

Evanston Township High School

Reading Comprehension of Word Problems and other mathematical text

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Mathematical Language

- Explanations of Procedures (step by step)
- Explanations of Concepts (relates ideas into bigger ideas, models situations, analogies, logically follows from other concepts, definitions)
- Problems

Procedures are easy to follow but hard to understand and transfer

- Step by step, Sequence matters, Set up matters
- Procedures are very brittle: prone to error and bugs
- Apply to narrow range of specific situations
- Algorithms are procedures that guarantee a result in finite steps
- Instructions are procedures (how to zoom a graph on the calculator is a procedure)

Concepts

- Examples: Even number, equation, constant of proportionality, slope, linear, rational number, equivalence
- Relates ideas into bigger ideas,
- Used in many situations
- Analogies
- Logically follows from other concepts, definitions
- Adds language and notation to mathematical language

The Problem with Word Problems

- Reading
- Comprehending Word Problems

Today: strategy

- How to make sense of the problem,
- solve it correctly,
- and learn mathematics from understanding what you did.

Comprehension

- Imagine a world
- Characters in the world
- Action in the world
- Mental Model

The Text: The Base

- Co-reference
- Cohesion
- Coherence

Story

- He was my best friend. Was. I still like him, but I can't trust him anymore. I also hate him.

You probably think it was his fault, or at least that I think it was his fault. But I don't think it was. Somebody else started the whole thing. We're both victims ... and I have an idea of whom. But he didn't have to treat me the way he did once it started. That's his own fault.

How many different people
referred to?

How many different people referred to?

- He = best friend = him = his
- My = I = me
 - NOTE: We = victims = He + I
- Somebody else = whom
- You
- Also refers to unknown event: it = the whole thing
- And: the way he treated me = that's

Co-reference in story text

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Cohesion in story text

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Mental model of story: What is happening?

- Characters:
 - Narrator
 - Former best friend
 - Suspected someone else
 - The reader
- Action
 - IT that is someone's fault
 - How former friend treated narrator

Why do students have to do word problems?

- a. to get answers because Homeland Security needs them, pronto
- b. I had to, why shouldn't they?
- c. so they will listen in class
- d. to learn mathematics

Understand the situation: mental model

- Once you understand the situation, it is much easier to answer questions about it.
- Part of imagining the situation: name all of the quantities in the problem and
- give them a value or represent them with a letter.

Multi-step Problems have Implicit Quantities

- Quantities can be referred to either explicitly or implicitly.
- *Explicit quantities* are mentioned “out loud” in the problem, but *implicit quantities* are not.
- Implicit quantities are found by imagining the situation clearly.
- Drawing a diagram can be helpful in finding quantities that are not mentioned.

Someone took his jacket. His Dad will be mad. But it wasn't his fault. He left on his chair. He saw his best friend, Johnny and ran up to him.

“Someone took my jacket. Let's find him and...”

“You just did”

Johnny reached into his backpack...

Referenced Identities

- Someone who took jacket = thief
- The boy whose jacket was taken = boy
- The boy's friend Johnny
Johnny =
- The boy's Dad = Dad

Who took the boy's jacket?

- Thief = Johnny
- “Co - reference” occurs when two different expressions refer to the same identity

A word problem describes a situation.

- To learn mathematics from the problem, a student must understand the situation described and then mathematize their understanding. To begin with, the student faces a reading comprehension challenge.

The reader must imagine the situation described in the problem,

- just like a reader imagines the situation when they read a story. In a story, the reader naturally focuses on the characters, their motives, the actions and reactions of characters.

Word problems are not about people

- What students learn about reading stories does not transfer without additional instruction to reading word problems. The action in a word problem is not between characters.

Word problems are about quantities

- The action in word problems is between quantities.
- a reader of a word problem has to recognize and keep track of what quantities are being referred to.... by name, phrase, clause, or silent implication. Further, readers have to track what is being said about the relationships among the quantities.

What is a quantity?

- A quantity is some particular feature of a situation that can be counted or measured. It consists of a unit or description of what is being measured or counted (inches, height in feet, Bea's weight in pounds, apples, dollars) AND a number (or letter representing an unknown or variable number)." John's height" is a quantity

A reader has to realize that what is being referred to is a quantity

- a feature of John to which a number and unit can be assigned. “John’s height is 64 inches”. This means that “john’s height equals 64 inches”, from the reading comprehension perspective: “john’s height” and “64 inches” are identical; they refer to the same quantity; they are co-referents.

