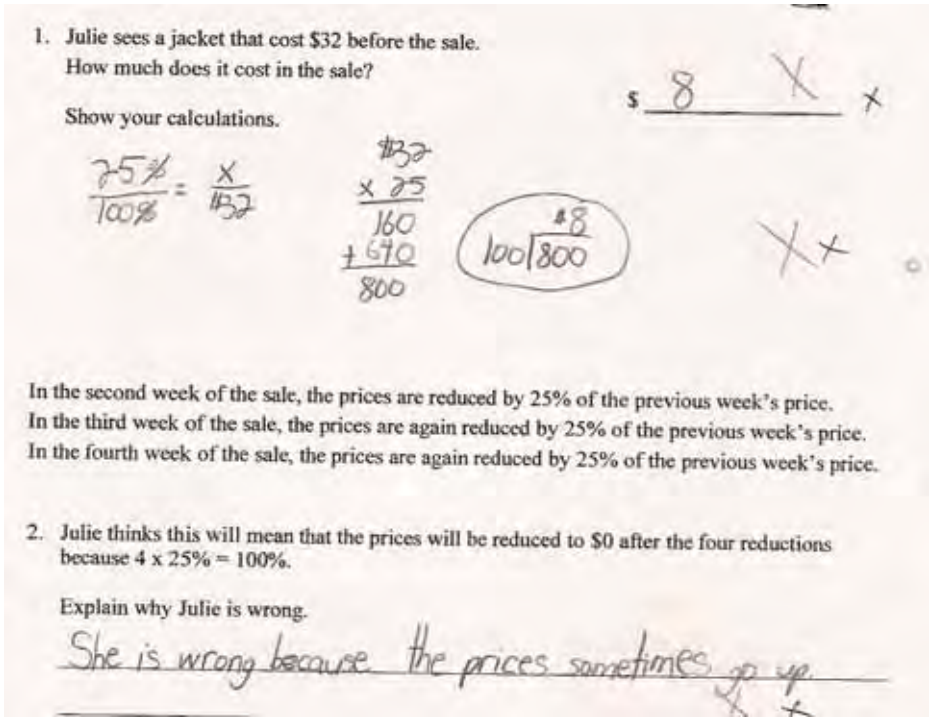
2. Julie thinks this will mean that the prices will be reduced to $0 after the four reductions because 4 x 25% = 100%.

   Explain why Julie is wrong.

you will have to pay tax and thats only 75% of which is still not x

Student F scored no points on the task. The student knew how to set up and solve a proportion, but did not realize that the discount needed to be subtracted from the original price to find the sale price. The student does not use mathematics to solve part two and does not attempt part three and four.

Student F



1. Julie sees a jacket that cost $32 before the sale.
How much does it cost in the sale?

Show your calculations.

$ 8

25% / 100% = x / 32

32 × 25 = 160 + 640 = 800

100 | 800 = 8

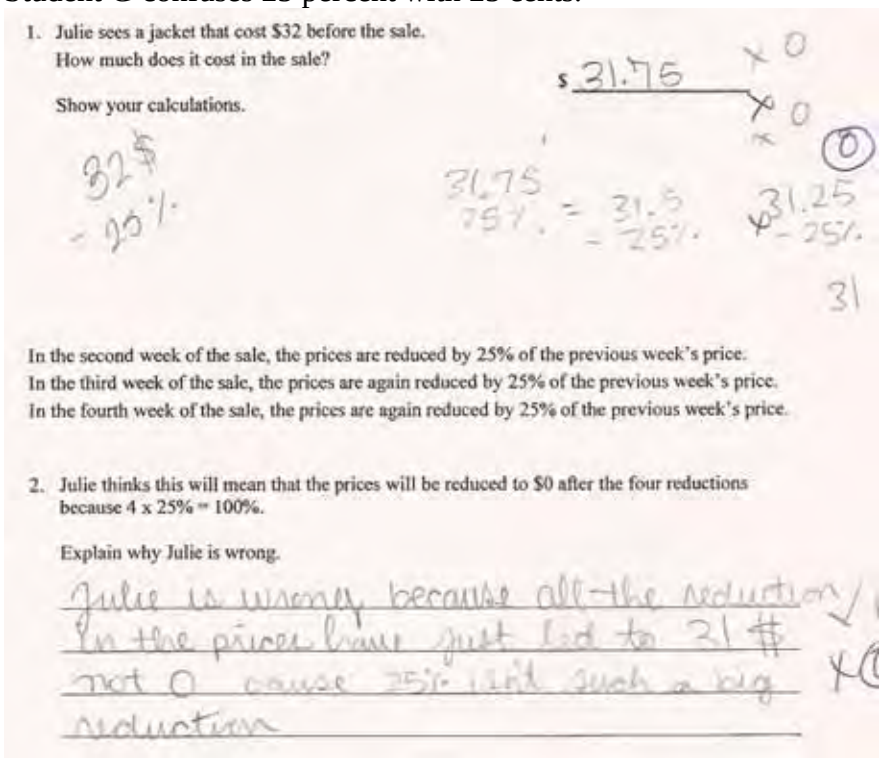
In the second week of the sale, the prices are reduced by 25% of the previous week's price.
In the third week of the sale, the prices are again reduced by 25% of the previous week's price.
In the fourth week of the sale, the prices are again reduced by 25% of the previous week's price.

2. Julie thinks this will mean that the prices will be reduced to $0 after the four reductions because 4 x 25% = 100%.

Explain why Julie is wrong.

She is wrong because the prices sometimes go up

Student G confuses 25 percent with 25 cents.



1. Julie sees a jacket that cost $32 before the sale.
How much does it cost in the sale?

Show your calculations.

$ 31.75

32 − 25¢

31.75 75% = 31.5 − 25%

31.25 − 25%

31

In the second week of the sale, the prices are reduced by 25% of the previous week's price.
In the third week of the sale, the prices are again reduced by 25% of the previous week's price.
In the fourth week of the sale, the prices are again reduced by 25% of the previous week's price.

