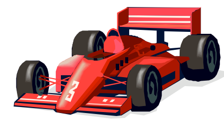
Take Off

This problem gives you the chance to:
• work with speed, time, and distance

1. A space rocket needs to move at 11.2 kilometers per second to escape from the earth's gravitational pull. This is called escape velocity.

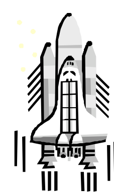


An Indy car races at up to 370 kilometers per hour.

How many times faster would an Indy car have to move to reach escape velocity?
Give your answer correct to the nearest whole number.

____

Show your work



2. Sound travels at about 340 meters per second.
Light travels at almost 300,000,000 meters per second.

If you **watched** a space rocket take off from a distance of 3.5 kilometers how much later would it be before you **heard** it take off?
Give your answer correct to the nearest second. ______________ seconds

Explain how you figured it out and show your calculations.

____

____

____

8

Task 5: Take Off		Rubric
The core elements of performance required by this task are: • work with speed, time and distance Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives a correct answer: **109** (accept 112) Shows correct work such as: 11.2 x 60 x 60 = or 370 ÷ 60 ÷ 60 = 0.1027 40320 / 370 11.2 ÷ 0.1027 =	1 1 1ft	3
2. Gives a correct answer: **10** seconds *Partial credit* Gives answer 10.29 Gives a correct explanation such as: You would see the take off straight away. 3.5 kilometres = 3500 metres 3500 / 340	2 (1) 1 1 1	5
Total Points		**8**

Take Off

Work the task. What are the big mathematical ideas being assessed?

How often do students in your class work with converting between units of measure? What are the classroom norms for using labels and dimensional analysis? Do you think students have enough understanding of algebra to understand the effect multiplying or dividing by measures with different units?
Did you see evidence that students understood that there were 3600 sec. in an hour? Did you see evidence that students knew they needed to either change meters to kilometers or kilometers to meters in part 2? How many of them showed evidence of knowing that a kilometer is larger than a meter?

Look at student work in part 1.

- How many of the students could solve the problem correctly? How many of these forgot to round their answers?
- How many made errors in the conversion, usually only multiplying or dividing by only 1 group of 60?
- How many of the students knew that this was a division situation? How many of the students tried to multiply instead?

Look at student work in part 2.

- Did you see any evidence of thinking about the speed of light?
- How many students recognized that the units did not match and attempted to make adjustments before dividing?
- How many students chose the wrong operation (subtraction, multiplication, addition)?

How does your textbook deal with the big mathematical idea of rate? Is it an extended topic to develop understanding or an afterthought in word problems scattered throughout the text? Is it taught as a generalization that can be applied to many situations or as a series of different formulas for every different type of rate?

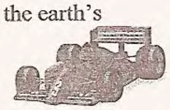
What would you like eighth graders to understand conceptually about rate?
How can you help students develop and use dimensional analysis on a regular basis to make sense of their calculations?

Looking at Student Work on Taking Off

Student A Is able to use labels to make sense of the conversion of units in part 1 and then compare the two rates. In part 2 the student converts the kilometers to meters and shows the time = distance / rate, which is probably where the 10 sec comes from. While the student doesn't say that you will see it right away, the comparison subtraction shows some thinking about the magnitude of difference between the two rates.

Student A

1. A space rocket needs to move at 11.2 kilometers per second to escape from the earth's gravitational pull. This is called escape velocity.



An Indy car races at up to 370 kilometers per hour.

How many times faster would an Indy car have to move to reach escape velocity?
Give your answer correct to the nearest whole number. 109 ✓

Show your work

$\frac{11.2\text{km}}{1\text{ sec}} \times \frac{60\text{ sec}}{1\text{ min}} \times \frac{60\text{ min}}{1\text{ hr.}} = 40320\text{ km/hr}$

$\left(\frac{370\text{ km/hr}}{370}\right) \times = \frac{40{,}320}{370} = 108.972973$

2. Sound travels at about 340 meters per second.
Light travels at almost 300,000,000 meters per second.

If you **watched** a space rocket take off from a distance of 3.5 kilometers how much later would it be before you **heard** it take off?
Give your answer correct to the nearest second. 10 ✓ seconds

Explain how you figured it out and show your calculations.

