Student E is able to find the coordinates for the intersections, probably by reading the graphs. The student has a very accurate graph for y=3x. The student uses substitution to show that the coordinates fit both equations. However, the student can't apply these strategies to a more general form of the equation because the thinking comes from visual representation.

Student E

2. Use the diagram to help you complete this statement:

   $2x$ is greater than x^2 when x is between 0,0 × and 2,4 +0

3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points. ✓

   a. Write down the coordinates of these two points. 0,0 2,4 +1

   b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross. × ×

   $0 = 0^2$ $2 \cdot 2 = 4$

   $0 = 2 \cdot 0$ $2^2 = 4$ +0

   +0

4.
   a. On the diagram, draw the graph of $y = 3x$. +1

   b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet?

   0,0 3,9 +1

   c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet? 0,0 × +0

   d. Use algebra to prove your answer. ×

   $0 = 0^2$

   $0 = n \cdot 0$ +0

Student F gives a generalization for how to solve for the intersections, including the idea that the two equations should be made equal to each other. (The paper is mis-scored on this mark). *Do you think the student makes the generalization? Is making generalizations a part of algebraic thinking? What do you think the student understands about algebra? How would you score this student?*

Student F

3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points.

   a. Write down the coordinates of these two points. (0,0) and (2,4) ✓

   b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross. I can use algebra to find the point of intersection by using a method called substitution. ^ 0
   You put each equation in y form and then remove 0
   the y= and set the two leftover pieces to eachother.
   Next you solve for x. That answer is your x coordinate.
   Then you replace the x in one of the equations with the answer you got, and solve for y. Your two answers together make up the coordinates of the intersection point.

4. a. On the diagram, draw the graph of $y = 3x$. ✓

   b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet? (0,0) (3,9) ✓ 1

   $\frac{x^2}{x} = \frac{nx}{x}$ $y = n^2$

   $x = n$

   c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet? (0,0) (n, n^2) ✓ 1

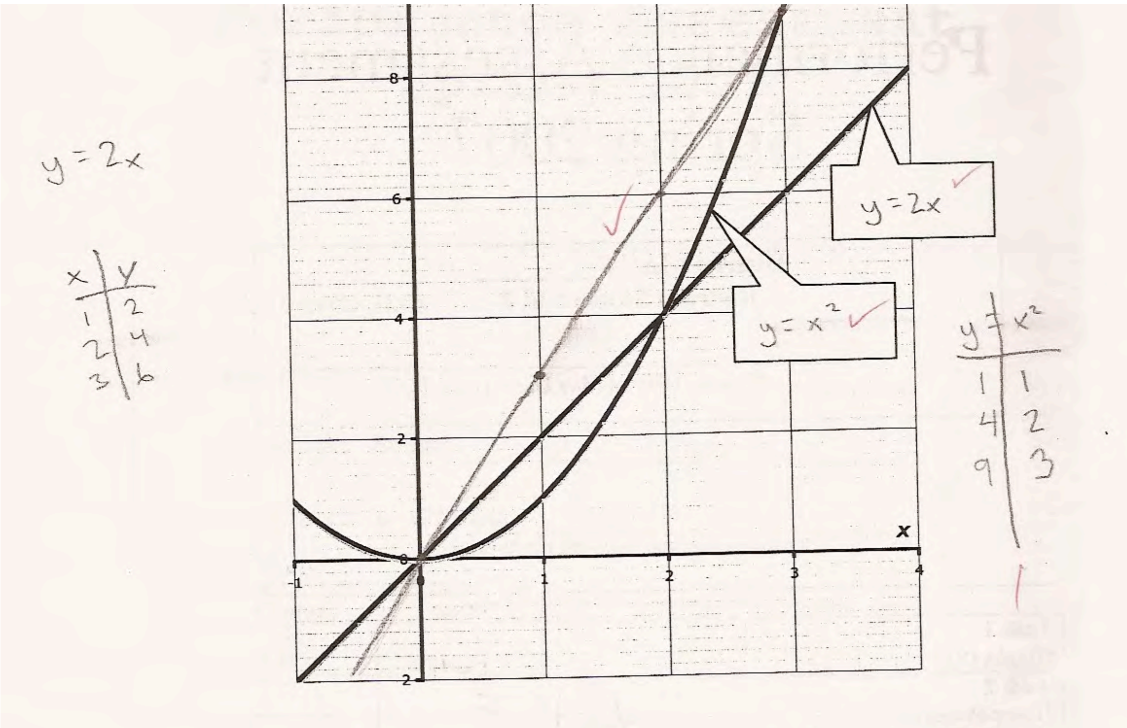
   d. Use algebra to prove your answer.