2. Julie thinks this will mean that the prices will be reduced to $0 after the four reductions because 4 x 25% = 100%.

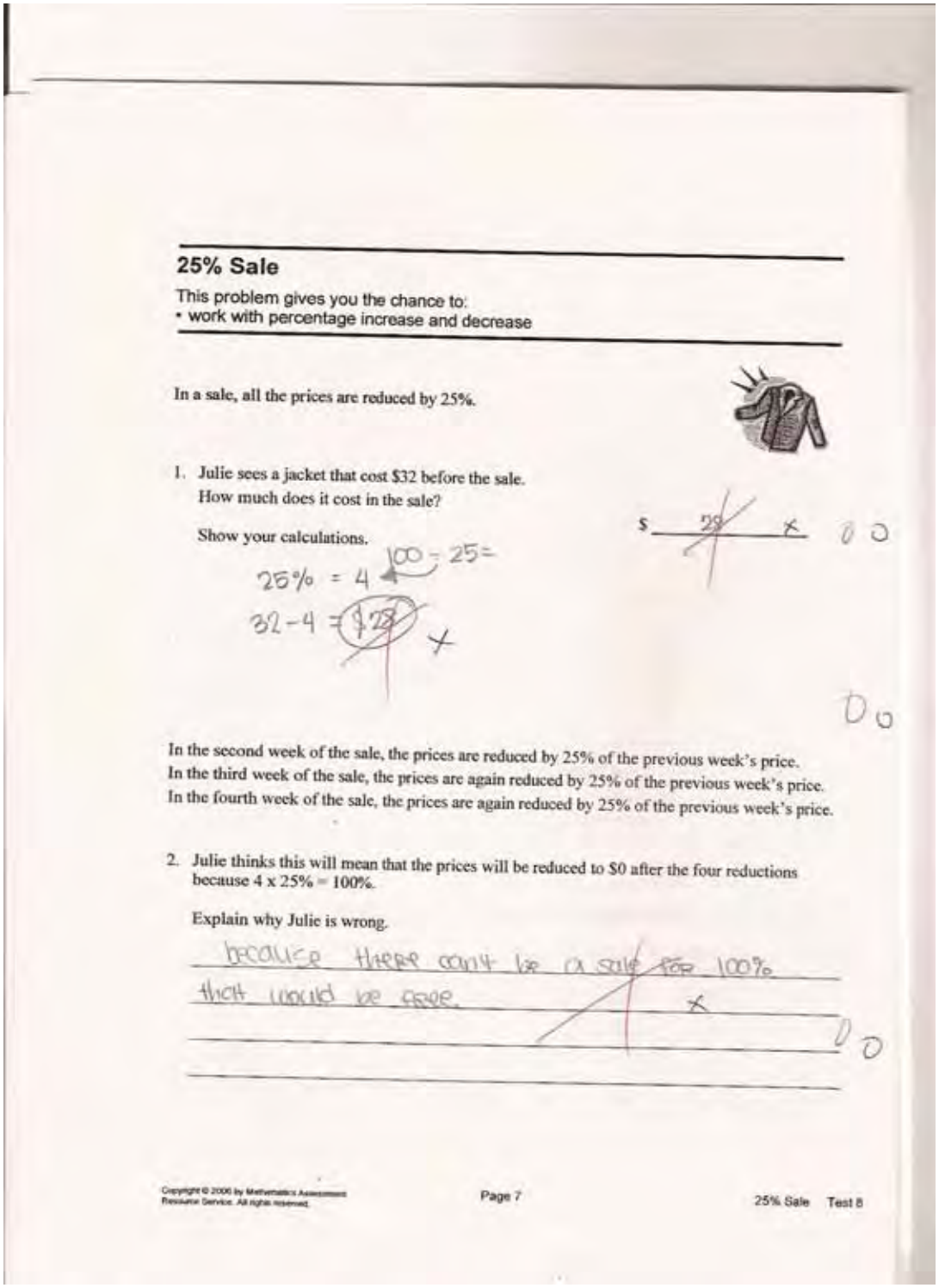
Explain why Julie is wrong.

Julie is wrong because all the reductions in the prices have just led to 31 ✓
not 0 cause 25¢ isn't such a big ✗
reduction

Thinking stays consistent with a new price of $31 in part 3.

Student H knows that 25 goes into 100 four times. But confuses the meaning of that four, which represents 1/4, with an absolute value of $4. Like many students working at this lower level, the work or logic of thinking is not consistent within the various parts of the task.

Student H



25% Sale

This problem gives you the chance to:
- work with percentage increase and decrease

In a sale, all the prices are reduced by 25%.

1. Julie sees a jacket that cost $32 before the sale.
   How much does it cost in the sale?

   Show your calculations.

   25% = 4 100 ÷ 25 =
   32 − 4 = $28

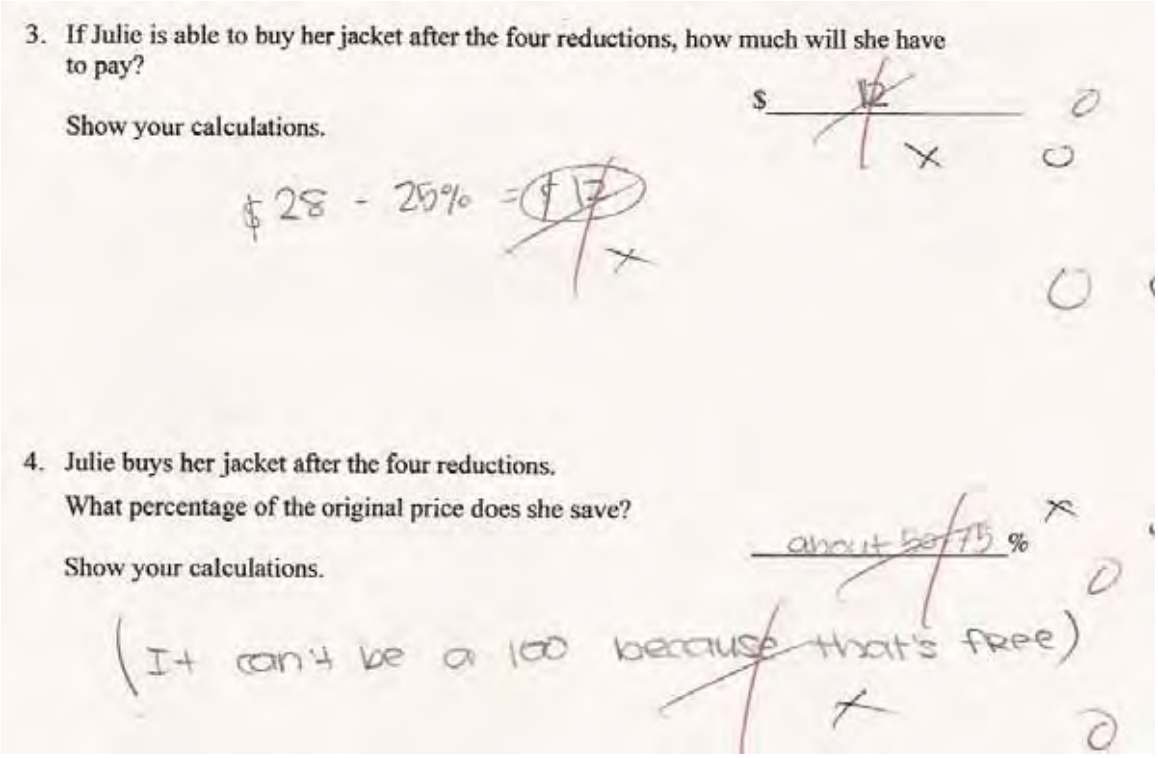
In the second week of the sale, the prices are reduced by 25% of the previous week's price.
In the third week of the sale, the prices are again reduced by 25% of the previous week's price.
In the fourth week of the sale, the prices are again reduced by 25% of the previous week's price.

2. Julie thinks this will mean that the prices will be reduced to $0 after the four reductions because 4 x 25% = 100%.

   Explain why Julie is wrong.

   because there can't be a sale for 100%
   that would be free.

Student H, continued:



3. If Julie is able to buy her jacket after the four reductions, how much will she have to pay?

   Show your calculations.

$ 12

$28 - 25% = $17

4. Julie buys her jacket after the four reductions.

   What percentage of the original price does she save?

about 50 75 %

   Show your calculations.

(It can't be a 100 because that's free)

Eighth Grade

8th Grade Task 4 25% Sale

Student Task	Work with percentage increase and decrease in the context of a sale. Develop a mathematical argument on the effects of decreasing price by 25% four times.
Core Idea 1 Number and Operation	• Work flexibly with fractions, decimals, and percents to solve problems.
Core Idea 2 Mathematical Reasoning	• Formulate conjectures and test them for validity • Verify and interpret results of a problem

Based on teacher observation, this is what eighth graders knew and were able to do:

- Find the 25% discount
- Find the sale price
- Reason about how successive reductions were based on the new price

Areas of difficulty for eighth graders:

- Calculating multiple reductions
- Organizing work
- Computing in multiplication, division, and subtraction accurately
- Understanding the difference between *percentage saved* and *percentage of original cost*

Strategies used by successful students:

- Using proportions
- Converting percents to decimals and multiplying
- Using benchmark percents
- Labeling work