3.km = 3,500 meters

t = see it = subtract

t = hear it

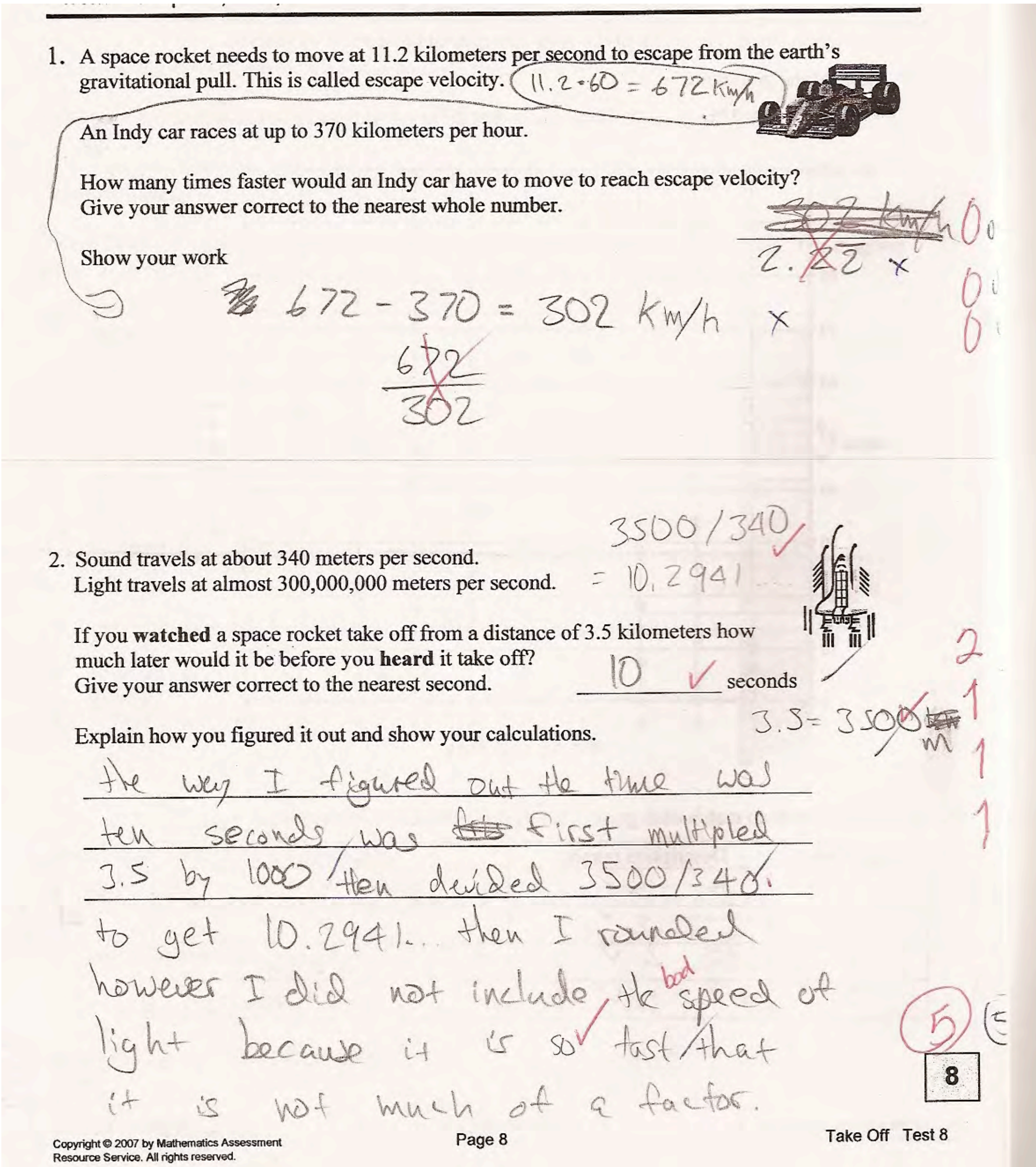
$t = d/r$

300,000,000
− 340
299999660

8

Student B makes the common error of only using one 60 when converting between per second and per hour measures. The student seems to realize that comparisons by can be made by subtraction and by division, and therefore does both. Notice that in part 2 the student is able to explain clearly where the 10 seconds come from and that the speed of light is too fast to make a difference.

Student B



1. A space rocket needs to move at 11.2 kilometers per second to escape from the earth's gravitational pull. This is called escape velocity. (11.2 • 60 = 672 Km/h)

An Indy car races at up to 370 kilometers per hour.