   Algebra says that if you use substitution (explained above), you can find intersection points. (0,0) is one ✓ 1
   because anything multiplied by zero is zero. Also (0,0) is a point on the $y=x^2$ graph. (n, n^2) is also one because it fits on the $y=x^2$ graph and because in $y=nx$, n and its multiples (such as n^2) would always work, so (0,0) and (n, n^2) are the points of intersection.

9 (7)

Student G uses a table of value to verify which graphs match the equations. In part 2 the student misses the lower end of the values but tries to show that the numbers need to be lower than 2. In part 3 the student gives more a description of the graph than directions to find the intersections. *In part 4, the student might be on the way to understanding that the intersections are when the two equations are equal. What does the student understand? What is the student struggling with?*

Student G



1. Fill in the labels to show which graph is which. Explain how you decided.

I made a table for "x" and "y" and I plugged in 3 sets of numbers that work. I plotted the points and the lines go through these points.

Student G, part 2

2. Use the diagram to help you complete this statement:

   $2x$ is greater than x^2 when x is between 1 and 1.$\bar{9}$

3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points.

   a. Write down the coordinates of these two points. (0,0) (2,4)

   b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross.

   y=2x means the equation will continually go up and right. y=x² means there will be squares meaning 4,9,16 which becomes a parabola.

4.
   a. On the diagram, draw the graph of $y = 3x$.

x	y
1	3
2	6
3	9
-1	-3

   b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet? (0,0) (3,9)

   c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet? (0,0) all squares

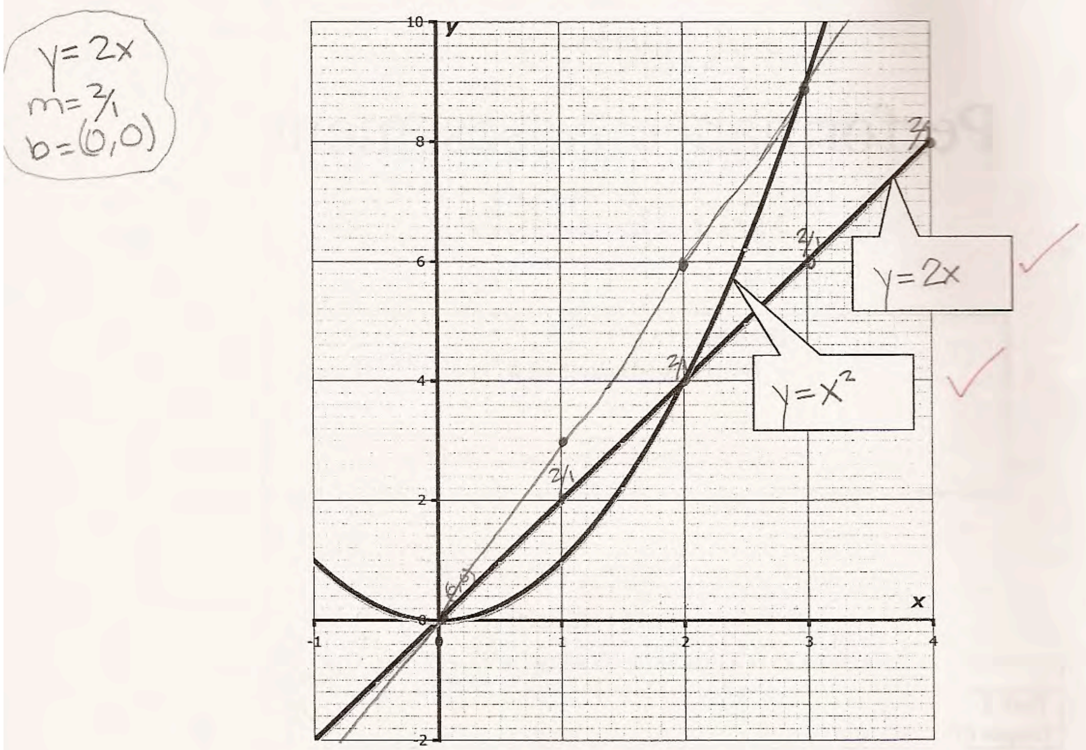
   d. Use algebra to prove your answer.

   y=x² means a number multiplied by itself. y=nx means n and x could have equal value such as this example. (4=2²) = (4=2(2)) if n and x have the same value it is exactly the same as squaring.

