

| | |
|---|---|
| Student Task | Find perimeters of shapes. Use Pythagorean theorem to find side lengths. |
| Core Idea 4 Geometry and Measurement | <p>Apply appropriate techniques, tools and formulas to determine measurements.</p> <ul style="list-style-type: none"> • Create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship. |

Based on teacher observations, this is what eighth graders know and are able to do:

- Find the perimeter of a rectangle
- Calculate the circumference
- Convert 6 inches to decimal notation

Areas of difficulty for eighth graders:

- Using Pythagorean theorem to find a side length
- Rounding numbers in context
- Confusing diameter and radius
- Confusing area and perimeter
- Making sense of a diagram, recognizing the added edge when cutting a circle in half

MARS Test Task 2 Frequency Distribution and Bar Graph, Grade 8

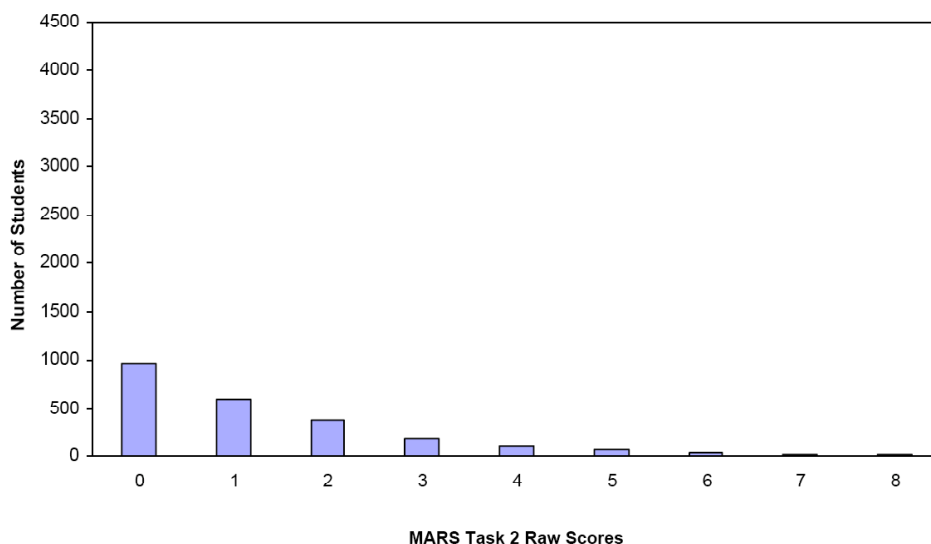
Task 2 - Rugs

Mean: 1.31 StdDev: 1.58

Table 41: Frequency Distribution of MARS Test Task 2, Grade 8

| Task 2 Scores | Student Count | % at or below | % at or above |
|---------------|---------------|---------------|---------------|
| 0 | 970 | 40.9% | 100.0% |
| 1 | 601 | 66.2% | 59.1% |
| 2 | 375 | 82.0% | 33.8% |
| 3 | 191 | 90.1% | 18.0% |
| 4 | 103 | 94.4% | 9.9% |
| 5 | 72 | 97.4% | 5.6% |
| 6 | 35 | 98.9% | 2.6% |
| 7 | 13 | 99.5% | 1.1% |
| 8 | 13 | 100.0% | 0.5% |

Figure 50: Bar Graph of MARS Test Task 2 Raw Scores, Grade 8



The maximum score available on this task is 8 points.

The minimum score for a level 3 response, meeting standards, is 3 points.

Many students, about 60%, could find the perimeter of a rectangle. Only 18% of the students could meet standard on this task usually getting the area of the rectangle and finding the circumference of a circle rounded to the nearest whole number. About 10% could also explain why the perimeter of a semi-circle is not half the circumference of a circle. Less than 1% of the students could meet all the demands of the task including using Pythagorean theorem to find the side length of an isosceles triangle, find the perimeter of a semicircle, and round the circumference of a circle to the nearest whole number. Almost 41% of the students scored no points on this task. 91% of the students with this score attempted the task.