How many times faster would an Indy car have to move to reach escape velocity?
Give your answer correct to the nearest whole number.

~~302 km/h~~ 2.22 ✗

Show your work

672 − 370 = 302 Km/h ✗

672 / 302

2. Sound travels at about 340 meters per second.
Light travels at almost 300,000,000 meters per second.

3500/340 = 10.2941

If you **watched** a space rocket take off from a distance of 3.5 kilometers how much later would it be before you **heard** it take off?
Give your answer correct to the nearest second. 10 ✓ seconds

3.5 = 3500 m

Explain how you figured it out and show your calculations.

the way I figured out the time was ten seconds was first multipled 3.5 by 1000 then divided 3500/340 to get 10.2941... then I rounded however I did not include the speed of light because it is so fast that it is not much of a factor.

8

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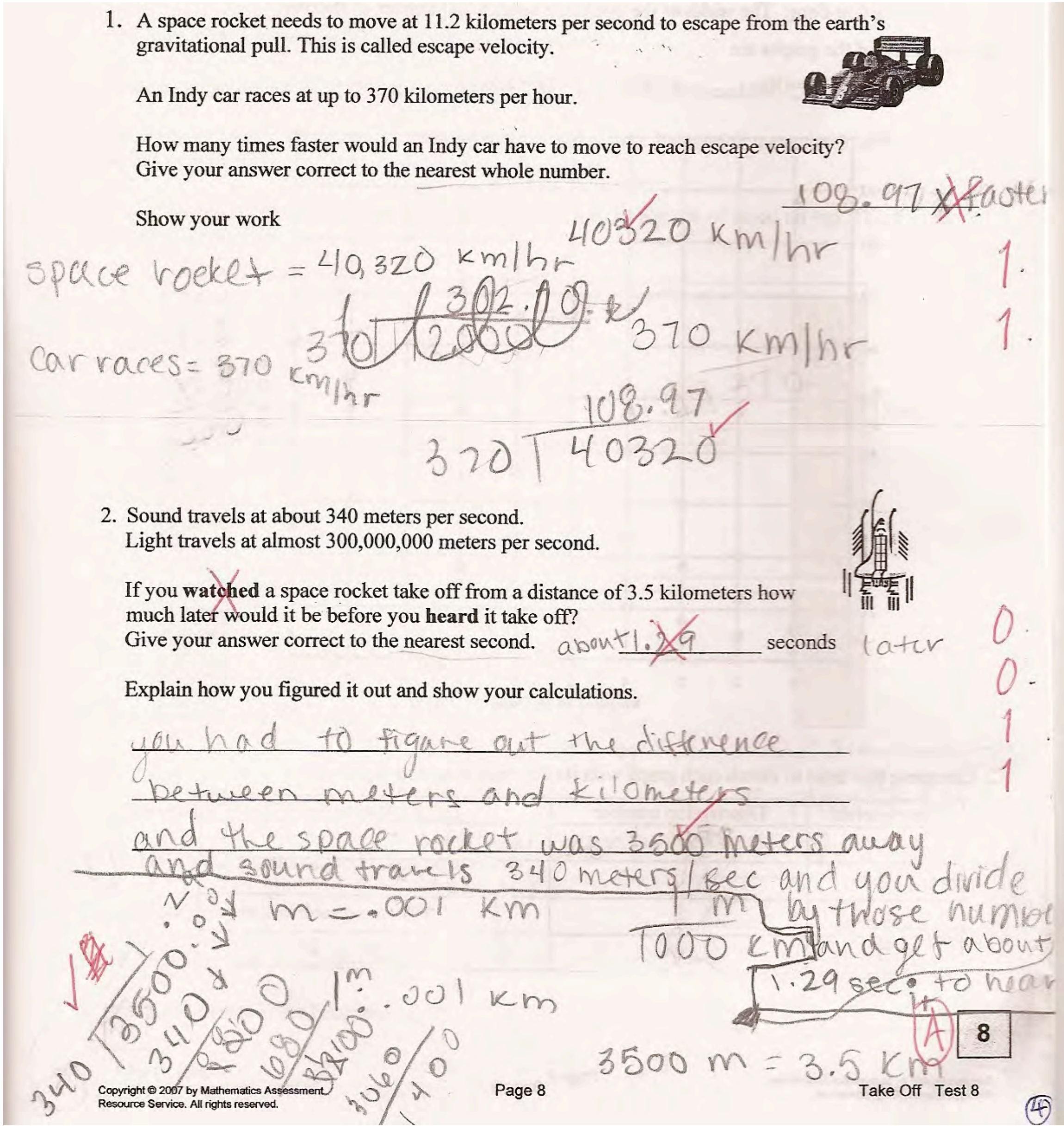
Student C is able to do the conversions. He forgets to round in part 1 and makes a decimal error in the division in part 2. The student does not consider the difference in speeds of the light and sound.