Student H appears to have a clear understanding of the equation y=mx+b and uses this information to identify the graphs in part 1 and to draw the graph in 4a. However the student seems to be confused about the slope in 3b and then can't think about the generalization in 4c. *What does this student need to help clarify ideas about slope? What do you think the student understands or doesn't understand about the intersections of two equations? What further instruction or experience does this student need?*

Student H

This diagram shows the graphs of $y = x^2$ and $y = 2x$.



1. Fill in the labels to show which graph is which. Explain how you decided.

On the equation y= 2x, slope= 2/1 and y-intercept=(0,0)
That leaves the other equation y=x² to be the
curved line.

Student H, part 2

2. Use the diagram to help you complete this statement:

   $2x$ is greater than x^2 when x is between (0,0) and (2,4) X 0

3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points.

   a. Write down the coordinates of these two points. (0,0) (2,4) ✓ 1

   b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross.

   You can combine ~~add~~ the slope (2,4) and combine X 0
   the y-intercept (0,0).

   X 0

4.
   a. On the diagram, draw the graph of $y = 3x$. ✓ 1
   m= 3/1 b=(0,0)

   b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet?
   (0,0) (3,9) ✓ 1

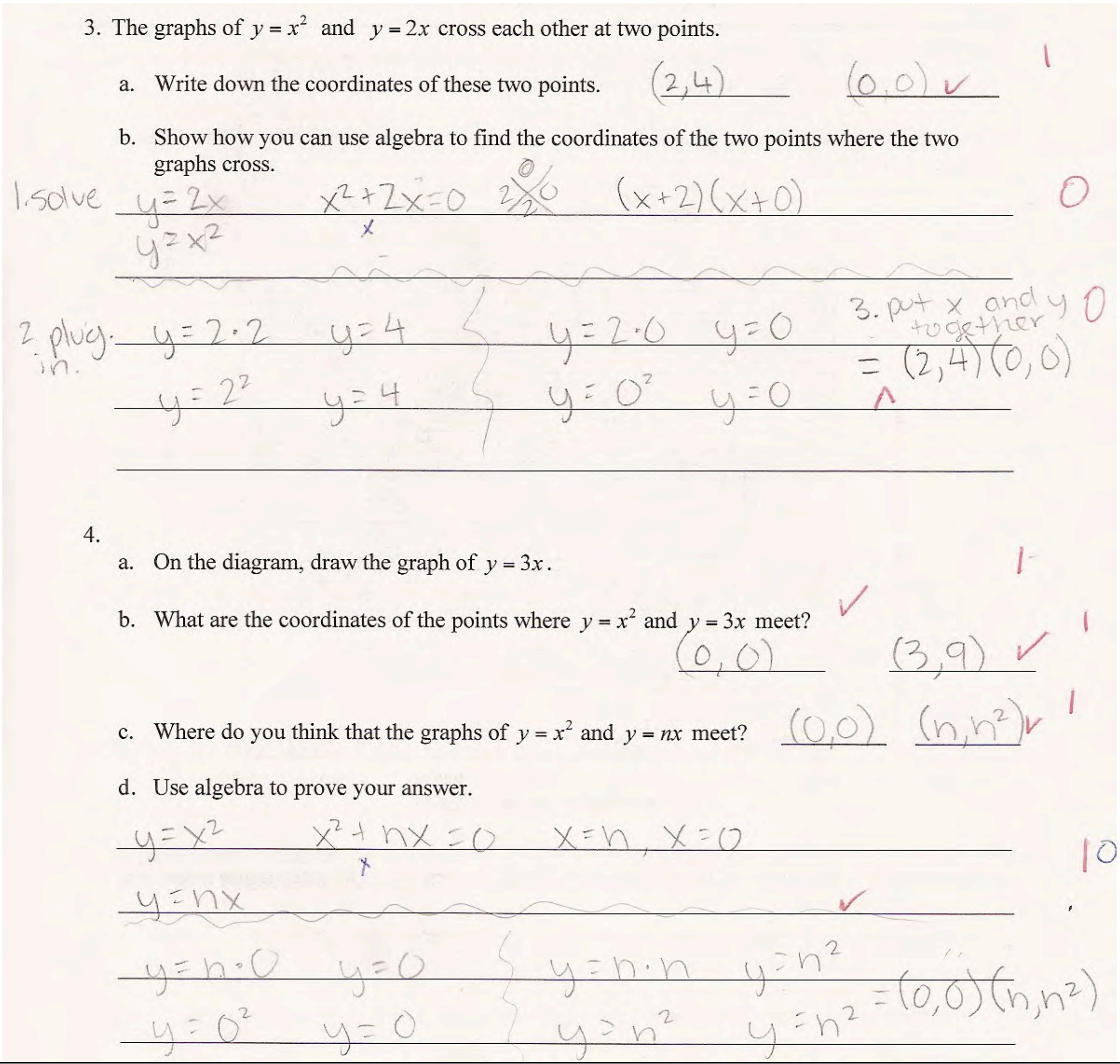
   c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet? (0,0) (2,4) X 0