Rugs

| Points | Understandings | Misunderstandings |
|----------|---|--|
| 0 | 91% of the students with this score attempted the task. | Students did not know how to find the perimeter of a rectangle. Almost 6% confused area and perimeter. The students who calculated area used 2.6 ft for 2 ft. 6 in. An additional 5% converted 2 ft. 6 in. to 2.6 ft when calculating perimeter. About 5% just added two dimensions when calculating area. Almost 7% ignored the 6 inches and just add $4+4+2+2$ to 12 inches for the perimeter. |
| 1 | Students could find the perimeter of a rectangle. | Students could not use the formula to find the circumference of a circle. 7.5% of the students did not attempt this part of the task. 12% used the diameter instead of the radius for calculating the circumference. 15% had an answer of 10. 18% did not round 15.7 to 16 ft. |
| 3 | Students could find the perimeter of a rectangle and calculate and round the circumference of circle when given the diameter. | Students could not reason about finding the perimeter of a semicircle. Almost 22% of the students did not attempt this part of the task. 9% of the students thought the answer would be 10. Almost 7% thought the perimeter would be exactly half the circumference of the circle. Almost 7% thought the perimeter would equal the diameter, ignoring the curved part of the rug. Other popular answers were 15, 2.5, and 8. |
| 4 | | Students could not find the perimeter of an isosceles triangle, given the base and height. 7% did not attempt this part of the task. 16% of the students used the height as the side length to get an answer of 7ft. 15% of the students multiplied base times height (area without the divide by 2). $\frac{2}{3}$ of this group used 1.6 ft. instead of $\frac{1}{5}$ ft. Other popular answers were 18 and 4. |
| 8 | Students could use Pythagorean theorem to find the side length of an isosceles triangle and calculate perimeter of a rectangle and triangle. Most could convert between units of measurement and decimal notation. Students could find the circumference of a circle given the diameter and find the perimeter of a semicircle. | |

Implications for Instruction

Students need more experiences with geometric figures. At this grade level students should be able to compose and decompose shapes into parts. For example, they should see that if you cut a circular rug in half, the new perimeter will be larger than half the circumference of a circle because the perimeter will also include the line across the center of the circle or the diameter. Students should then be able to calculate the perimeter of a semi-circle.

Students at this grade level should have experience working with Pythagorean Theorem and applying it in problem situations. An interesting task for interpreting diagrams is the 2001 Course 2 task, Writing Desks.

Many students still struggled with the idea of area and perimeter. Some students confused radius and diameter. Students need practice applying these ideas to problems to help them sort through or make sense of the differences.

Ideas for Action Research

Because 41% of the students scored no points on this task and most of them attempted the problem. It is important to think about what are the mathematical gaps in their thinking. What types of experiences do these students need?

Often when planning remediation or helping students who are behind, teachers think about the students who are almost there. What are the few steps they need to be successful? But what is it that the students who are at the lowest end of the spectrum need? How are their issues different?

Sit down with colleagues and examine the following pieces of student work or examples from your own students. Consider the following questions:

1. What are the strengths, if any, that the student has? What are the concepts the students understand about the situation? How might these strengths be used to help build their understanding of the whole situation?
2. How did students use representations? Were the representations accurate? Why or why not? What would have helped the student to improve their representation?
3. What misunderstandings does the student have? What skills is the student missing? What does this suggest about a specific course of action to help this student?
4. How are the needs of each of these students the same or different?

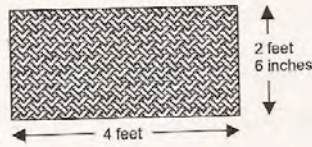
After you have carefully looked at each piece of student work, see if you can devise a plan of experiences/ discussions/ tools that might help these students to make more sense of these situations. While you don't have these exact students in your class, each member of the group will probably have students with similar misunderstandings. Identify students who you think are low and plan different approaches for attacking the problems outlined here. Have each person in the group try out a different course of action and report back on the how the lesson or series of lessons effected the targeted students. See if you can all use some similar starting problems and bring work of the students to share. What types of activities or experiences made the most noticeable improvement in student work?

Anna

Hank works at a factory that makes rugs.

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.



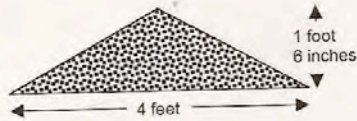
How much braid will Hank need to cut to go all the way around this rug?