Suppose we are also told

- that John's height is 10 inches more than Sonia's. If we have comprehended that 64 inches and John's height refer to the same quantity, then ordinary language comprehension will tell us that 64 inches is 10 more inches than Sonia's height

Imagine the “height” situation

- Finding Sonia’s height will be easy for anyone who comprehends the situation. Students who get this wrong have reading comprehension problems. They have failed to imagine the situation. They need to be taught how to do this with word problems.

Problem as Stories

A piece of glass with an initial temperature of 99°C is cooled at a rate of $3.5^{\circ}\text{C}/\text{min}$. At the same time, a piece of copper with an initial temperature of 0°C is heated at a rate of $2.5^{\circ}\text{C}/\text{min}$.

Assignment: Solo: Write a math question about this situation

Partner: Show question to partner and discuss

What is this Problem Situation about?

Mrs. Jackson has 27 stickers. She needs 7 more in order to give 2 stickers to each student in her class.

What are the quantities in this situation?

- List quantities without worrying about whether they are known or unknown
- Are there any quantities implied?

How do the quantities relate to each other?

- In words
- In diagram
- In equations

Your mental model of the situation

- Base from text: The quantities referred to and how they relate to each other
- Mental model: diagram, equation, word picture (tie these representations together by pointing out loud to the correspondences)

Catching Up

- Entering grade 6, on-level students have learned 6 years of mathematics. Students two years behind have learned 2 years less of mathematics.
 - What will the gap be at the end of grade 12 if each group continues at its same rate?
 - How fast would the behind group have to learn to catch up by the end of grade 8?

Catching Up References to Quantities

- Entering grade 6,
6 years of mathematics. {*Entering grade 6*}
2 years less of mathematics.
the gap the end of grade 12
its same rate?:
– How fast
the end of grade 8?

References

- “entering grade 6”, “end of grade 12” and “end of grade 8” are three points on the same numberline representing elapsed grades
- Behind math. learned in 6 grades = $(6 - 2)$ yrs.
- “6 years of mathematics” and “2 years less of mathematics” imply two ‘per 6 grades’ rates:
 - 6yrs/6grades and 4yrs/6grades
 - “each ... its same rate” refers to these two rates

Catching Up

- Entering grade 6, on-level students have learned 6 years of mathematics. Students two years behind have learned only 4 years of mathematics. What will the gap be at the end of grade 12 (ignore pre-K) under each of these conditions?:
 - Each group continues at its same rate
 - The behind group accelerates to the same rate as the on-level group
 - The behind group accelerates to a rate fast enough to catch up by the end of grade 8
- Graph and table each condition.

Catch-up II

- If the on level students average 4 pages a day for 150 days a year, how many pages a day would it take for the behind students to catch up in one year? Assume they had to cover the same pages.

Mixture Problems

- Why are they so hard?