   d. Use algebra to prove your answer.

   I added 1 onto the x-point and 5
   onto the y point. X 0

Student I tries to combine the two equations and use factoring, but uses a positive instead of a negative in both 3b and 4d. However the student ignores that work and is able to find the coordinates using substitution and number sense or guess and check. *What does the student know about algebra? What is the student confused about?*

Student I



3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points.

   a. Write down the coordinates of these two points. (2,4) (0,0) ✓ 1

   b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross.

1.solve $y = 2x$ $x^2 + 2x = 0$ $\frac{2}{2}\cancel{x}0$ $(x+2)(x+0)$ 0

$y = x^2$

2. plug in. $y = 2 \cdot 2$ $y = 4$ $y = 2 \cdot 0$ $y = 0$ 3. put x and y together 0

$y = 2^2$ $y = 4$ $y = 0^2$ $y = 0$ $= (2,4)(0,0)$

4.

   a. On the diagram, draw the graph of $y = 3x$. 1

   b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet? ✓ (0,0) (3,9) ✓ 1

   c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet? (0,0) (n,n^2) ✓ 1

   d. Use algebra to prove your answer.

$y = x^2$ $x^2 + nx = 0$ $x = n, x = 0$ 10

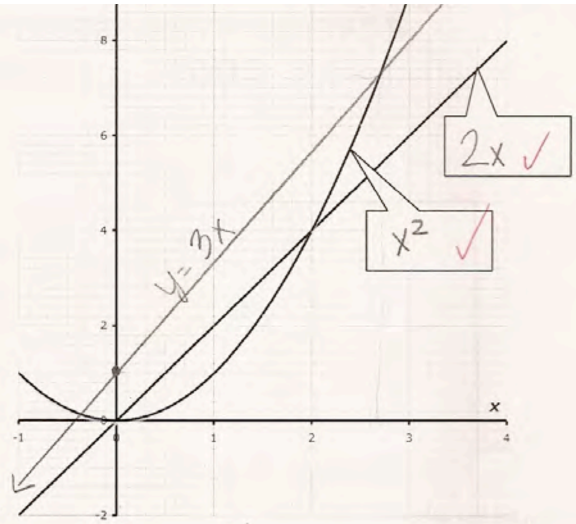
$y = nx$ ✓

$y = n \cdot 0$ $y = 0$ $y = n \cdot n$ $y = n^2$

$y = 0^2$ $y = 0$ $y = n^2$ $y = n^2$ $= (0,0)(n,n^2)$

Student J does not understand that the graph for y=3x should go through the origin. While it is unclear how the student made the graph, it does appear to go through the point (1,3). The student is able to get the coordinates for 3a, probably by looking at the graph. The student is able to make the 2 graphs equal, but doesn't know how to use algebraic skills to solve the equation. It is interesting that the student doesn't use the graphs to find the coordinates in 4b. *What types of experiences does this student need? Where does his understanding of graphing break down? Where does his understanding of solving equations break down?*

Student J



1. Fill in the labels to show which graph is which. Explain how you decided.

I decided the line thats going like this is a parabala the x² because it's on a negative and positive line and it curves like a parabola. The 1 that's just going straight up is a line because in every graph function if there is a number and an x we know it will be a line.

