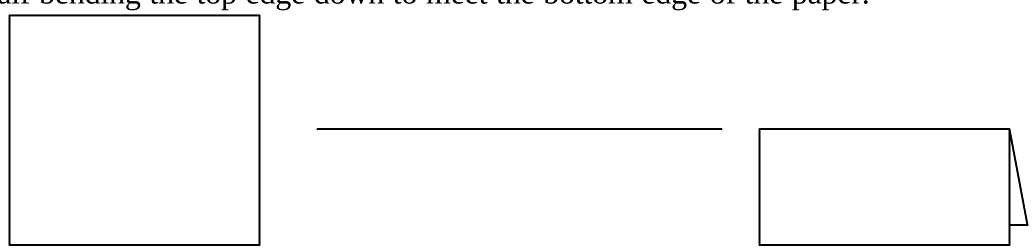
Problem of the Month

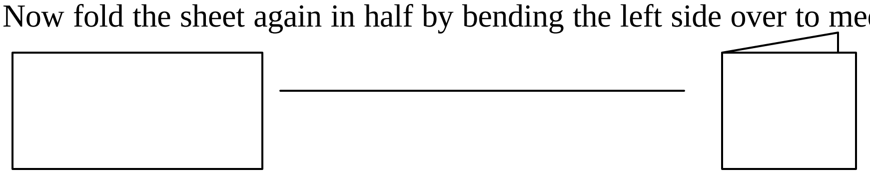
Cut It Out

Level A:

Start with a square piece of paper that is 8 inches in length. Fold the piece of paper in half bending the top edge down to meet the bottom edge of the paper.

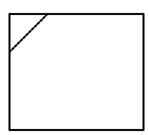


Now fold the sheet again in half by bending the left side over to meet the right side.



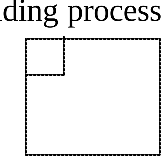
How does the shape and size of the folded paper compare to the original sheet of paper? Describe its dimensions and area.

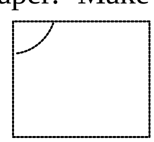
Now make the cut (dotted line) at the top left-hand corner of the folder paper.



Without unfolding the paper, draw and describe what the paper will look like when unfolded. Explain how you know.

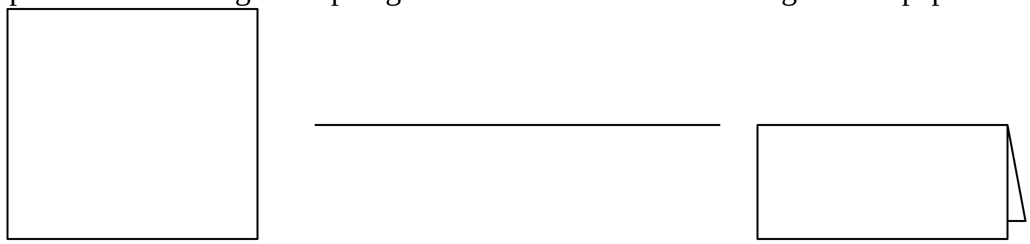
Repeat the folding process with new sheets of paper. Make the following cuts.



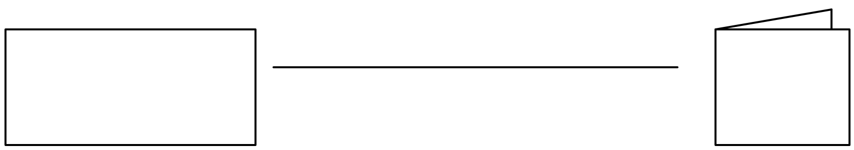


Before unfolding the paper, predict what the original paper will look like after the cut. Draw an illustration and explain your reasoning.

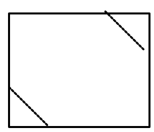
Level B:
Once again, start with a square piece of paper that is 8 inches in length. Fold the piece of paper in half bending the top edge down to meet the bottom edge of the paper.



Now fold the sheet again in half by bending the left side over to meet the right side.

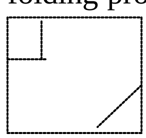


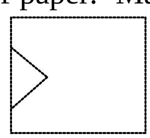
Now make the cuts (dotted lines) at the top right-hand corner and bottom left-hand corner of the folded paper.