MARS Test Task 4 Frequency Distribution and Bar Graph, Grade 8

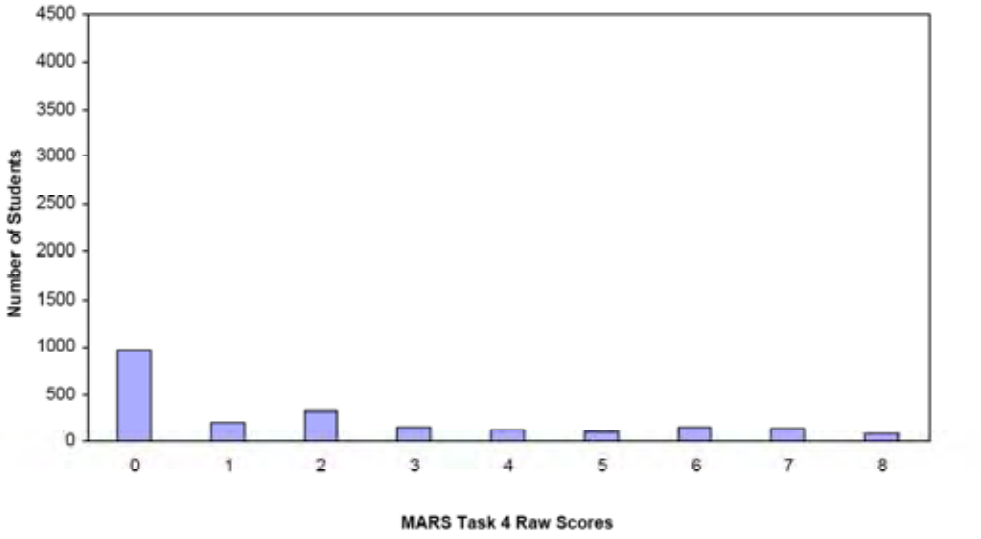
Task 4 - 25% Sale

Mean: 2.16 StdDev: 2.56

Table 43: Frequency Distribution of MARS Test Task 4, Grade 8

Task 4 Scores	Student Count	% at or below	% at or above
0	972	43.5%	100.0%
1	202	52.5%	56.5%
2	321	66.9%	47.5%
3	153	73.7%	33.1%
4	115	78.9%	26.3%
5	94	83.1%	21.1%
6	148	89.7%	16.9%
7	140	96.0%	10.3%
8	90	100.0%	4.0%

Figure 52: Bar Graph of MARS Test Task 4 Raw Scores, Grade 8



The maximum score available for this task is 8 points.
The minimum score for a level 3 response, meeting standards, is 4 points.

More than half the students, 56%, could discuss why taking 25% off 4 times did not give a price of "free". A little less than half the students, 47%, could find the discount in part one and use that to calculate the sale price. Some students, 26%, could find the sale price, talk about 25% as being on a new price for each reduction, and calculate the first 3 reductions. 17% of the students could find the new price after all 4 reductions. 4% of the students could meet all the demands of the task, including finding the percentage saved after the reductions. Almost 44% of the students scored no points on this task. 80% of the students with this score attempted the task.

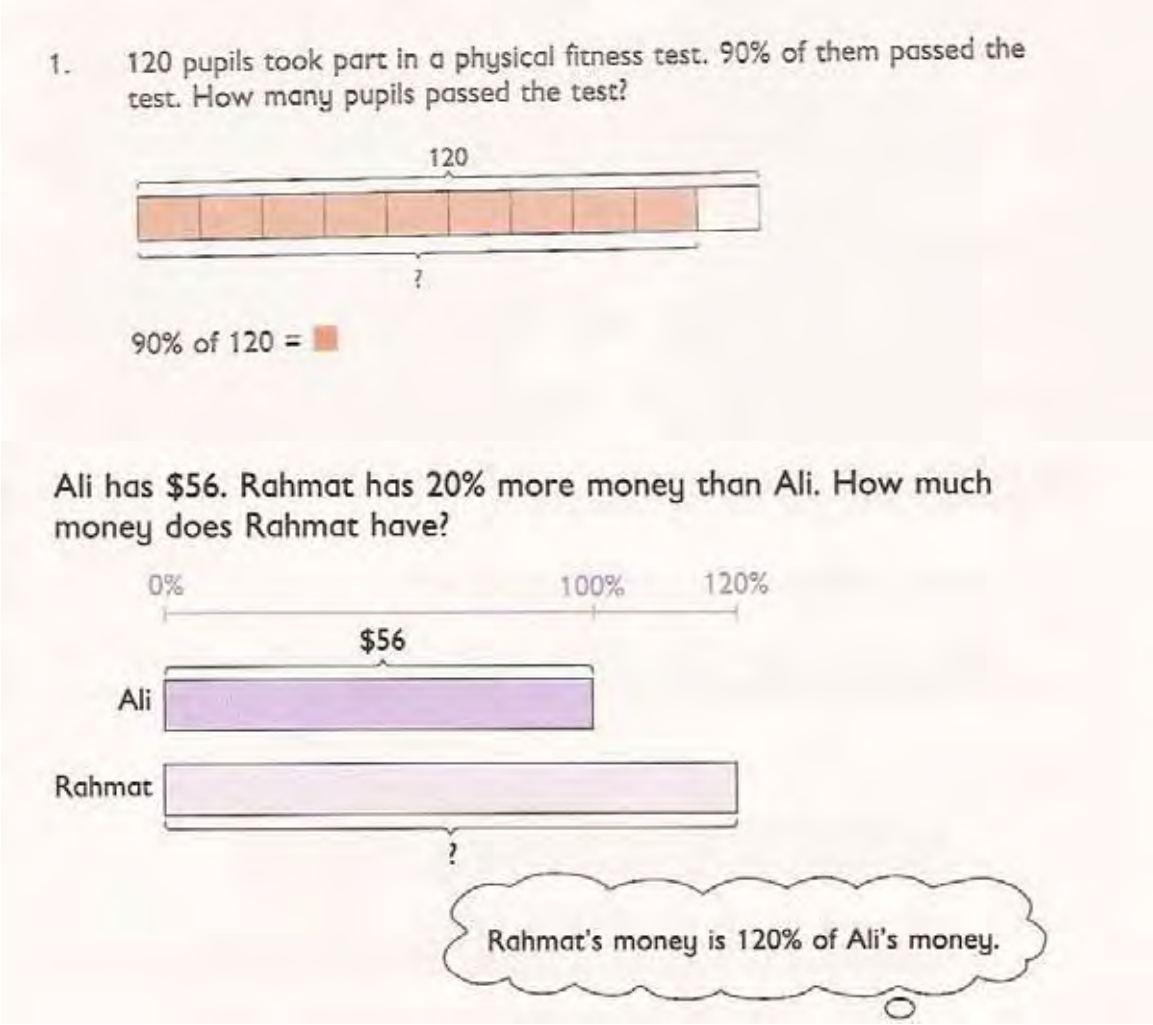
25% Sale

Points	Understandings	Misunderstandings
0	80% of the students with this score attempted the task.	12% of the students calculated the 25% reduction, but then didn't find the sale price. 3% thought that 25% was $4. 3% confused percent with cents and subtracted $0.25.
1	Students could explain that each 25% needed to come off the new price.	9% of all students did not attempt this part of the task. 7% thought that things can't be free.
2	Students could calculate the discount and find the sale price.	They struggled with the 4 successive reductions. 13% of the students did not attempt this part of the task. 3% thought the answer was still $24. 6% made calculation errors in multiplication. 3% thought the price was still $32. 2% thought it would be free.
4	Students could find the sale price, talk about 25% as being on a new price for each reduction, and calculate the first 3 reductions.	20% of the students did not attempt to find the percentage saved. 5% thought the savings was 75%. 13% found the percentage of the new price to the original price.
6	Students could find the sale price, talk about 25% as being on a new price for each reduction, calculate the first 3 reductions, and find the cost after 4 reductions.	
8	Students could find the sale price, talk about 25% as being on a new price for each reduction, calculate the first 3 reductions, and find the cost after 4 reductions. Students could calculate the percentage saved.	