 Page 2 Graphs Test

2. Use the diagram to help you complete this statement:

$2x$ is greater than x^2 when x is between 1 and 2 x 0

3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points.

a. Write down the coordinates of these two points. 0, 0 2, 4 ✓ 1

b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross.

$x^2 = 2x + 0$ ✓ 1

$-2x \quad -2x$

x 0

4.

a. On the diagram, draw the graph of $y = 3x$. x 0

b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet?

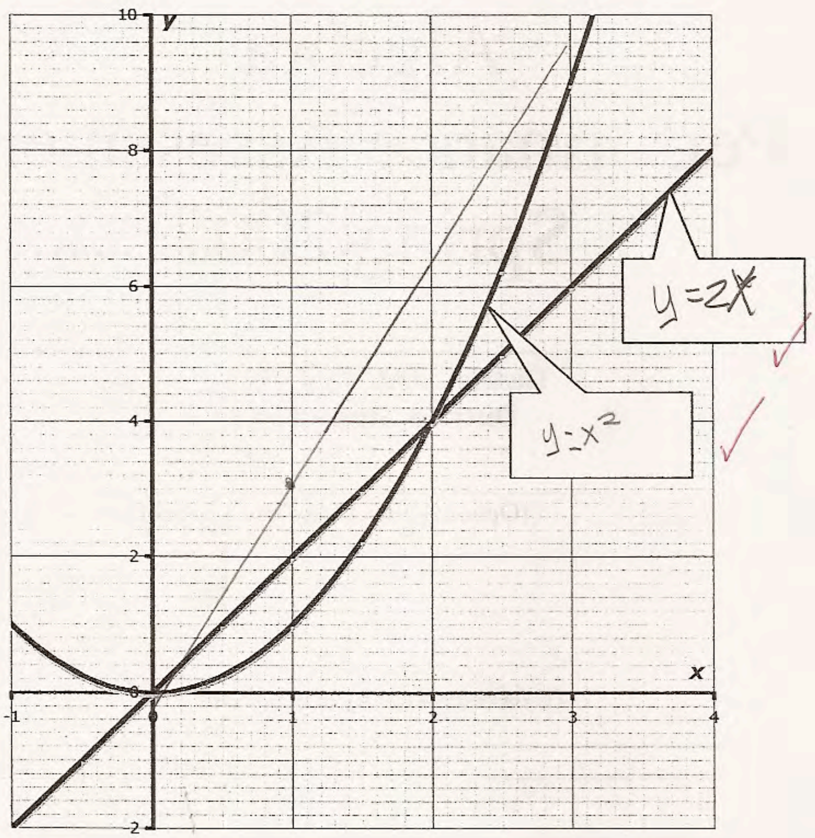
-1, 1 2⅓, 7 x 0

c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet? 0, 0 3.7, 14 x 0

d. Use algebra to prove your answer.

Student K has only 2 of the 9 marks on this task, but there is evidence of understanding. In part 1 the student makes a t-table to help identify the graphs. However the student doesn't give enough information about how it was used. In drawing the graph for y=3x, there appears to be evidence of thinking about y=mx+b. (The graph is off slightly, but clearly attempts to go through(0,0) and (1,3). However by the end of the line it appears the line might go through (3,10) instead of (3,9). The student doesn't pick up on this discrepancy in answering 4b. The student seems to think that looking at the graph should give you the solution, so why bother with algebra. But, in 4d can think about the equations in 4d going through the origin, but can't or isn't willing to attempt a second solution. *How can you help build on these understandings?*

Student K



1. Fill in the labels to show which graph is which. Explain how you decided.

~~[illegible]~~ · y=x² y=2x

1	1
2	4
3	9

Student K, part 2

2. Use the diagram to help you complete this statement:

   $2x$ is greater than x^2 when x is between ___2___ and ___x^2___ x 0

3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points.

   a. Write down the coordinates of these two points. ___2,4___ ___0,0___ ✓ 1

   b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross. x 0

   Well look where the intersect and write it down x 0
   were they cross

4.

   a. On the diagram, draw the graph of $y = 3x$. y=0 3/ x 0 1

   b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet?
   ___10___ ___3___ x 0