12 ~~x~~ ~~x~~ 8 ~~x~~ feet

Show your work.

$$\begin{array}{r} 6 \\ + 6 \\ \hline 12 \end{array}$$

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.



How much braid will Hank need to cut to go all the way around this rug?

7 ~~x~~ ~~x~~ 4 ~~x~~ feet

Show your work.

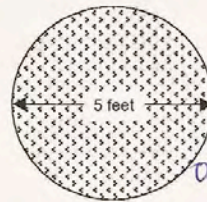
$$\begin{array}{r} 6 \\ + 1 \\ \hline 7 \end{array}$$

The factory also makes a circular rug that has a diameter of 5 feet.

How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

The circumference of a circle = $2\pi r$
The area of a circle = πr^2

10 ~~x~~ feet



Show your work.

$$\begin{array}{r} 5 \\ \times 2 \\ \hline 10 \end{array}$$

4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

Explain why Hank is not correct.

I think he is correct ~~x~~ ~~x~~

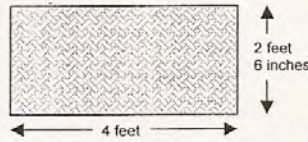
How much braid will this rug need?

3 ~~x~~ feet

Bradley

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.



How much braid will Hank need to cut to go all the way around this rug?

Show your work.

$$6 + 4 = 10$$

$$2 \overline{) 10}$$

$$\underline{4}$$

$$6$$

$$\underline{10.5} \text{ feet}$$

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.



How much braid will Hank need to cut to go all the way around this rug?

Show your work.

$$1 + 1 = 2$$

$$\begin{array}{r} 1 \\ 5 \overline{) 5.1} \\ \underline{5} \\ 1 \end{array}$$

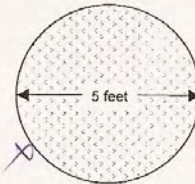
$$\underline{5.1} \text{ feet}$$

3. The factory also makes a circular rug that has a diameter of 5 feet.

How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

The circumference of a circle = $2\pi r$
The area of a circle = πr^2

$$\underline{15.70} \text{ feet}$$



Show your work.

$$A = \pi r^2 = (3.14)(5) = 15.70$$

4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

Explain why Hank is not correct.

Hank is wrong because they are different measurements.

How much braid will this rug need?

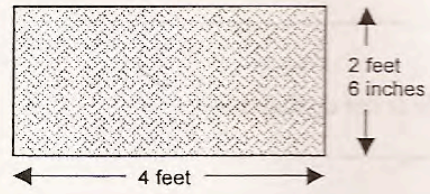
$$\underline{15.70} \text{ feet}$$

Carrie

Hank works at a factory that makes rugs.

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.



How much braid will Hank need to cut to go all the way around this rug?

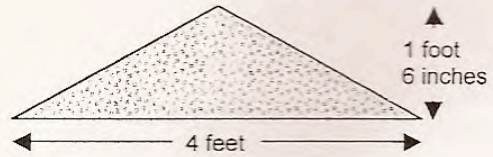
Show your work.

$$4 + 4 = 8 \quad \frac{12}{12}$$

$$2\frac{6}{6} + 2\frac{6}{6} = 4\frac{12}{12}$$

$$\underline{12} \times 0 \text{ feet}$$

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.



How much braid will Hank need to cut to go all the way around this rug?

Show your work.

$$4 + 1 + 1 = 6$$

$$\underline{6} \times 0 \text{ feet}$$

0

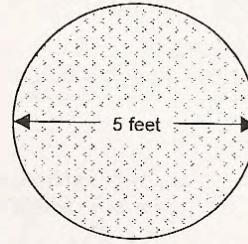
0

Carrie, part 2

3. The factory also makes a circular rug that has a diameter of 5 feet.

How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

The circumference of a circle = $2\pi r$
 The area of a circle = πr^2



16.7 feet

Show your work.

$$\begin{array}{r} 3.14 \\ \times 2 \\ \hline 6.28 \end{array}$$

$$2 \times 3.14 \times 2.5 = 6.28$$

$$\begin{array}{r} 6.28 \\ \times 2.5 \\ \hline 3140 \\ 13560 \\ \hline 15700 \end{array}$$

16.700

4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

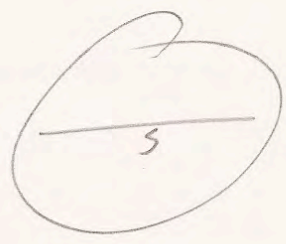
Explain why Hank is not correct.