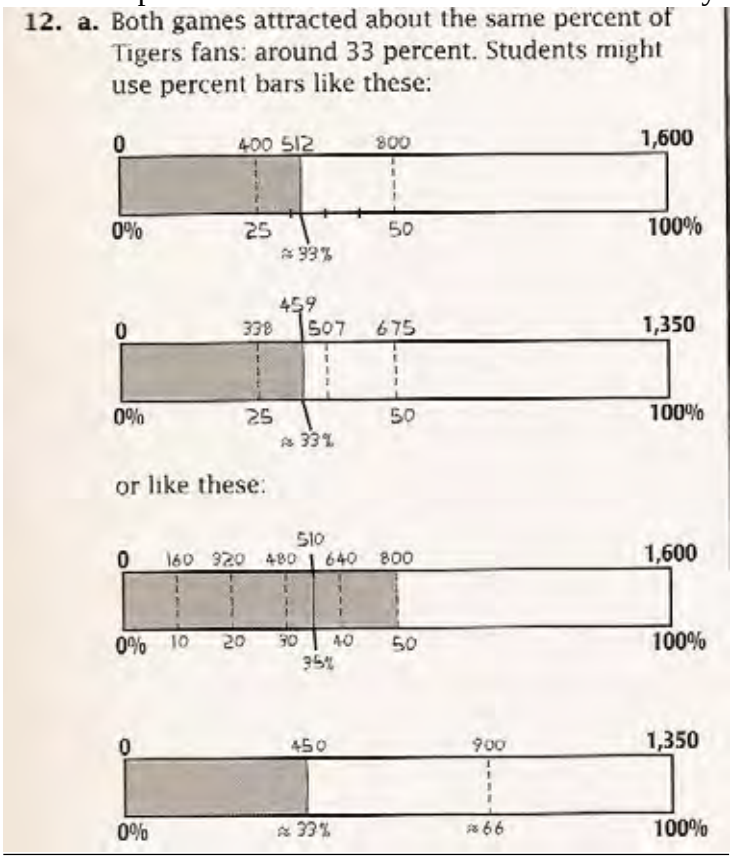
Implications for Instruction

Students at this grade level should have a variety of strategies for finding percents. Some benchmark percents, like 50%, 25%, 33 1/3%, or 20%, should be readily convertible to fraction and decimal equivalents. At this grade level, students should be able to take simple percentages as mental math problems and easily be able to solve them in math talks. Consider a problem like 65% of 84. Students might be able to think that 10% is 8.4. So 60% is 48 + 2.4 = 50.4. Then 5% would be 4.2. Therefore the final answer would be 54.6. Students might also think that 1/4 or 25% of 84 would be 21. So, 75% would be 63. If 10% is 8.4, then 65% would be 63 – 8 = 55 and 55 – 0.4 = 54.6. Frequent experiences with these types problems helps students to build a sense of percents and a basic idea about relative size. This is a way to build a good understanding of the concept before reaching for more efficient strategies. Combining number talks with models such as the double bar model or double line model help students to see percents as a measuring or dividing up of a quantity and gives them a tool for thinking about the action of a problem.

Here are examples of bar models to help students think about percents, as shown in the Primary Mathematics Series from Singapore:



Here is an example of a double line model being used for estimation, but it could easily be adapted to finding exact answers or for showing student thinking in Number Talks. This example is from Mathematics in Context Series by Britannica, Per Sense:



Students at this level should have a variety of strategies for calculating percents. While proportions make sense and seem easiest to adults. Research suggests that this strategy is the most difficult for students to understand. It doesn't connect to mental math, intuitive thinking, or estimation. Students at this grade level do better with relating percents to fractions and decimals. So, it is important to give students a variety of ways to grasp the big ideas of percents.

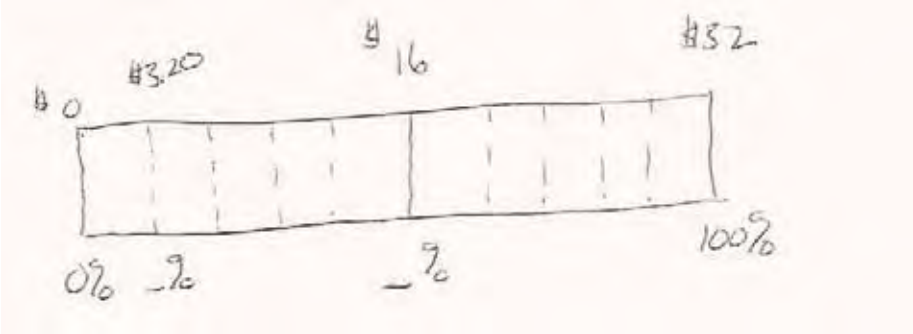
Ideas for Action Research

Number Talks

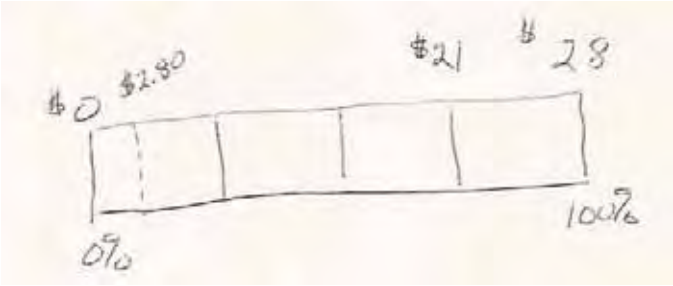
Design a series of number talks dealing with percents. Start with friendly numbers, but build to more challenging examples like the 65% of 84 above. As students do the number talks, try introducing some of the models to illustrate their thinking and see if students start to use it on their own. Another approach might be to show a model for solving a problem and ask students to work in pairs to explain why it works. For example,

Samantha and Freddy were shopping for a present for their mother. The sign for the picture frame Samantha liked was on sale for 45% of the original price, $32. The sign for the picture frame Freddy liked said the price was reduced 35% from the original price of $28.

Samantha:



Fred:



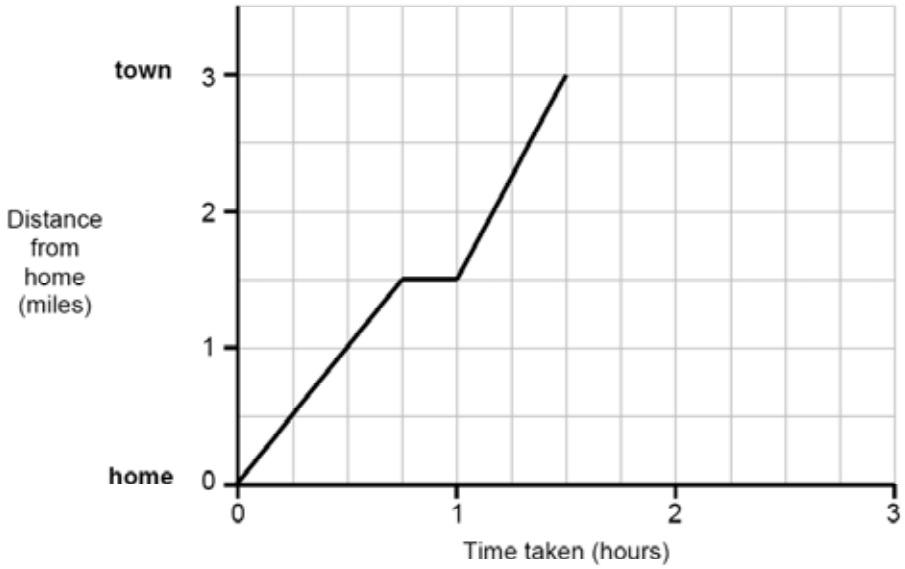
Can you figure out what Samantha and Freddy were doing? What do they need to do to find the to finish finding the price of the picture frame? Whose frame was cheaper?

After working with the number talks for a week or two, what do you notice that is different about students' approaches to problems? How do you think their big idea about understanding percents has changed or deepened?

Going to Town

This problem gives you the chance to:
• interpret and complete a distance/time graph for a described situation

Craig and James walk from home to town during the school holidays.
The distance/time graph below shows their journey into town.
They set off from home at 10:30 a.m.



1. At what speed did they walk for the first part of their journey?

   _____ miles per hour

   Show how you figured it out.

2. What do you think they do after they have travelled $1\frac{1}{2}$ miles?

3. After an hour the graph is steeper. What does this tell you about Craig and James's walking speed?

Craig and James stay in town for an hour and then catch a bus home.
The bus averages 12 miles per hour.