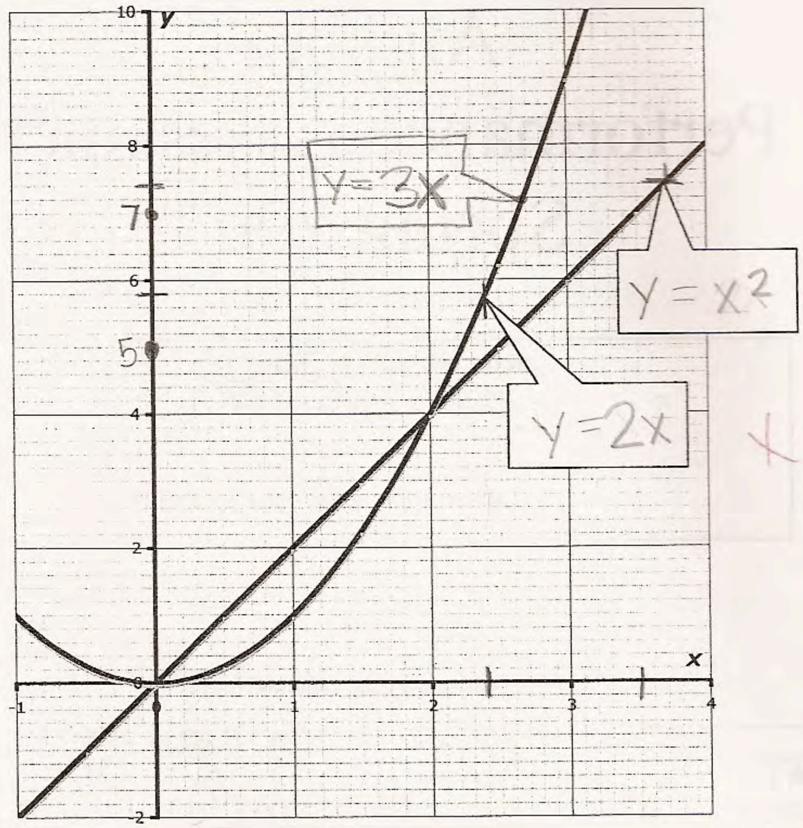
   c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet? ___0,0___ ___0,0___ x 0

   d. Use algebra to prove your answer.

   y=0 x=0 So they would meet at x 0
   0,0

Students L and M both struggle at just the entry level question on graphing. Student L knows the definition of linear and quadratic equations and their lines, but can't link that information with the equations. The student tries to make y=3x part of parabola. Student M does not see the link between the equations and the graphs, but instead tries to pinpoint the value of x or y at the exact place on the graph that the arrow is pointing. Neither of these students is in high school (thereby repeating algebra for the second time) and both are from districts that have participated in MAC since the beginning of the project.