I think that you would only need half not half more.

How much braid will this rug need?

2.5 feet

$$\begin{array}{r} 2 \\ 3.14 \\ \times 2.5 \\ \hline 15700 \\ 6280 \\ \hline 20980 \end{array}$$



$$5 \div 2 = 2.5$$

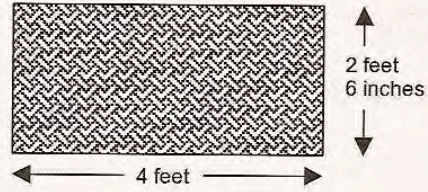
8

David

Hank works at a factory that makes rugs.

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.



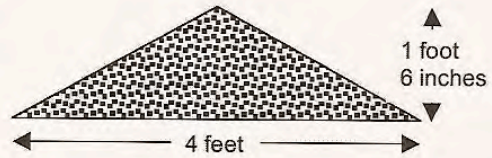
How much braid will Hank need to cut to go all the way around this rug?

21 ~~x~~ feet

Show your work.

$4 \times 4 = 16 + 2.5 \times 2.5 = 21$ he needs 21ft ~~x~~
 2ft 6in basically equals = 2.5 on a ruler

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.



How much braid will Hank need to cut to go all the way around this rug?

19 ~~x~~ feet

Show your work.

$4 \times 4 = 16$ ~~x~~
 $1 + 1 = 2$ ft + 6in + 6in = 1ft = 3 ~~x~~

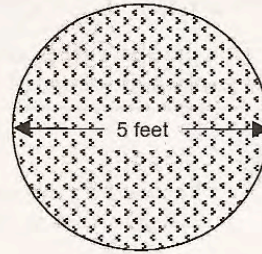
David, part 2

3. The factory also makes a circular rug that has a diameter of 5 feet.

How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

The circumference of a circle = $2\pi r$
The area of a circle = πr^2

15.7 feet



Show your work.

$$5 \div 2 = 2.5 = \text{radius}$$

$$2.5 \times 3.14 = 7.85 \times 2 = 15.7 \text{ feet}$$

4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

Explain why Hank is not correct.

He is not correct because you can't make a semi-circular rug with the amount of material that you made circular rug with.

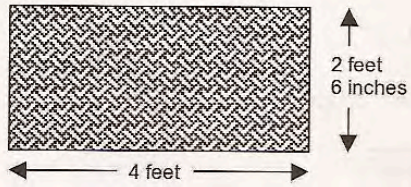
How much braid will this rug need?

around 8 feet

Ellie

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.

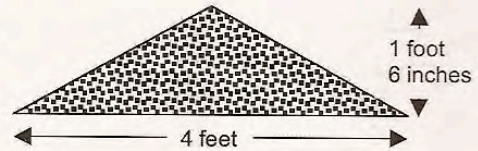


How much braid will Hank need to cut to go all the way around this rug?

Show your work.

Handwritten student work for the rectangular rug problem. It includes a diagram of the rug with dimensions 4 FT and 2 FT 6 IN. Next to it are two addition problems: $4 + 4 = 8$ and $2.6 + 2.6 = 5.2$. A red checkmark is next to the number 13, and the word "BKT." is written in red.

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.



How much braid will Hank need to cut to go all the way around this rug?

Show your work.

Handwritten student work for the triangular rug problem. It includes a diagram of the triangle with a base of 4 FT and a height of 1 FT 6 IN. Next to it are calculations: $4 + 1 \text{ FT } 6$, $2.12 = 3$, and $4 + 3 = 7$. A red checkmark is next to the number 7.

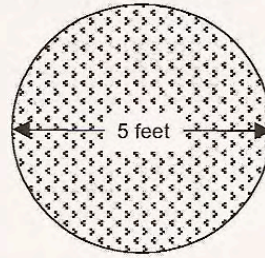
Ellie, part 2

The factory also makes a circular rug that has a diameter of 5 feet.