Student C



1. A space rocket needs to move at 11.2 kilometers per second to escape from the earth's gravitational pull. This is called escape velocity.

An Indy car races at up to 370 kilometers per hour.

How many times faster would an Indy car have to move to reach escape velocity?
Give your answer correct to the nearest whole number.

108.97 x faster

Show your work

40320 km/hr

space rocket = 40,320 km/hr

370 km/hr

car races = 370 km/hr

108.97

370 | 40320

2. Sound travels at about 340 meters per second.
Light travels at almost 300,000,000 meters per second.

If you **watched** a space rocket take off from a distance of 3.5 kilometers how much later would it be before you **heard** it take off?
Give your answer correct to the nearest second. about 1.29 seconds later

Explain how you figured it out and show your calculations.

you had to figure out the difference between meters and kilometers and the space rocket was 3500 meters away and sound travels 340 meters/sec and you divide by those numbers and get about 1.29 sec to hear it

1 m = .001 km

1 m / 1000 km

3500 m = 3.5 km

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Take Off Test 8

Student D divides by two groups of 60; first getting 6.16 and then getting the 0.11 (*which should have been 1 pt.*). This now converts the two rates to kilometers per second so that they can be compared. The student chooses comparison subtraction, how much larger, instead of comparison by division, how many times larger. The student doesn't round in part 2 and does not consider the speed of light.

Student D

1. A space rocket needs to move at 11.2 kilometers per second to escape from the earth's gravitational pull. This is called escape velocity.

An Indy car races at up to 370 kilometers per hour.

How many times faster would an Indy car have to move to reach escape velocity? Give your answer correct to the nearest whole number.

10.09 K/S

Show your work

$60 \overline{)370}$ = 6.16   0.11 $11.20 - 0.11 = 10.09$

0
0
0

2. Sound travels at about 340 meters per second.
Light travels at almost 300,000,000 meters per second.

If you **watched** a space rocket take off from a distance of 3.5 kilometers how much later would it be before you **heard** it take off?
Give your answer correct to the nearest second. 10.30 seconds

Explain how you figured it out and show your calculations.

Since it takes 340 meters a second to travel you have to change 3.5Km to meters then you divide it by 340 & then you get about 10.29 but rounding with the excess # you would get 10.30.

3,500 m $340 \overline{)3500}$

0
0
1
1

(2)

Student E actually shows a correct strategy for finding the solution in part 1, which is not recognized by the scorer. The student converts the kilometers per hour into kilometers per second by diving by 60 twice. Now the rates can be compared. Multiplying by 109 is the same as dividing the 11.2 by the 0.1027. The students thinking falls apart in part 2 where the student divides the kilometers by 1000 instead of multiplying by 1000 to find the distance in meters.

Student E

1. A space rocket needs to move at 11.2 kilometers per second to escape from the earth's gravitational pull. This is called escape velocity.

An Indy car races at up to 370 kilometers per hour.

How many times faster would an Indy car have to move to reach escape velocity? Give your answer correct to the nearest whole number.

109 times faster ✓ 1

Show your work

370 ÷ 60 ÷ 60 × 109 = 11.20

0

0

2. Sound travels at about 340 meters per second.
Light travels at almost 300,000,000 meters per second.

If you **watched** a space rocket take off from a distance of 3.5 kilometers how much later would it be before you **heard** it take off?
Give your answer correct to the nearest second. 35 seconds

Explain how you figured it out and show your calculations.