4. At what time did they get home? _____
Show how you figured it out.

5. Continue the graph on the previous page to show the rest of this information.

8

Going to Town	Rubric	
The core elements of performance required by this task are: • interpret and complete a distance/time graph for a described situation Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives a correct answer: **2** mph Shows correct work such as: 1 1/2 miles in 3/4 hour equals 2 miles in an hour, or uses the line on the graph	1 1	2
2. Gives a correct answer such as: Stop for a rest.	1	1
3. Gives a correct answer such as: They walk faster.	1	1
4. Gives a correct answer: **1:15** p.m**.** or **13:15** Shows correct calculations such as: 3/12 = 1/4 3/4 + 1/4 + 1/2 + 1 + 1/4 = 2 3/4 10:30 plus 2 3/4 hours = 13:15	1 1	2
5. Draws correct graph: Horizontal line for 1 hour. Line down to axis taking 1/4 hour.	1 1	2
Total Points		**8**

Going to Town

Work the task and look at the rubric. What are the big mathematical ideas being assessed in this task?

What strategies might a student use to find the speed in part one of the task? Look at student work in part one.

- How many of your students thought the speed was 1 1/2 miles per hour? How do you think they got that answer?
- How many of your students thought the speed was 3 mph? What might they have been thinking?
- How many of your students thought the speed was 1 mph?

Look at student work for part two. How many of your students thought Craig and James:

Rested or stopped	Went faster	Kept walking	Went at a constant speed	Slowed down	Went straight

What do these errors show that students are not understanding about graphs? the difference between a graph and the action of a story?

Now look at student work in part 4 and 5. Were your students able to:

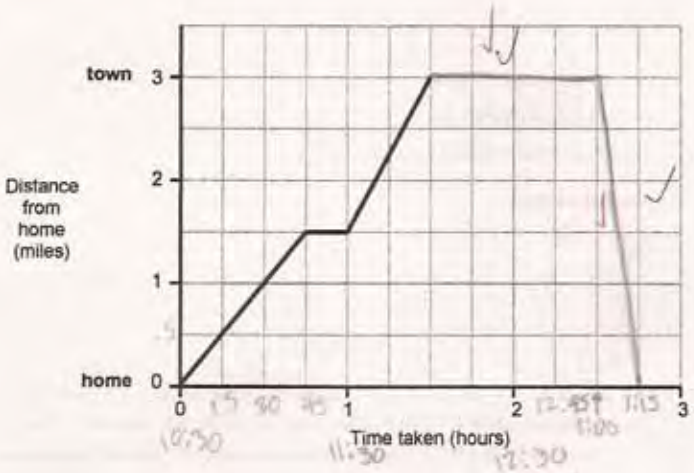
- Graph the hour for staying in town?
- Graph the trip home? How many of your students continued the graph in an upward direction? How many of your students tried to connect the graph back to the origin? How many of your students had difficulty finding the slope for the bus travel?
- How many of your students could calculate the time for the bus trip?
- Did your students have difficulty converting measurements of time?

What are the implications for instruction?

Looking at Student Work on Going to Town:

Student A does a good job of labeling the graph to show how it was used in solving different parts of the task. The student has a clear understanding of rate.

Student A



1. At what speed did they walk for the first part of their journey?

2 miles per hour

Show how you figured it out.

1 · 2 = 2
½ · 2 = 1 hr

they walked 1 mile in ½ an hour, so to make it one whole mile we multiply by 2.

2. What do you think they do after they have travelled $1\frac{1}{2}$ miles?

I think they take a break and stop walking for 15 min.

3. After an hour the graph is steeper. What does this tell you about Craig and James's walking speed?

They are walking faster.

Craig and James stay in town for an hour and then catch a bus home.
The bus averages 12 miles per hour.

4. At what time did they get home?
Show how you figured it out.

1:15

3 mi to home

12 mi per hr (bus)

12 ÷ 3 = 4
it will take them ¼ hr to get home

5. Continue the graph on the previous page to show the rest of this information.

Student B is able to use the graph to find the speed in part one and can calculate the time for the trip home. The student forgets about the 1 hour rest when drawing the graph and gives two different versions of the bus trip home. *What is the student not understanding or confused about the bus trip home? How is the graph different a telling a story of the situation?*

Student B

Craig and James walk from home to town during the school holidays.
The distance/time graph below shows their journey into town.
They set off from home at 10:30 a.m.

1. At what speed did they walk for the first part of their journey?

   2 miles per hour

   Show how you figured it out.

2. What do you think they do after they have travelled $1\frac{1}{2}$ miles?

   I think they may have stopped and gotten some food or something then had more energy for the second half.

3. After an hour the graph is steeper. What does this tell you about Craig and James's walking speed?

I tells me that it increased from 2 miles per hour to 3 miles per hour.

Craig and James stay in town for an hour and then catch a bus home.
The bus averages 12 miles per hour.

4. At what time did they get home? 1:15
Show how you figured it out.

1:00 when they leave for home
+0:15
1:15

Bus
12 mph, and 3 miles from home
12 ÷ 4 = 3, 60 min (1 hour) ÷ 4 = 15

Finding the rate was difficult for many students. Student C is able to use the graph to make sense of the rate in part 1. Although the student calculates the time for the bus trip correctly, the student continues the graph upward for the trip home. *What part of the graph is confusing the student? What is being misunderstood?*

Student C

1. At what speed did they walk for the first part of their journey?

2 miles per hour

Show how you figured it out.

in one hour the would have gone 2 miles if they hadn't stoped

Student D calculates the speed as 3 mph. *Can you think about how the student got the numbers 15 and 5?*

Student D

1. At what speed did they walk for the first part of their journey?

3x miles per hour

Show how you figured it out.

$5\overline{)15}$ = 3

2. What do you think they do after they have travelled $1\frac{1}{2}$ miles?

They slowed down

Many students, like Student E and F, got a speed of 1.5 mph. *Can you explain how student E got his answer? How do you think Student F used the graph to get her answer?*

Student E

1. At what speed did they walk for the first part of their journey?

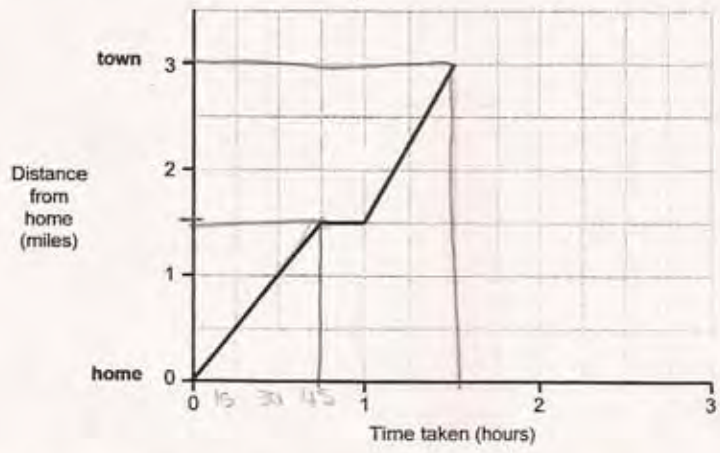
1.5 miles per hour

Show how you figured it out.

I looked at the graph, so since they took a 15minute break, they ended up going 1.5 mph.

Student F

They set off from home at 10:30 a.m.