Without unfolding the paper, draw and describe what the paper will look like when unfolded. Explain how you know.

Repeat the folding process with new sheets of paper. Make the following cuts.



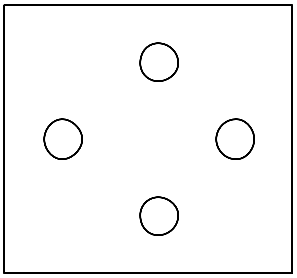


Before unfolding the paper, predict what the original paper will look like after the cut. Draw an illustration and explain your reasoning.

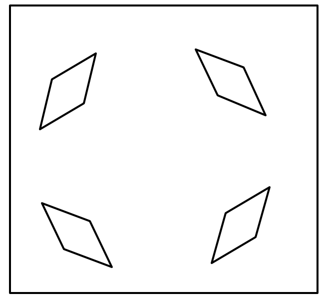
Level C:

Examine each of the following images. Each image is a blank sheet of paper that contains a set of holes. The paper was folded a **number of times** and then **one continuous cut** was made to produce the image. Determine how the paper was folded and how the cut was made to produce each of the six images below:

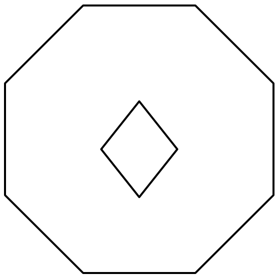
A.



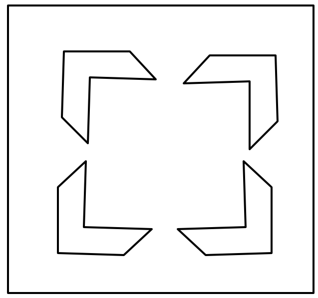
B.



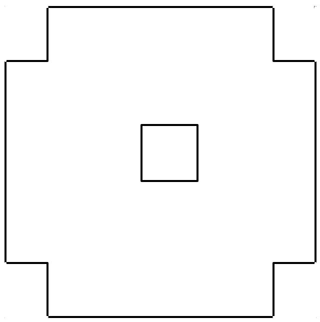
C.



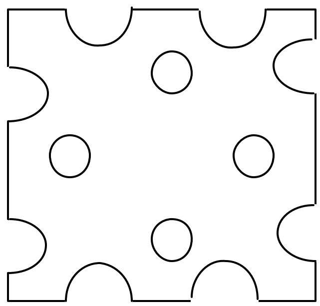
D.



E.



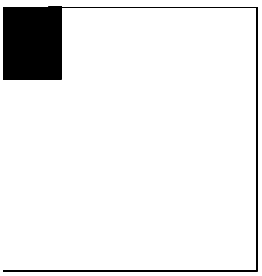
F.



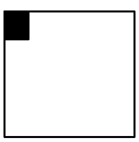
Level D:

A fractal is an image that has self-similarity. In this activity, create a fractal. Start with a square sheet of tissue paper 8 inches on each side. List the area and perimeter of the paper.

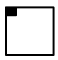
1. Fold the paper by bringing the left side over to the right. Then bring the top down to the bottom. If you were to open the paper up, it would be divided into 4 congruent squares. Now imagine cutting a square measuring 1 inch on a side out of the most folded corner (top left corner) of the folded paper. What do you think the paper will look like when it is opened up? Draw a picture to represent the paper after the square has been cut out. Cut the square out of the upper left corner of the folded paper and check to determine how your prediction compared to the actual result. Determine the new area of the paper (excluding the hole). If we define perimeter to be the boundary around the area of the remaining paper, then calculate the new perimeter (the distance around the outside of the paper, plus the distance bordering each hole). How do the area and perimeter compare to the original paper?



2. Take the folded sheet and fold it again by bringing the left side over to the right and the top down to the bottom. If you were to open the paper up, it would be divided into 16 squares. Now imagine cutting a square measuring 1/4 inch on a side out of the most folded corner (top left corner) of the folded paper. What do you think the paper will look like when it is opened up? Draw a picture to represent the paper after the square has been cut out. Cut the square out of the upper left corner of the folded paper and check to determine how your prediction compared to the actual result. Determine the new area of the paper (excluding the holes). If we define perimeter to be the boundary around the area of the remaining paper, then calculate the new perimeter (the distance around the outside of the paper, plus the distance around each hole). How do the area and perimeter compare to the original paper?