3.5 ÷ 1000 = 0.00035

0

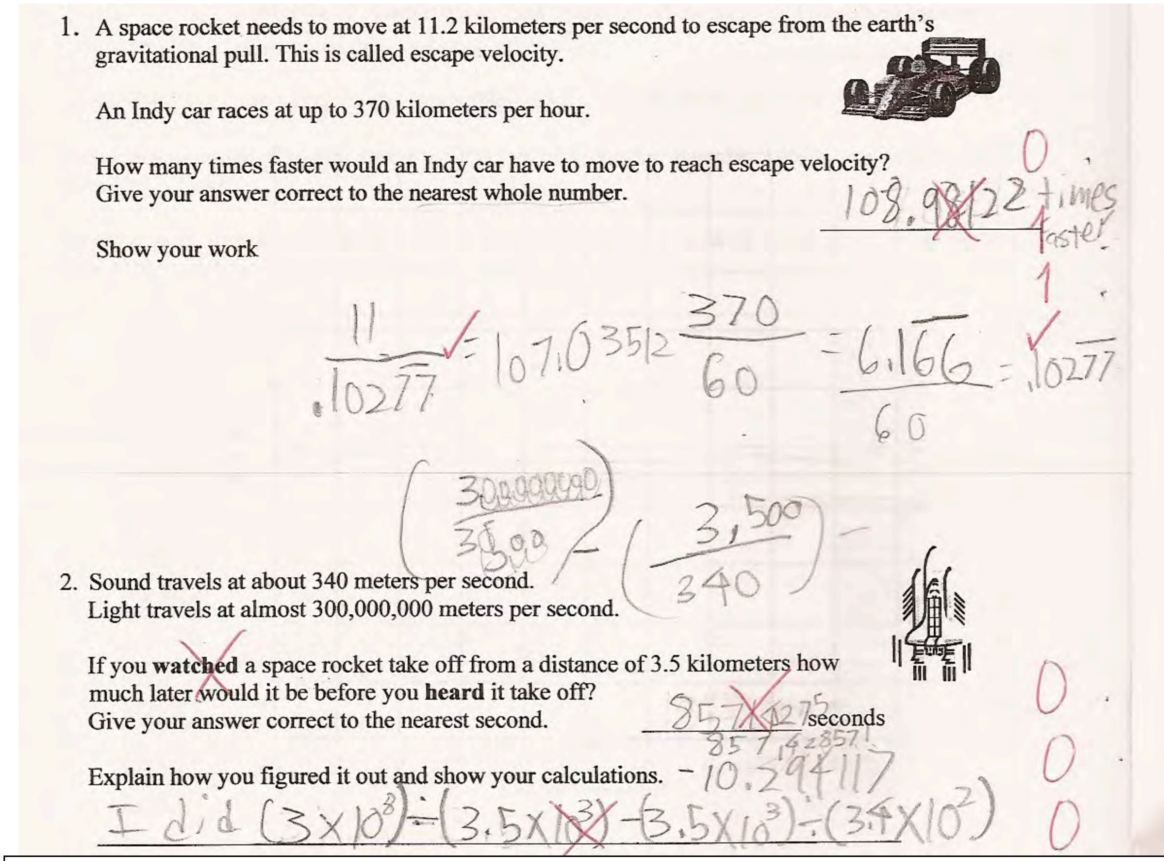
0

0

0

Student G actually shows thinking about the difference in the speed of light and the speed of sound by writing out a subtraction equation. The student also has the correct number of seconds but forgot to round. Rounding is a major issue for students. They want to have the level of accuracy of their calculators, which does not match the accuracy of the instruments used or the information needed by the problem. *How do we help students think about this matter and make sensible decisions about place value and significant digits?*

Student G



1. A space rocket needs to move at 11.2 kilometers per second to escape from the earth's gravitational pull. This is called escape velocity.

   An Indy car races at up to 370 kilometers per hour.

   How many times faster would an Indy car have to move to reach escape velocity? Give your answer correct to the nearest whole number.

   108.9822 times faster

   Show your work

2. Sound travels at about 340 meters per second.
   Light travels at almost 300,000,000 meters per second.

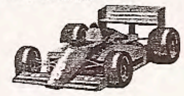
   If you **watched** a space rocket take off from a distance of 3.5 kilometers how much later would it be before you **heard** it take off?
   Give your answer correct to the nearest second. 857.1275 seconds

   Explain how you figured it out and show your calculations.

Student H attempts to compare to ratios 340:3600 and 11.2:1. In order to make a comparison between ratios one of the quantities needs to be fixed. Either the numerators need to be the same so the denominators can be compared or the denominators need to be the same some the numerators can be compared. This is a big mathematical generalization that students at this grade level should be making. In part two the student multiplies the meters by 1000 instead of dividing by 1000 to convert to kilometers. *Do you think students in your class come to you with the basic logic of making conversions? What do you do to help students develop an understanding of conversions?*

Student H

1. A space rocket needs to move at 11.2 kilometers per second to escape from the earth's gravitational pull. This is called escape velocity.