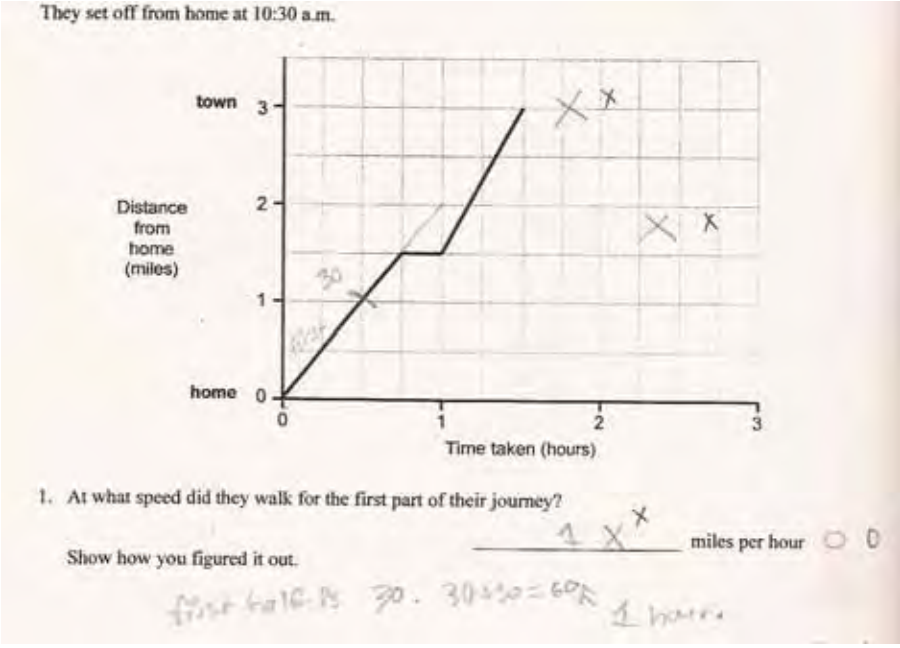
1. At what speed did they walk for the first part of their journey?

$1\frac{1}{2}$ miles per hour

Show how you figured it out.

Student G also uses the graph to help solve the problem. *What would have helped the student get from 30 to 2 mph? What does that student not understand?*

Student G


They set off from home at 10:30 a.m.

town
home
Distance from home (miles)
Time taken (hours)

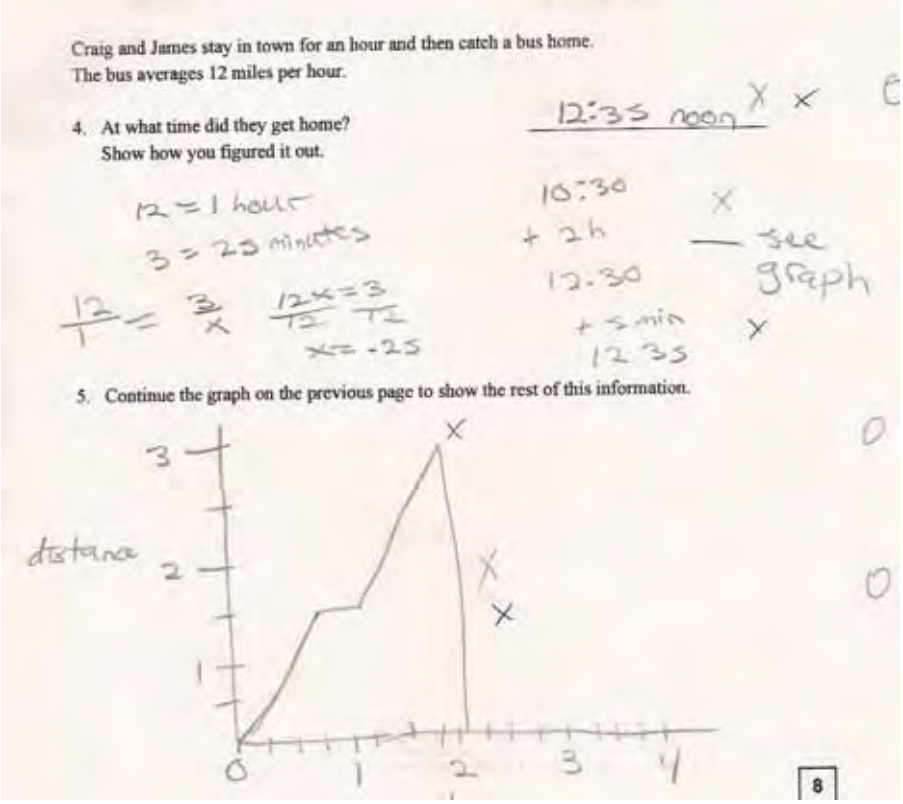
1. At what speed did they walk for the first part of their journey?

Show how you figured it out.

1 miles per hour

Student H has difficulty going from speed to time in part four. *What does the .25 really represent? What kind of hint might help this student?*

Student H


Craig and James stay in town for an hour and then catch a bus home.
The bus averages 12 miles per hour.

4. At what time did they get home?
Show how you figured it out.

12:35 noon

12 = 1 hour
3 = 25 minutes
12/1 = 3/x
12x = 3
x = .25

10:30
+ 2h
12:30
+ 5 min
12 35

see graph

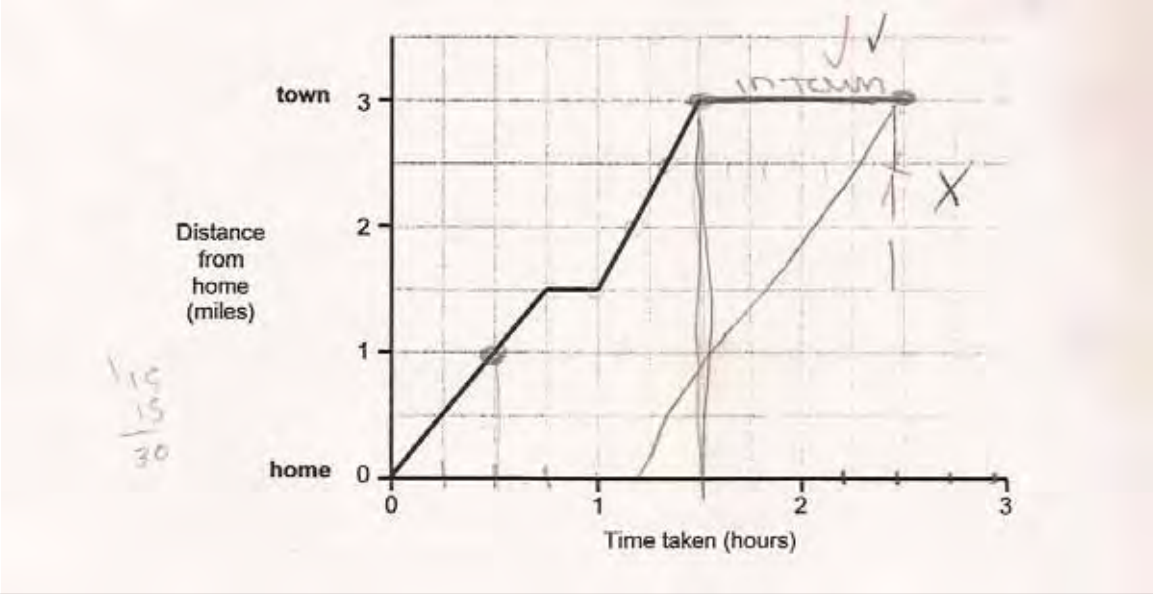
5. Continue the graph on the previous page to show the rest of this information.