How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

The circumference of a circle = $2\pi r$
 The area of a circle = πr^2

19.63 feet



Show your work.

$$\begin{array}{r} 2.5 \\ \times 3.14 \\ \hline 7.85 \\ \times 2 \\ \hline 15.7 \end{array}$$

$$\begin{array}{r} 6.25 \\ \times 3.14 \\ \hline 19.62 \approx 19.63 \end{array}$$

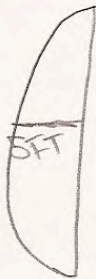
4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

Explain why Hank is not correct.

BECAUSE SINCE THE CIRCLE IS CUT
 IN HALF THE DPT. CANT BE THE RADIUS
 BECAUSE ITS ALREADY CUT IN HALF

How much braid will this rug need?

54.95 feet



$$\begin{array}{r} 31.4 \\ 78.5 \\ \hline 109.9 \end{array}$$

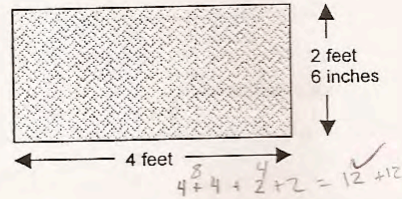
$$\begin{array}{r} 54.95 \\ 2 \overline{) 109.9} \\ \underline{10} \\ 09 \\ \underline{08} \\ 19 \\ \underline{18} \\ 10 \end{array}$$

Fred

Hank works at a factory that makes rugs.

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.

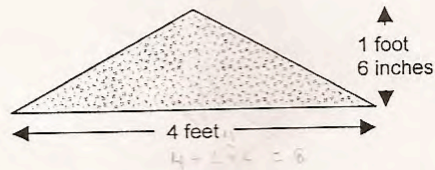


How much braid will Hank need to cut to go all the way around this rug?

Show your work.

13 ✓ feet

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.



How much braid will Hank need to cut to go all the way around this rug?

Show your work.

$3 + 8 = 11$ X feet

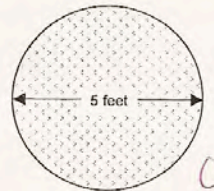
Fred, part 2

3. The factory also makes a circular rug that has a diameter of 5 feet.

How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

The circumference of a circle = $2\pi r$
The area of a circle = πr^2

2.5 X feet



Show your work.

$$2.14 \times 5^2 = 53.75$$

$$\begin{array}{r} 53.75 \\ \times 2 \\ \hline 107.50 \\ \hline 107.50 \\ \hline 215.00 \end{array}$$

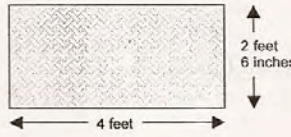
4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

Explain why Hank is not correct.

Geena

The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.

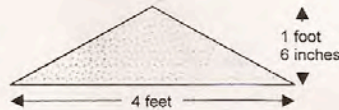
1. The factory makes a rectangular rug that is 4 feet long and 2 feet 6 inches wide.



How much braid will Hank need to cut to go all the way around this rug? 8 ft 6 inches

Show your work.

2. The factory makes a triangular rug. It is an isosceles triangle 4 feet wide with a perpendicular height of 1 foot 6 inches.



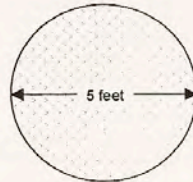
How much braid will Hank need to cut to go all the way around this rug? 4 ft 6 inches

Show your work.

3. The factory also makes a circular rug that has a diameter of 5 feet.

How much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet.

The circumference of a circle = $2\pi r$
The area of a circle = πr^2



Show your work.

$$\begin{array}{r} 3.14 \times 2.5 \\ \hline 7.85 \\ 3.14 \times 2.5 \\ \hline 7.85 \\ \hline 15.70 \end{array}$$

7.85 feet

4. There are plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug.

Explain why Hank is not correct.

because he might need more rug because just because the rug is smaller doesn't mean it needs less braid.

How much braid will this rug need?

256400.00 feet

$$\begin{array}{r} 1256 \\ 3140 \\ \hline 4396 \\ 4396 \\ \hline 8792 \end{array}$$

0

8