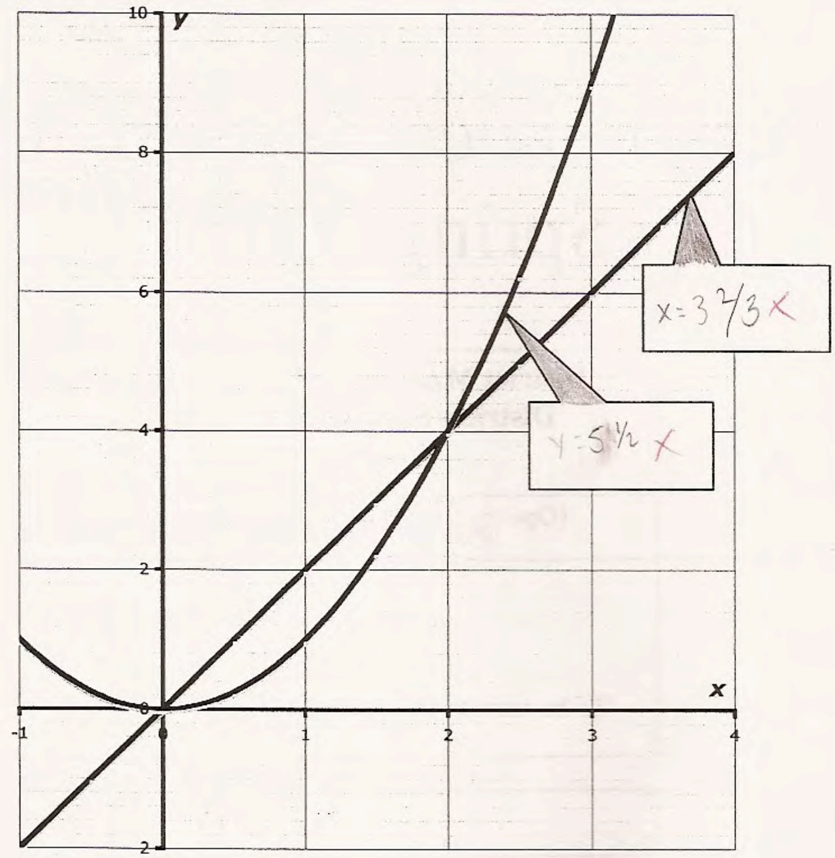
Student L



1. Fill in the labels to show which graph is which. Explain how you decided.

I decided because one line is linear (the straight one) ✓ and the quadratic (the curved one). I think that it makes more sense because the number can be made to even on the linear line.

Student M



1. Fill in the labels to show which graph is which. Explain how you decided.

Because they are lined up on that number

Algebra

Algebra Task 1 Graphs

Student Task	Work with linear and quadratic functions, their graphs, and equations.
Core Idea 1 Functions and Relations	**Understand patterns, relations, and functions.** • Analyze functions of one variable by investigating local and global behavior including slopes as rates of change, intercepts, and zeros.
Core Idea 3 Algebraic Properties and Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** • Understand the meaning of equivalent forms of expressions, equations, inequalities, or relations. • Write equivalent forms of equations, inequalities and systems of equations and solve them. • Use symbolic algebra to represent and explain mathematical relationships.

Mathematics in this task:
- Distinguish between linear and quadratic equations and their graphical representations
- Ability to graph a linear equation
- Ability to locate points on a graph and interpret their meaning
- Use algebra to find the intersections of two equations

Based on teacher observation, this is what algebra students knew and were able to do:
- Find the coordinates where the graphs intersect
- Give a reason for connecting equations with their graphs
- Draw a graph of y=3x

Areas of difficulties for algebra students:
- Finding values for x, where one graph or equation is less than another
- Using algebra to find the points of intersection for two equations
- Knowing that the equations should equal each other at the points of intersection
- Using factoring as a tool to solve a quadratic equation
- Understanding that you can't divide by 0

Strategies used by successful students:
- Making a table of values to help them graph
- Understanding y=mx+b and using it to help them graph
- Substitution