3. Take the folded sheet and fold it again by bringing the left side over to the right and the top down to the bottom. If you were to open the paper up, it would be divided into 64 squares. Now imagine cutting a square measuring 1/16 inch on a side out of the most folded corner (top left corner) of the folded paper. What do you think the paper will look like when it is opened up? Draw a picture to represent the paper after the square has been cut out. Cut the square out of the upper left corner of the folded paper and check to determine how your prediction compared to the actual result. Determine the new area of the paper (excluding the holes). If we define perimeter to be the boundary around the area of the remaining paper, then calculate the new perimeter (the distance around the outside of the paper, plus the distance around each hole). How do the area and perimeter compare to the original paper?



4. Imagine taking the folded sheet and folding it again for the fourth time using the same process. How many sub-squares would the folded paper contain? Now imagine cutting a square measuring 1/64 inch on a side out of the upper left corner of the folded paper. What do you think the paper will look like when it is opened up? Draw a picture to represent the paper after the square has been cut out. Determine the new area and perimeter of the paper. How do the area and perimeter compare to the original paper?

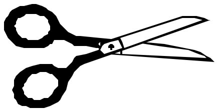
5. Examine the process you followed in the previous steps. A fractal contains an infinite number of iterations (steps). Explain what the fractal would ultimately look like. Draw a diagram of the fractal. Explain the size of the fractal at the first five iterations (steps). Determine the actual size of the fractal in terms of area and perimeter. Discuss how you found your answers and explain your mathematical reasoning.

Level E:

A fractal is a geometric figure that has self-similarity, that is created using a recursive process and that is infinite in structure. There are two categories of fractals – geometric and random. A geometric fractal is an endless generating pattern of self-similarity. The pattern continually replicates itself in smaller versions. Thus, when a small portion of a geometric fractal is magnified, it looks exactly like the original version. A random fractal also contains self-similar images of itself, only in a disorderly, non-predictable pattern. Beautiful computer-generated images are examples of these fractals such as the Mandelbrot Set.

Design a poster/object that contains a fractal. The fractal could be a self-similar collage, a series of pictures inside a picture, a self-similar geometric design, or another self-similar unique creation. It must be an original drawing or design. Your poster may contain photographs, pictures from periodicals, enlargements and reductions from copiers, and/or computer-generated designs. The fractal may be created using a random (chaos) technique or a self-similar drawing. You may produce a 3-dimensional model of a fractal. Your design must contain at least four iterations of a process that produces some self-similar shapes.

Write a report that describes the fractal and the process that you used to create the design. Be sure to describe the relationship between similar objects in your design. Identify the self-similar shapes or pictures that you used in the fractal. Demonstrate a procedure for finding the size (length, area, volume, angular distance, etc.) of the self-similar objects at any given level of the fractal.



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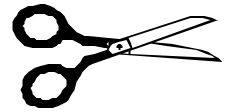
Cut It Out



Primary Version Level A

Materials: Square tissue paper and scissors for the teacher to use during discussion and square tissue paper for every student to hold and examine.

Discussion on the rug: (Teacher holds up the square tissue paper) "We are going to think hard and make a picture in our brains. Follow me in folding our pieces of paper. First fold the paper in half like a hot dog bun. Next, fold this same paper in half again but this time like a hamburger. If we unfolded the paper what will it look like?" (Students think about the folded parts that make four squares. After soliciting some guesses, students unfold the paper to examine the results.) "Why did that happen?"

In small groups: (Students have re-folded their paper. Teacher demonstrates cutting the folded corner with a diagonal cut.) "If I unfold the paper what will it look like now?" (Students make a drawing of what they visualized. The teacher unfolds the paper and asks students to explain how they pictured the altered paper. The process is repeated, with a new paper and the teacher cutting out a small square in the folded corner. The third time, teacher cuts a quarter of a circle out of the folded corner.)

At the end of the investigation: Students either discuss or dictate a response to this summary question.) "Explain how you

know what the paper will look like after we fold it and the paper is cut.”