An Indy car races at up to 370 kilometers per hour.

How many times faster would an Indy car have to move to reach escape velocity?
Give your answer correct to the nearest whole number.

X 3,600 times faster

Show your work

370km= 1hr
1hr = 60m
60m= 3600 sec

370Km = 3,600 sec
11.2km = 1 sec

X
X

2. Sound travels at about 340 meters per second.
Light travels at almost 300,000,000 meters per second. 340000 3.5

If you **watched** a space rocket take off from a distance of 3.5 kilometers how much later would it be before you **heard** it take off?
Give your answer correct to the nearest second. 1000 X seconds

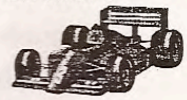
Explain how you figured it out and show your calculations.

If it travels at 340 meters per second, that would equal 340000 kilometers per sec. I divided 340 from 340000

Student I does not notice the difference in units of the two rates in part 1. The student attempts to make an absolute comparison in part 1 by subtracting, instead of comparison of how many times larger, which would use division. *How can we help students learn the meaning of operations and understand the difference between the two types of comparison?* Notice again in part 2, the student does not pick up on the difference in units between meters per second and kilometers.

Student I

1. A space rocket needs to move at 11.2 kilometers per second to escape from the earth's gravitational pull. This is called escape velocity.



An Indy car races at up to 370 kilometers per hour.

How many times faster would an Indy car have to move to reach escape velocity? Give your answer correct to the nearest whole number.

258 kilometer

0

Show your work

370 − 11.2 = 258

370 × 11.2 = 740, 3700, 37000 … 38110

2. Sound travels at about 340 meters per second.
Light travels at almost 300,000,000 meters per second.

If you **watched** a space rocket take off from a distance of 3.5 kilometers how much later would it be before you **heard** it take off?
Give your answer correct to the nearest second. 96 x seconds

Explain how you figured it out and show your calculations.

I divided x

3.5 ⟌ 3.40 = 96
315, 250, 210, 30

33.5 × 6 = 210

0

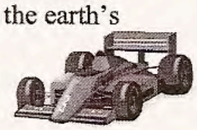
0

8

Student J attempts to make a comparison, how many times larger in part 1. The failure to pay attention to units makes the answer unsensible. So the student then just adds the two rates making a decimal error in the process. In part 2 the student again ignores the difference in units and chooses an incorrect operation for situation.

Student J

1. A space rocket needs to move at 11.2 kilometers per second to escape from the earth's gravitational pull. This is called escape velocity.



An Indy car races at up to 370 kilometers per hour.

How many times faster would an Indy car have to move to reach escape velocity?
Give your answer correct to the nearest whole number. 48.2 X

Show your work

11.2 370) 11.2

370 kilometers
+11.2

2. Sound travels at about 340 meters per second.
Light travels at almost 300,000,000 meters per second.

340
x3.5
102.G

If you **watched** a space rocket take off from a distance of 3.5 kilometers how much later would it be before you **heard** it take off?
Give your answer correct to the nearest second. 102.0 X seconds

Explain how you figured it out and show your calculations.

Well I multiplyed 340 by 3.5 and I got 102.0

8th Grade Task 5 Take Off

Student Task	Work with speed, time and distance. Make conversions between measures in the metric system. Recognize significant digits and number size in a practical situation.
Core Idea 2 Mathematical Reasoning	**Employ forms of mathematical reasoning and justification appropriately to the solution of a problem.** • Extract pertinent information from situations and determine what additional information is need. • Verify and interpret results in a problem.

Based on teacher observation, this is what eighth graders know and are able to do:

- Most students made some attempt to convert from hours to seconds.
- Many students knew that comparison could be done with subtraction or division.