MARS Test Task 1 Frequency Distribution and Bar Graph, Course 1

Task 1 - Graphs

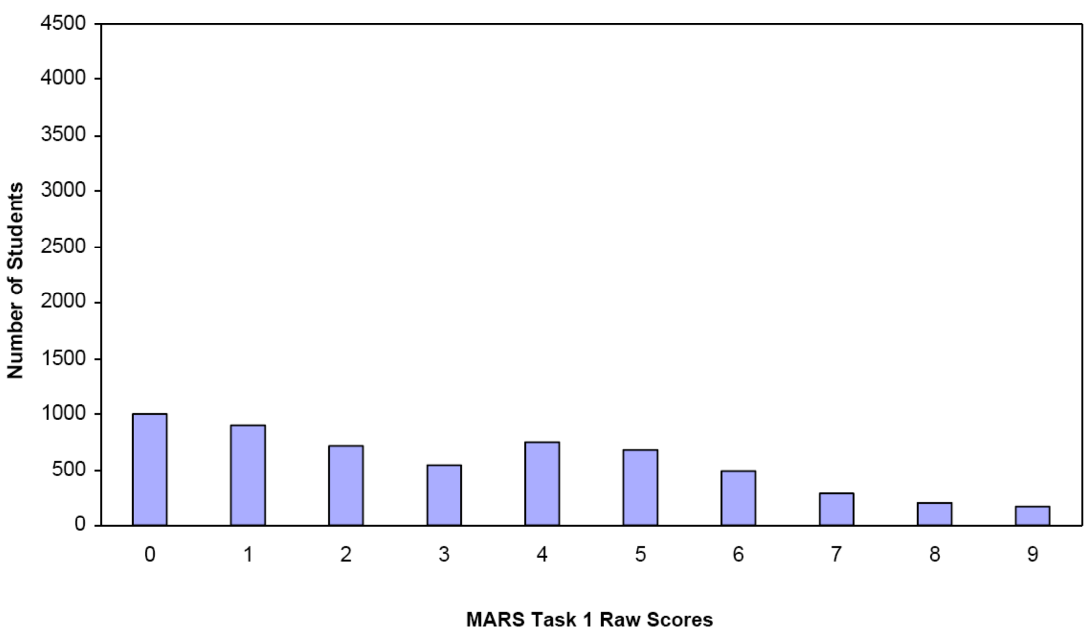
Mean: 3.23 StdDev: 2.54

Table 45: Frequency Distribution of MARS Test Task 1, Course 1

Task 1 Scores	Student Count	% at or below	% at or above
0	1003	17.5%	100.0%
1	892	33.0%	82.5%
2	714	45.4%	67.0%
3	543	54.9%	54.6%
4	746	67.9%	45.1%
5	676	79.6%	32.1%
6	500	88.3%	20.4%
7	292	93.4%	11.7%
8	202	96.9%	6.6%
9	176	100.0%	3.1%

•

Figure 54: Bar Graph of MARS Test Task 1 Raw Scores, Course 1



The maximum score available for this task is 9 points.
The minimum score for a level 3 response, meeting standards, is 4 points.

Most students, 83%, could identify the coordinates for the points of intersection of the two graphs. More than half the students, 67%, could identify the equations with their graphs and find the points of intersection. Almost half the students, 45%, could also draw a graph of $y=3x$ and find the coordinates where it intersects with $y=x^2$. About 9% of the students could meet most of the demands of the task, but struggled with locating the coordinates of intersection for $y= x^2$ and $y=3x$ giving values for x in part 2. Almost 18% of the students scored no points on this task. 95% of the students in the sample with this score attempted the task.

Graphs

Points	Understandings	Misunderstandings
0	95% of the students with this score attempted the task.	Students did not understand the blanks and put the coordinates on separate lines, not giving both intersections.(6%) Some reversed x and y coordinates (3%)
1	Students could find the coordinates for the intersections of the two graphs.	Students had trouble giving reasons for matching the equations and their graphs. 11% reversed the graphs. Some tried to pinpoint the location of the arrows.
2	Students could match equations with their graphs and find the points of intersection.	25% did not attempt to draw the graph for y=3x. 6% made graphs that were too low. 5% drew parabolas. 5% made graphs that did not pass through the origin.
4	Students could identify the graphs and intersections, draw a graph from an equation, and find the intersection of their graph with $y=x^2$.	Students had difficult using symbolic algebra to find the points of intersection. 10% said just look at the graph. 12% made a table of value to find where the values were the same for each equation. 12% just substituted values from the graph into the given equations to show the values were equal. 12% tried to use slope to make a justification.
6	Students could meet all the demands of the task except use algebra to justify the points of intersection.	10% could set the two equations equal to ach other but couldn't solve the equation. They may have divided by x (x could and does equal 0). They tried to take the square root of 2x or 3x.
8	Students could match equations to graphs, make their own graph of an equation, find the intersection points for 2 equations including 2 generalized equations. Successful students looked at patterns to solve the problem, used number theory, and number sense and mathematical reasoning.	No student in the sample solved a quadratic equation.

Implications for Instruction

Students need more opportunities to work rich tasks that provide them with the opportunity to synthesize and to use all the knowledge that they are accumulating about a particular algebraic topic. The problems should allow them to use a variety of tools to make sense of the situation. Teachers need to think about which tools the students feel comfortable enough to use and to use well. Also, which expected tools are never used by students? What types of instruction do students need in order to feel comfortable applying these tools to new situations?

Action Research

Sit down with colleagues to think about what are the big ideas in Algebra. Many in the mathematics world are talking about redefining what it means to learn Algebra. In Europe for example and in many junior colleges, books are focusing on a functional approach to algebra, often eliminating all those sections on factoring and starting quadratics with the quadratic formula. In the task Graphs, many issues about what is valued in learning and understanding graphs are raised. How do we teach algebra in a way that allows students to develop practical skills as well as procedures? What are the underlying ideas that we want students to understand about graphs and their equations? about graphs and their intersections? What does the solution to a quadratic equation actually mean in practical terms?

Use the tool kit for graphs to probe the thinking of your group as you grapple with defining the issues and implications.

What are some of the other issues that this task raised within the group? What tasks can you give to students that might allow you think more deeply about these issues or allow you to gather more data?