distance

8

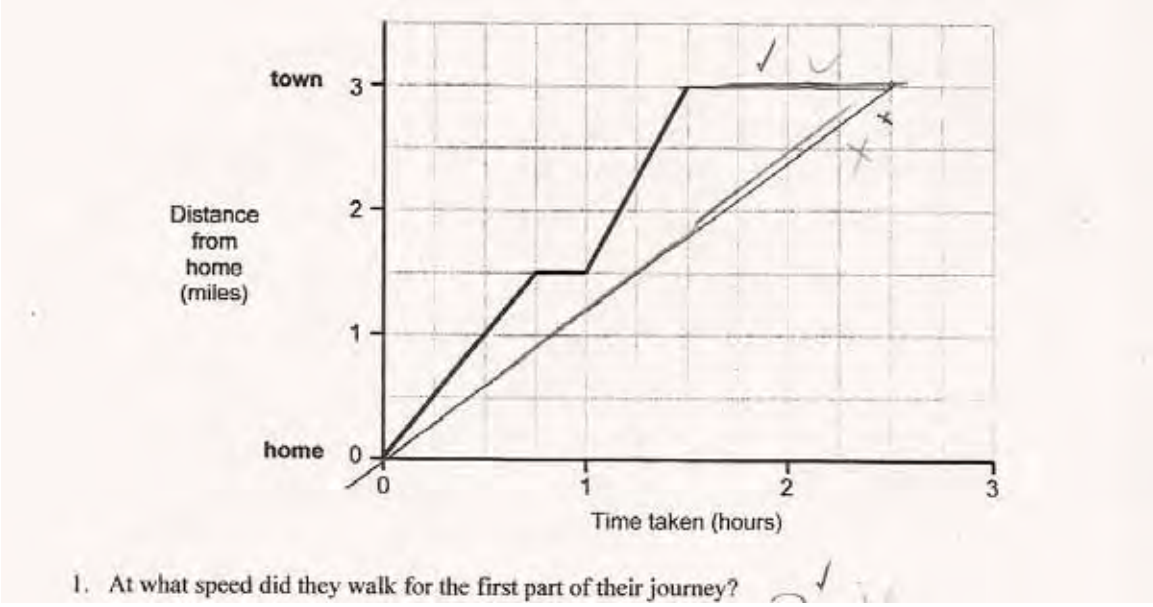
Many students were confused between the action of a story and the graph of the time and distance. Student J shows a graph with the line going toward home, which actually represents time travel or going back in time. The student thought the 12 mph meant 12 minutes, so there is also a problem with understanding scale.

Student J



Student K actually knew that the bus trip lasted 15 minutes and that the ending time was 1:15, but equates starting time with the origin rather than the distance from home.

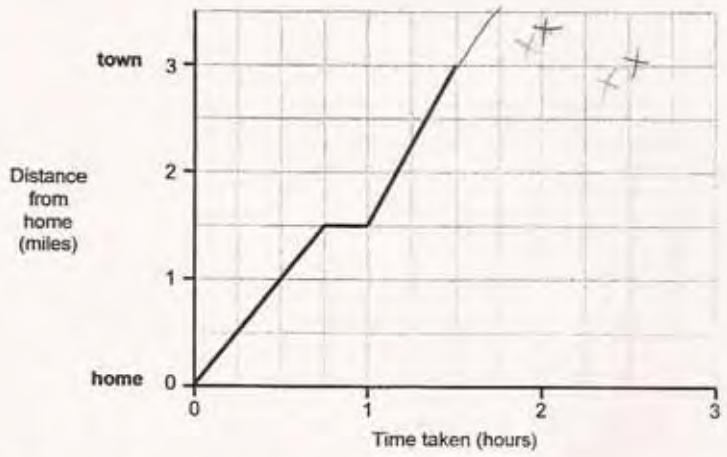
Student K



Student L is an example of a student who does not make sense of graphing or even basic number operations. Look at part one and three. *Can you tell where the numbers come from? How might you help this student?*

Student L

They set off from home at 10:30 a.m.



1. At what speed did they walk for the first part of their journey?

1090 miles per hour

Show how you figured it out.

10 30 (60) = 1090

2. What do you think they do after they have travelled $1\frac{1}{2}$ miles?

I think they walk home.

3. After an hour the graph is steeper. What does this tell you about Craig and James's walking speed?

I think the graph got steeper because probably Craig and James walked a lot more which increased the distance from home (miles)

Craig and James stay in town for an hour and then catch a bus home.
The bus averages 12 miles per hour.

4. At what time did they get home?

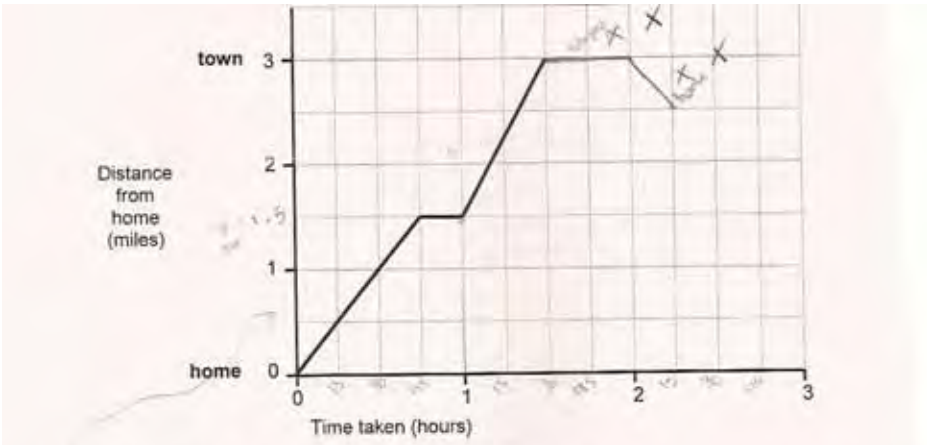
7 8 30

Show how you figured it out.

12 (3) (2) (1) = 7 8 30

Student M is able to label the scale on the graph, but has difficulty maintaining the idea of scale when making the graph for part 4. Also notice the work for calculating time in part four. *What is the student misunderstanding about combining units of time?*

Student M



1. At what speed did they walk for the first part of their journey?

1.5 miles per hour

Show how you figured it out.

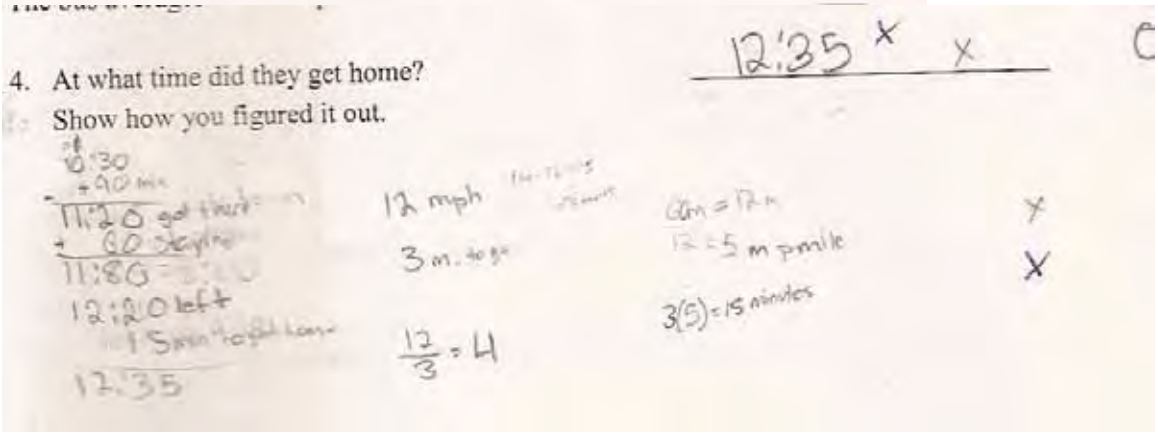
2. What do you think they do after they have travelled $1\frac{1}{2}$ miles?

After they've traveled 1.5 miles I think they start walking faster because to get to 3 miles it only took them 30 minutes

4. At what time did they get home?

12:35

Show how you figured it out.