Areas of difficulty for eighth graders:

- Understanding how to make a conversion or how to use labels to understand how operations effect units
- Choosing significant digits or level of accuracy when making a calculation with a calculator
- Using division to find out how many times larger something is
- Understanding that in order to compare ratios, either the numerators or denominators need to be set to an equal amount
- Choosing operations in a complex problem
- Willingness to preserve or attempt a task with large numbers

MARS Test Task 5 Frequency Distribution and Bar Graph, Grade 8

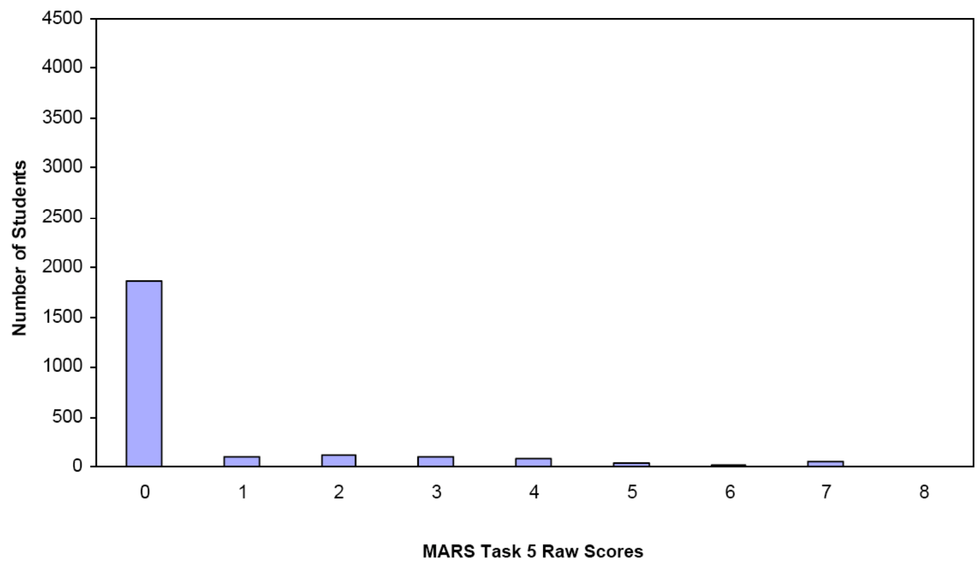
Task 5 - Take Off

Mean: 0.69 StdDev: 1.58

Table 44: Frequency Distribution of MARS Test Task 5, Grade 8

Task 5 Scores	Student Count	% at or below	% at or above
0	1865	78.6%	100.0%
1	109	83.2%	21.4%
2	111	87.9%	16.8%
3	95	91.9%	12.1%
4	87	95.5%	8.1%
5	30	96.8%	4.5%
6	22	97.7%	3.2%
7	50	99.8%	2.3%
8	4	100.0%	0.2%

Figure 53: Bar Graph of MARS Test Task 5 Raw Scores, Grade 8



The maximum score available on this task is 8 points.
The minimum score available for a level 3 response, meeting standard, is 3 points.

Some students, about 21% were able to make the conversion from kilometers per second to kilometers per hour or convert kilometers per hour to kilometers per second. About 12% were able to use this information to find out how many times faster the space rocket is than the Indy car. About 2% of the students could also reason how long it would take to hear something from 3.5 kilometers, including converting between kilometers and meters before making the comparison. Less than 1% of the students could talk about the difference in speed of light compared to sound. Almost 79% of the students scored no points on this task. 68% of the students with this score attempted the task.

Take Off

Points	Understandings	Misunderstandings
0	68% of the students with this score attempted the task.	Students did not pay attention to the difference of units in the two rates. 11% of the students divided kilometers per hour by kilometers per second (33). Many students only divided or multiplied by 1 group of 60 when attempting to make the conversion.
1	Students could make the conversion between km/sec to km/hr or vice versa.	Students did not know to take that information and use it to make a comparison. Many students did comparison subtraction instead of using division to find out how many times larger.
3	Students could convert between different rate measurements and do comparison division to find how many times larger one rate is than the other.	Students did not know how to use a rate and distance to find time. They could not convert between meters and kilometers.
6	Students could convert between different rate measurements and do comparison division to find how many times larger one rate is than the other. Students could also convert between kilometers and meters.	They could not make the final step to find the time given a rate and distance. Students also struggled with rounding in context, not making answers to the nearest whole number. Students at this score point generally did not make a comment about the difference between the speed of light and the speed of sound.
8	Students could convert between different units of rate to make comparisons and use the time= distance/ rate formula to find how long it would take to hear a rocket that took off 3.5 km away.	