Milkfat

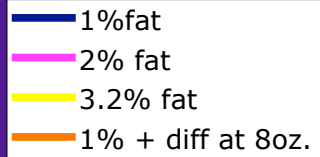
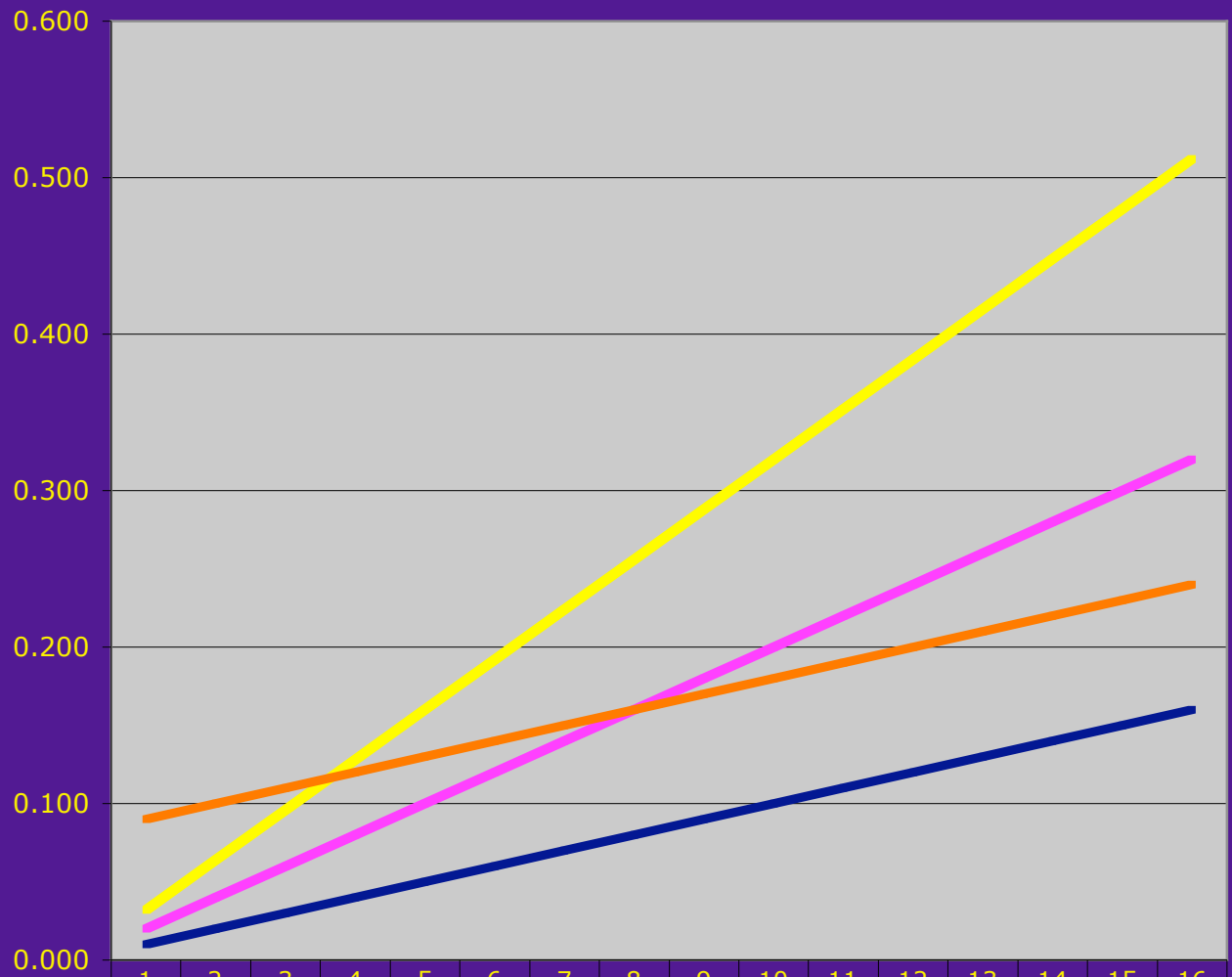
- Antoine wants 8 ounces of 2% milk. He only has 1% milk and regular milk (3.2%). How much of each should he mix to get 8 oz. of 2% milk?
 - Diagram
 - Equations
 - Table
 - How can you solve this graphically?

Running off the Fat

- Annie wants to average 2 minutes per lap on an indoor track. She wants to get her pace right: she wants to warm up at a pace of 3.2 minutes per lap and then run 1 minute laps. If she runs 8 laps, how long should she run at each pace to average 2 minutes per lap?

mixtures

How can we graph mixture situations where
two mixtures are mixed?



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1%fat	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16
2% fat	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32
3.2% fat	0.03	0.06	0.09	0.12	0.16	0.19	0.22	0.25	0.28	0.32	0.35	0.38	0.41	0.44	0.48	0.51
1% + diff at 8oz.	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23	0.24

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Density of Rice

From a book on the chemistry of cooking we learn that in cooking rice the suggested proportion of water to rice is 1.7 to 1 if measured by weight, and 1.4 to 1 if measured by volume.

- What is the density of rice?
- [note: density is mass / volume; for example: gm. / cm.³]

Problem Situation

- The kitchen where you just started working has a vertical cylindrical hot water tank that is 1.5 feet across on the inside. The depth of the water in the tank is 4 feet.

Problem Situation

- The farm where you just started working has a vertical cylindrical oil tank that is 2.5 feet across on the inside. The depth of the oil in the tank is 2 feet.

Question from Work Keys Level 7

- The farm where you just started working has a vertical cylindrical oil tank that is 2.5 feet across on the inside. The depth of the oil in the tank is 2 feet. If 1 cubic foot of space holds 7.48 gallons, about how many gallons of oil are left in the tank?

Work keys 6

- You are preparing to tile the floor of a rectangular room that is $15\frac{1}{2}$ feet by $18\frac{1}{2}$ feet in size. The tiles you plan to use are square, measure 12 inches on each side, and are sold in boxes that contain enough tiles to cover 25 square feet. How many boxes of tiles should you order?

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Catching Up

- Students with history of going slower are not going to catch up without spending more time and getting more attention
- Goal: not AP but complete prerequisites for college credit mathematics by HS graduation.