Eighth Grade

8th Grade Task 5 Going to Town

Student Task	Interpret and complete a distance/time graph for a described situation. Work with rates in the context of slope.
Core Idea 3 Algebra and Functions	**Use models to solve problems involving quantity and change.** • Identify and describe situations with constant or varying rates of change and compare them. • Use proportional reasoning
Core Idea 2 Mathematical Reasoning	**Employ forms of mathematical reasoning and justification appropriately to the solution of a problem.**

Based on teacher observation, this is what eighth graders knew and were able to do:

- Recognize and graph a stop or rest break on a time/distance graph
- Interpret slope as faster or slower

Areas of difficulty for eighth graders:

- Using a graph to calculate slope or speed
- Combining units of time
- Interpreting scale on a graph
- Working from mph to time for a journey

MARS Test Task 5 Frequency Distribution and Bar Graph, Grade 8

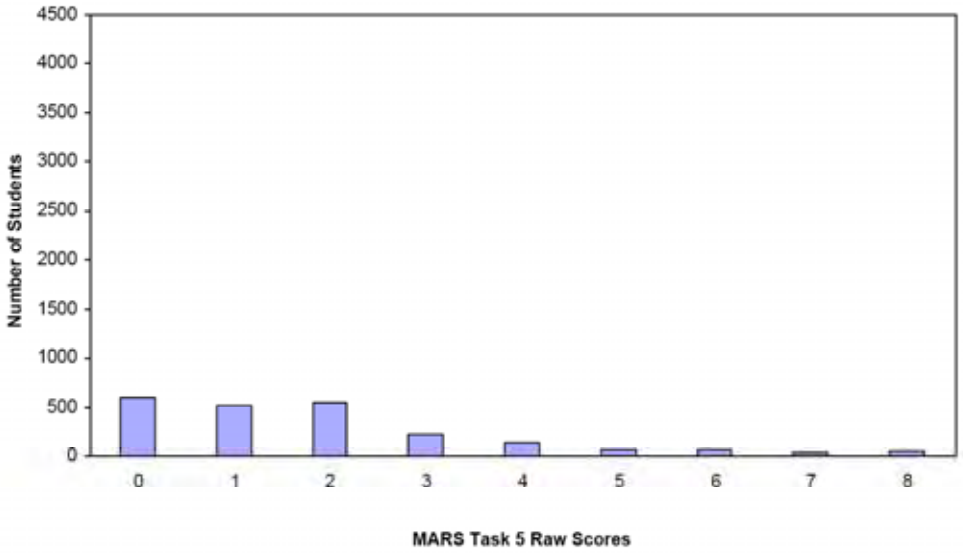
Task 5 - Going to Town

Mean: 1.86 StdDev: 1.85

Table 44: Frequency Distribution of MARS Test Task 5, Grade 8

Task 5 Scores	Student Count	% at or below	% at or above
0	596	26.7%	100.0%
1	516	49.8%	73.3%
2	545	74.1%	50.2%
3	221	84.0%	25.9%
4	142	90.4%	16.0%
5	75	93.7%	9.6%
6	71	96.9%	6.3%
7	26	98.1%	3.1%
8	43	100.0%	1.9%

Figure 53: Bar Graph of MARS Test Task 5 Raw Scores, Grade 8



The maximum score available on this task is 8 points.
The minimum score for a level 3 response, meeting standards, is 3 points.

Many students, 73%, could reason about the slope and increased speed in part three of the task. Half the students could reason about a horizontal line on a time/distance graph as a stop or rest and about the relationship between slope and speed. Only 1/4 of the students could also graph a stop of 1 hour on the graph. Less than 20% of the students could calculate the speed for part one. Only 2% of the students met all the demands of the task, including calculating speed and using speed to calculate time, graphing information about time and distance, and reasoning about slope in relationship to the context. Almost 27% of the students scored no points on this task. 50% of the students with this score attempted the task.

Implications for Instruction

Students should be able to determine rates from looking at time/distance graphs by using division or extending lines to make unit rates. Students should be ale to think about the relationship between the steepness of the slope and the speed of travel. Common misconceptions include thinking that a flat or horizontal line represents a constant speed rather than a speed of zero or that the horizontal line represents the action "going in a straight line". Students also confuse steepness of the graph with going up a hill. Students had difficulty with the distance scale, confusing it with how you would read a bar graph; so each intersection on the graph equals a new distance traveled. Students also confused the distance scale with speed. Students need opportunities to make their own graphs to help them reason about the logic of a graph. A good exercise is to give them a story and have them graph the action of the story. It is important to give them situations where the look on the graph will be different from the action of a story. For example the distance traveled on a roller coaster or ferris wheel will not go up or down like the movement of the ride, but will continue to increase with changes in slope.

Ideas for Action Research

Making Graphs for Stories

Having students make their own graphs about situations helps them to understand the logic of the graph and see how the lines do not represent the action of the story. Consider giving your students some story situations and have them make a graph of the general situation, not necessarily dealing with the issues of an exact scale. For example:

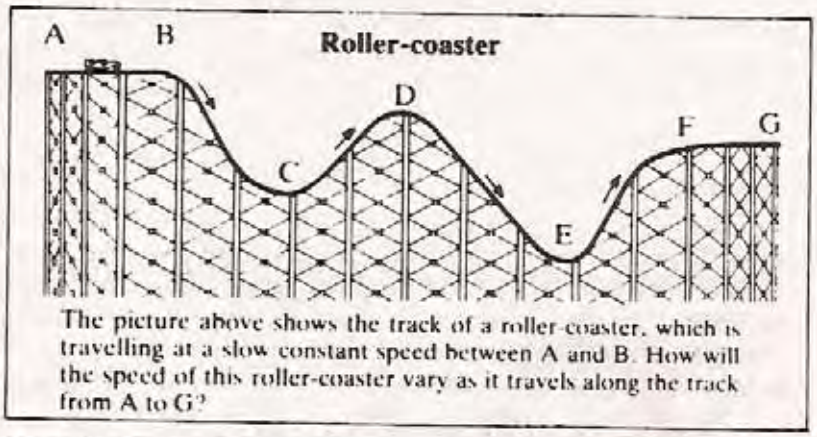
A factory cafeteria contains a vending machine selling drinks.
On a typical day:
- The machine starts half full.
- No drinks are sold before 9 a.m. or after 5 p.m.
- Drinks are sold at a slow rate throughout the day, except during the morning and lunch breads (10:30-11 am and 102 pm) when there is a greater demand.
- The machine is filled up just before the lunch break. (It takes about 10 minutes to fill).

Make a sketch to the graph to show how the number of drinks in the machine might vary from 8 am to 6 pm.

What does the student have to understand about graphing to do this task? What do you think the graph might look like? What errors do you anticipate students might make? How does this help you think about how to process this activity? What are the mathematics you want to bring out or highlight as students discuss their work?

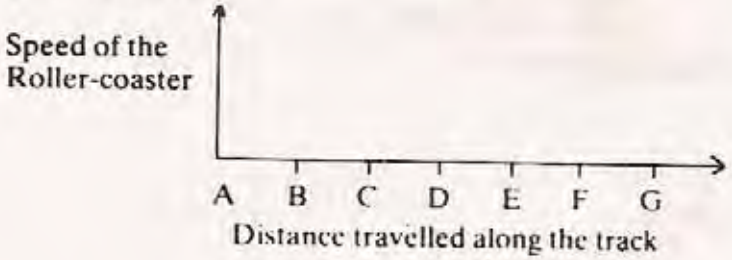
Now consider another task from the Shell Centre book, The Language of Functions and Graphs.

Now try the problem below:



The picture above shows the track of a roller-coaster, which is travelling at a slow constant speed between A and B. How will the speed of this roller-coaster vary as it travels along the track from A to G?

Describe your answer both in words and by sketching a graph in your book.



2

This next activity will help you to see how well you have drawn your sketch graph.

Fold this booklet so that you cannot see the picture of the roller-coaster track.

Try to answer the following questions using *only your sketch graph*.

* Along which parts of the track was the roller-coaster travelling quickly? slowly?
* Was the roller-coaster travelling faster at B or D? D or F? C or E?
* Where was the roller-coaster accelerating (speeding up)? decelerating (slowing down)?

Check your answers to these questions by looking back at the picture of the roller-coaster track. If you find any mistakes, *redraw* your sketch graph. (It is better to use a fresh diagram than to try and correct your first attempt.)

* Now invent some roller-coaster tracks of your own. Sketch a graph for each one, on a separate sheet of paper. Pass *only the sketch graphs* to your neighbour.

  Can she reconstruct the shape of the original roller-coaster tracks?

* Do you notice any connection between the shape of a roller-coaster track, and the shape of its graph? If so write down an explanation.

  Are there any exceptions?

How did this activity add to student understanding of line graphs? What did the student have to think about to be successful? What evidence of understanding did you see in student work?