Implications for Instruction

Students need more experience with converting units of measurement. Students should be able to understand how to move from hours to seconds. Students should also be comfortable with speed, time, and distance problems. Students need to know and be able to apply the formula distance = rate x time in a practical context.
(See MARS conversion tasks: 2002 7th grade – Leaky Faucets, 2003 7th grade – Yogurt)
Students at this grade level need opportunities to think about rounding in context. What are reasonable significant digits for a specific situation? Why don't the number of digits on the calculator make sense for different situations or imply an unreasonable level of accuracy? Students at this grade level should also be thinking about different types of comparison. Comparison can be done with subtraction: How much more allowance does Jane get than Jack? Comparison by multiplication: The skyscraper was ten times higher than the bookstore. Comparison using percents (to compare items with different totals): Comparing population densities for two different countries or percentage of students meeting standards in different classrooms.

Ideas for Action Research –
Investigating Comparisons by Re-engagement - Cereal

One useful strategy when student work does meet your expectations is to use student work to promote deeper thinking about the mathematical issues in the task. In planning for re-engagement it is important to think about what is the story of the task, what are the common errors and what are the mathematical ideas I want students to think about more deeply. Then look through student work to pick key pieces of student work to use to pose questions for class discussion. Often students will need to have time to rework part of the task or engage in a pair/share discussion before they are ready to discuss the issue with the whole class. This reworking of the mathematics with a new eye or new perspective is the key to this strategy.

Taking Off revealed student misunderstandings about making comparisons. A good task for examining the idea of comparison is the 2004 7th grade task, Cereal. Give your class the task to work on individually. Look at their papers. What are some of the key mathematical ideas around proportions and comparison? How strategies did students use to solve the problem? What questions could you pose to plan a class discussion to get students to refocus on the key mathematics? For example, you might pose this question for the class:

Fred says, " I think we can use common denominators to figure out which has a higher proportion of protein." What might be good common denominators? Why? How does this help Fred solve the problem?

How does this question get all students rethinking the mathematics of the task? How did this prompt stimulate class discussion? What big mathematical ideas came up?

For further questions or prompts, consider:

Kira says, "I got the same result as Fred but I used common numerators instead of common denominators." What do you think Kira means? Try it and see if you understand what she did? How does this help Kira solve the problem?

How does this prompt build on the thinking from the first task? How does this help build to a generalization on making comparisons?

Now pose a question, like:

Samantha says, "I solved the problem by dividing: 12/100 = 0.12 and 5/45 = 0.11" George says, "I disagree. I divided 45/5 =9 and 100/12 = 8.333" Who is correct? Why does that method make sense?"

The heart of the process of re-engagement is in the discussion, controversy, and convincing of the big mathematical ideas. This is where students have the opport8nitiy to clarify their own thinking, confront their misconceptions to see the errors in logic, use mathematical vocabulary for a purpose, and make generalizations and connections.

Reflecting on the Results for 8th Grade as a Whole?

Think about student work through the collection of tasks and the implications for instruction. What are some of the big misconceptions or difficulties that really hit home for you?

If you were to describe one or two big ideas to take away and use for planning for next year, what would they be?

What were some of the qualities that you saw in good work or strategies used by good students that you would like to help other students develop?

Five areas stood out for the Collaborative. These included:

1. Making and Interpreting Diagrams – Students could not interpret the diagrams in Rugs. They did not distinguish between the height and the side measure in the triangle. They were unsure of the difference between a diameter and a radius when looking at the circle diagram. Many did not make diagrams to help them think about cutting a circular rug in half. In triangles, many students drew equilateral or isosceles triangles as a generic iconic model for all triangles. These models may limit the types of thinking students do in trying to find solutions to problems. In Shelves, many students interpreted the diagram as a picture for a partially made book shelf. Students did not consider drawing a diagram for the whole book case as a thinking tool for solving the problem. Students did not consider the dimensions of all the parts of the diagram when trying to calculate height.
2. Converting between Measurements – Many students incorrectly converted between measurement notation and decimal notation in Rugs. Students had difficulty converting between time measurements or distance measurements in Take Off.
3. Significant Digits/ Level of Accuracy – In both Rugs and Take Off students did not round answers. Many students want to use all the digits that appear on their calculators implying a level of accuracy or precision not justified by the context or instruments used for original measurements.
4. Relating symbolic notation to context – Students had difficulty matching the symbolic generalizations in Number Calculations with their investigations and generalizations in the earlier part of the task. Students could match the graph to the equations in Shelves, but struggled when trying to match either the graph or the equation to a description of the context being represented.
5. Comparisons – Students struggled with the idea of making a comparison. They didn't understand that the units needed to be the same in order to compare ideas. Some students tried to make an absolute comparison, how much larger, using subtractions; rather than making a multiplicative or relative comparison, how many times larger is a than b.